

# 11. Design and Detailing for Earthquake Effects

## 11.1. Introduction

IBC 1613.1 requires that all structures, in whole and in part, be designed to resist the effects from earthquake motions. This chapter covers the design and detailing requirements for reinforced concrete members in structures located in areas of moderate and high seismic risk. Prior to presenting the Code requirements, information is provided on seismic design criteria [including seismic design category (SDC)] and seismic force-resisting systems (SFRSs) for reinforced concrete.

According to IBC 1613.1, the effects of earthquake motion on structures and their components are to be determined in accordance with ASCE/SEI 7-10, excluding Chap. 14 (Material Specific Seismic Design and Detailing Requirements) and Appendix 11A (Quality Assurance Provisions). These chapters from ASCE/SEI 7 have been excluded because the IBC includes quality assurance provisions in Chap. 17 and structural material provisions in Chaps. 19 through 23.

In general, a structure must have complete vertical and seismic force-resisting systems that are capable of providing adequate strength, stiffness, and energy-dissipation capacity when subjected to the design ground motion, which is assumed to occur along any horizontal direction of a structure. Without sufficient strength and stiffness, large displacements can occur, which could lead to local or overall instability, or both. A continuous load path with adequate strength and stiffness must be provided to transfer all forces from the point of application to the final point of resistance (usually the ground). If all of the components of a building are not adequately tied together, including nonstructural components, the individual members will move independently and can pull apart from one another, which could lead to partial or total collapse.

Seismic forces are generated by the dead weight of a structure. These inertial forces are created by the motion of the ground that supports a structure's foundation. The response of a structure resulting from such ground motion is influenced by the nature of the motion and the properties of the structure and its foundation. The horizontal components of ground motion typically have a more significant effect on a building than the vertical components. Thus, earthquake-resistant design concentrates more on the effects of the horizontal forces although vertical load effects must also be considered as part of the earthquake load effect  $E$ .

[Section 3.2.10](#) of this book provides a description of how earthquake effects are resisted by reinforced concrete structures and gives an overview of how earthquake motion is translated into loads that get applied to a structure. In general, earthquake loads are a function of the design accelerations at the site, the type of seismic force-resisting system utilized in the structure, the risk category of the structure, and the stiffness characteristics of the structure. Brief discussions of these parameters are given in the following sections. [Reference 1](#) provides comprehensive explanations for all of these parameters along with detailed descriptions and examples on how to calculate seismic forces using the methods prescribed in ASCE/SEI 7-10.

The general philosophy of earthquake-resistant design is to allow some structural and nonstructural damage while minimizing hazard to life. This is to be achieved by utilizing the inelastic deformability of a structure and allowing dissipation of the earthquake energy. It is expected that structures will undergo relatively large deformations when subjected to a design-level earthquake and that yielding would occur in some members of the structure. Possessing sufficient inelastic deformability through ductile detailing of critical members enables the structure to survive without collapse when subjected to several cycles of deformation into the inelastic range.

Detailing requirements to achieve the necessary levels of inelastic deformability and energy dissipation are directly related to the SDC that is assigned to a structure. An explanation of SDC and the methods prescribed in the IBC and ASCE/SEI 7 on how to determine it are given in [Section 11.2.3](#) of this book.

IBC 1901.2 requires that reinforced concrete members be designed and constructed in accordance with the provisions in ACI 318-14.<sup>2</sup> Chap. 18 in ACI 318-14 contains the seismic requirements that need to be satisfied for various types of members

based on the SDC. These requirements, which are applicable for structures assigned to SDC B, C, D, E, and F, are covered in this chapter.

## 11.2. Seismic Design Criteria

### 11.2.1. Seismic Ground Motion Values

The seismic forces that are applied to a structure are directly related to the magnitudes of the ground motion accelerations expected at the site. In general, the energy that is released during an earthquake radiates out in the form of random vibrations in all directions from the source, which is usually an area where slippage has occurred along fault lines. Ground shaking is felt on the surface due to these vibrations. As the ground displaces, a building will move and undergo a series of oscillations. Depending on the size of the earthquake, ground shaking can last from a few seconds to several minutes.

IBC Figs. 1613.3.1(1) and 1613.3.1(2) and ASCE/SEI Figs. 22-1 and 22-2 contain contour maps of the conterminous United States giving  $S_S$  and  $S_1$ , which are the mapped risk-targeted maximum considered earthquake ( $MCE_R$ ) spectral response accelerations at periods of 0.2 second and 1.0 second, respectively, for a Site Class B soil profile and 5% damping. Site classes, which are based on the soil properties at the site of the structure, are defined in ASCE/SEI Table 20.3-1 and are discussed later in this section. Similar contour maps are also provided for Alaska, Hawaii, Puerto Rico, and the United States Virgin Islands.

The mapped spectral accelerations are based on procedures developed by the U.S. Geological Survey (USGS) and are the smaller of the probabilistic risk-based and deterministic ground motion values obtained at a particular site. In lieu of the maps,  $MCE_R$  spectral response accelerations can be obtained by using the U.S. Seismic Design Maps web application, which can be accessed on the USGS website (<http://earthquake.usgs.gov/designmaps/us/application.php>). Accelerations are output for a specific latitude and longitude, which is input by the user.

The short-period acceleration  $S_S$  has been determined at a period of 0.2 second because it was concluded that 0.2 second was reasonably representative of the shortest effective period of buildings and structures that are designed using these requirements. The 1-second acceleration  $S_1$  is used because spectral response accelerations at periods other than 1 second typically can be derived from the acceleration at 1 second. These two acceleration parameters are sufficient to define an entire response spectrum for the range of periods that are applicable for most buildings and structures.

Once  $S_S$  and  $S_1$  have been determined, they must be modified for the particular soil profile present at a site. Six site classes are defined in ASCE/SEI Table 20.3-1 (see Table 11.1). A site is to be classified as one of these six classes based on the soil properties measured over the top 100 ft (30 m) of the site:

- Average shear wave velocity at small shear strains  $\bar{v}_s$
- Average field standard penetration  $\bar{N}$  or average standard penetration resistance for cohesionless soil layers  $\bar{N}_{ch}$
- Average undrained shear strength  $\bar{s}_u$

Table 11.1 Site Classification

Site Class	$\bar{V}_s$ (ft/s)	$\bar{N}$ or $\bar{N}_{ch}$	$\bar{S}_u$ (psf)
A—Hard rock	>5,000	NA	NA
B—Rock	2,500 to 5,000	NA	NA
C—Very dense soil and soft rock	1,200 to 2,500	>50	>2,000
D—Stiff soil	600 to 1,200	15 to 50	1,000 to 2,000
	<600	<15	<1,000
E—Soft clay soil	Any profile with more than 10 ft (3 m) of soil with the following characteristics: <ul style="list-style-type: none"> <li>• Plasticity index <math>PI &gt; 20</math></li> <li>• Moisture content <math>w \geq 40\%</math></li> <li>• Undrained shear strength <math>\bar{S}_u &lt; 500</math> psf</li> </ul>		
F—Soils requiring site response analysis in accordance with ASCE/SEI 21.1	See ASCE/SEI 20.3.1		
In SI: 1 ft/s = 0.3048 m/s; 1 psf = 0.0479 kN/m <sup>2</sup> .			

Site Class A is hard rock, which is typically found in the eastern United States. Site Class B or C is a softer rock, including various volcanic deposits, sandstones, shales, and granites; these types are commonly found in western parts of the country. Very dense sands and gravels as well as very stiff clay deposits usually qualify as Site Class C. The most common site class throughout the United States is Site Class D, which consists of sites with relatively stiff soils, including mixtures of silty clays, silts, and sands. Sites that are located along rivers or other waterways that are underlain by deep soft clay deposits are classified as Site Class E. Site Class F indicates soil so poor that a site response analysis is required to determine site coefficients; these are sites where soils are subject to liquefaction or other ground instabilities. Site-specific ground motion procedures for seismic design are given in Chap. 21 of ASCE/SEI 7.

Once the mapped  $MCE_R$  spectral accelerations and site class have been established, the risk-targeted  $MCE_R$  spectral response acceleration for short periods  $S_{MS}$  and at a 1-second period  $S_{M1}$  adjusted for site class effects are determined by IBC Eqs. (16-37) and (16-38), respectively, or by ASCE/SEI Eqs. (11.4-1) and (11.4-2), respectively:

$$S_{MS} = F_a S_S$$

(11.1)

$$S_{M1} = F_v S_1$$

(11.2)

In these equations,  $F_a$  is the short-period site coefficient determined from IBC Table 1613.3.3(1) or ASCE/SEI Table 11.4-1 and  $F_v$  is the long-period site coefficient determined from IBC Table 1613.3.3(2) or ASCE/SEI Table 11.4-2 (see Tables 11.2 and 11.3). As expected the values of  $F_a$  and  $F_v$  are equal to 1.0 for Site Class B irrespective of the seismicity at the site. For site classes other than Site Class B, an adjustment to the mapped spectral response accelerations is necessary. Straight-line

interpolation for intermediate mapped accelerations in the tables is permitted. For Site Class F soils, values of  $F_a$  and  $F_v$  are to be determined using the methods prescribed in ASCE/SEI 11.4.7.

**Table 11.2** Values of Site Coefficient  $F_a$

Site Class	$S_S \leq 0.25$	$S_S = 0.50$	$S_S = 0.75$	$S_S = 1.00$	$S_S \geq 1.25$
A	0.8	0.8	0.8	0.8	0.8
B	1.0	1.0	1.0	1.0	1.0
C	1.2	1.2	1.1	1.0	1.0
D	1.6	1.4	1.2	1.1	1.0
E	2.5	1.7	1.2	0.9	0.9

**Table 11.3** Values of Site Coefficient  $F_v$

Site Class	$S_1 \leq 0.10$	$S_1 = 0.20$	$S_1 = 0.30$	$S_1 = 0.40$	$S_1 \geq 0.50$
A	0.8	0.8	0.8	0.8	0.8
B	1.0	1.0	1.0	1.0	1.0
C	1.7	1.6	1.5	1.4	1.3
D	2.4	2.0	1.8	1.6	1.5
E	3.5	3.2	2.8	2.4	2.4

Typically, ground motion is amplified in softer soils (Site Classes C through E) and attenuated in stiffer soils (Site Class A). This can be observed in the tables where the magnitudes of  $F_a$  and  $F_v$  increase going from Site Class A to E for a given mapped ground motion acceleration. The only exception to this occurs for short periods where  $S_S \geq 1.00$  and the Site Class changes from D to E. Very soft soils are not capable of amplifying the short-period components of subsurface rock motion; in fact, deamplification occurs in such cases.

Design spectral response accelerations at short periods  $S_{DS}$  and at a 1-second period  $S_{D1}$  are determined by IBC Eqs. (16-39) and (16-40), respectively, or by ASCE/SEI Eqs. (11.4-3) and (11.4-4), respectively:

$$S_{DS} = \frac{2}{3} S_{MS}$$

(11.3)

$$S_{D1} = \frac{2}{3} S_{M1}$$

(11.4)

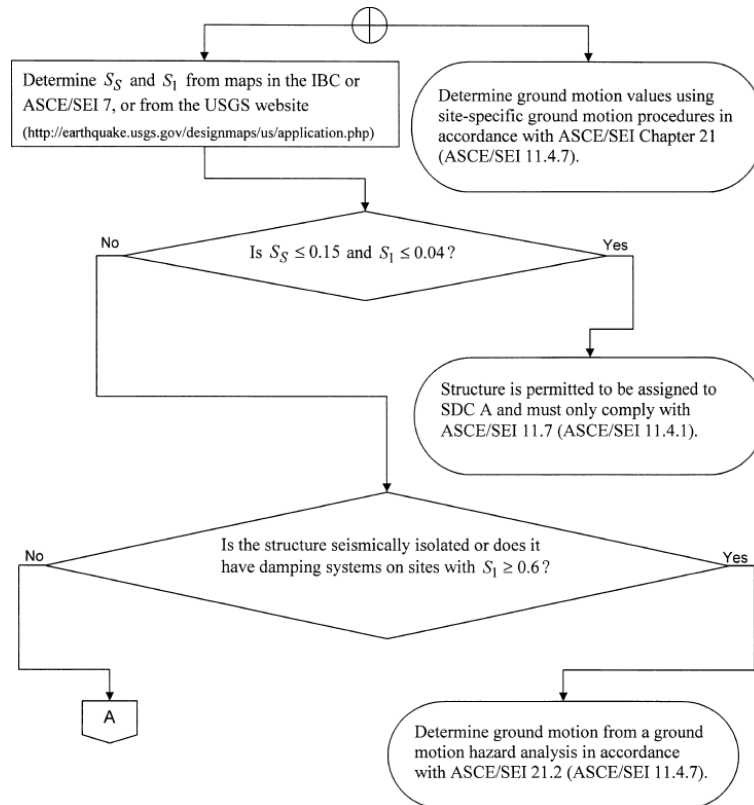
These accelerations are used in determining, among other things, the SDC and the magnitude of the effects on a structure created by a seismic event. In previous editions of ASCE/SEI 7, the lower bound margin against collapse that was inherent in buildings designed by the seismic provisions in that standard was judged, based on experience, to correspond to a factor of

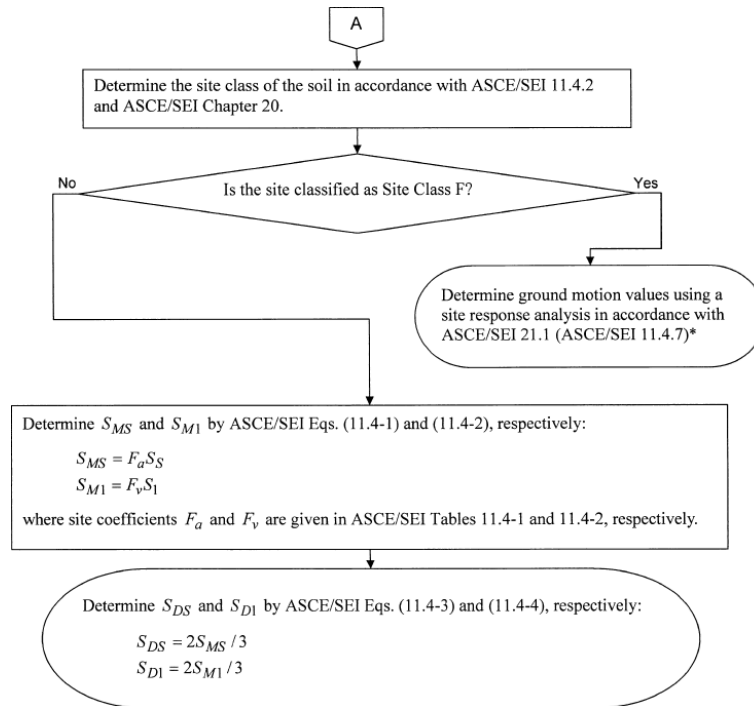


about 1.5 in ground motions, which corresponds to design spectral response accelerations equal to approximately  $1/1.5 = 2/3$  of the soil-modified design accelerations. The uncertainty in this margin is accounted for in ASCE/SEI 7 in the collapse fragility defined in ASCE/SEI 21.2.1.2. Regardless of this, the design earthquake ground motion in ASCE/SEI 7-10 is based on two-thirds of the  $MCE_R$  ground motion for consistency with previous editions.

Figure 11.1 provides a step-by-step procedure on how to determine the design spectral accelerations for a site in accordance with ASCE/SEI 11.4.

Figure 11.1 Seismic ground motion values.





\* A site response analysis in accordance with ASCE/SEI 21.1 is required for structures on Site Class F sites unless the exception in ASCE/SEI 20.3.1(1) is satisfied for structures with periods  $T \leq 0.5$  sec.

## 11.2.2. Importance Factor and Risk Category

Risk categories are defined in IBC Table 1604.5 and ASCE/SEI Table 1.5-1. These categories are used to relate the criteria for maximum environmental loads or distortions that are specified in the code or referenced standards to the consequence that would occur to the structure and its occupants if such loads were exceeded. Prior to the 2012 edition of the IBC, "Occupancy Category" was used. The term "occupancy" relates primarily to issues associated with fire and life safety protection as opposed to the risks associated with structural failure. As such, "Risk Category" was adopted to more clearly identify the nature of the categorization.

Risk Category I buildings and structures are those that are usually unoccupied and, as such, result in negligible risk to the public should they fail. Included are agricultural facilities (such as barns), certain temporary facilities, and minor storage facilities.

The vast majority of buildings and structures, including most residential, commercial, and industrial facilities, fall under Risk Category II. According to IBC Table 1604.5, any building or structure that is not listed in Risk Category I, III, or IV is assigned to Risk Category II.

Included in Risk Category III are buildings and structures that house large numbers of persons, including places of public assembly, educational facilities, and institutional facilities. Also included are structures associated with utilities that are required to protect the health and safety of a community, such as power-generating stations and water and sewage treatment facilities. Buildings and other structures that contain certain amounts of toxic or explosive materials also fall within this risk category.

Risk Category IV includes buildings and structures that are essential for a community to cope with emergency situations. Hospitals, fire stations, police stations, rescue facilities, and designated emergency shelters are some of the types of structures included in this risk category. Power-generating stations and other public utility facilities required as emergency backup facilities and buildings or structures that house certain quantities of highly toxic materials also fall in this risk category.

Importance factors are directly related to the risk category of a structure. The magnitude of an importance factor is different for different risk categories and is based on the statistical characteristics of the environmental loads and the manner in which a building or structure responds to these loads. In general, larger importance factors are assigned in situations where the consequence of failure may be severe.

A seismic importance factor  $I_e$  is assigned to a building or structure in accordance with ASCE/SEI Table 1.5-2 based on its risk category. Larger values of  $I_e$  are assigned to more important risk categories, such as assembly and essential facilities, to increase the likelihood that such structures would suffer less damage and continue to function during and following a design earthquake. The risk category of a structure is also used in determining the SDC (see Section 11.2.3 of this publication).

### 11.2.3. Seismic Design Category

All buildings and structures must be assigned to a SDC in accordance with IBC 1613.3.5 or ASCE/SEI 11.6. In general, the SDC is a function of the risk category and the design spectral accelerations at the site.

Structures are categorized according to the seismic risk they could pose. Six SDCs are defined ranging from A (minimal seismic risk) to F (highest seismic risk). As the SDC of a structure increases, so do the strength and detailing requirements.

The SDC of a structure is assigned as follows where  $S_1$  is greater than or equal to 0.75:

- Structures classified as Risk Category I, II, or III are assigned to SDC E
- Structures classified as Risk Category IV are assigned to SDC F

Where  $S_1$  is less than 0.75, the SDC is determined twice: first as a function of  $S_{DS}$  by IBC Table 1613.3.5(1) or ASCE/SEI Table 11.6-1 and second as a function of  $S_{D1}$  by IBC Table 1613.3.5(2) or ASCE/SEI Table 11.6-2 (see Tables 11.4 and 11.5). The more severe SDC of the two governs.

**Table 11.4** Seismic Design Category Based on  $S_{DS}$

Value of $S_D$	Risk Category	
	I, II, or III	IV
$S_{DS} < 0.167$	A	A
$0.167 \leq S_{DS} < 0.33$	B	C
$0.33 \leq S_{DS} < 0.50$	C	D
$0.50 \leq S_{DS}$	D	D

**Table 11.5** Seismic Design Category Based on  $S_{D1}$

Value of $S_D$	Risk Category	
	I, II, or III	IV
$S_{D1} < 0.067$	A	A
$0.067 \leq S_{D1} < 0.133$	B	C
$0.133 \leq S_{D1} < 0.20$	C	D
$0.20 \leq S_{DS}$	D	D

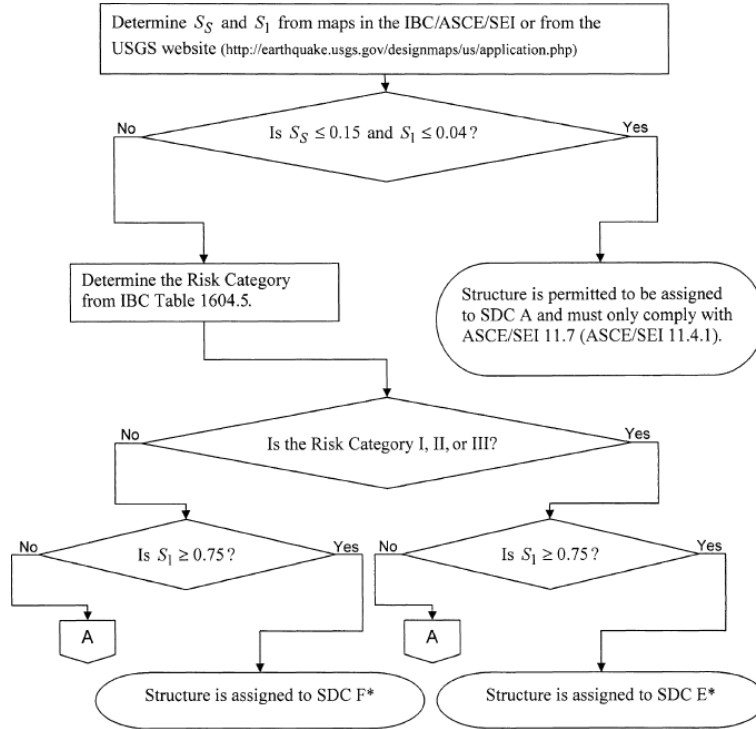
The SDC may be determined by IBC Table 1613.3.5(1) or ASCE/SEI Table 11.6-1 based solely on  $S_{DS}$  in cases where  $S_1$  is less than 0.75 provided that all of the conditions listed under IBC 1613.3.5.1 or ASCE/SEI 11.6 are satisfied. These conditions are primarily applicable to stiff, low-rise buildings. This exception should always be considered because it makes it possible for a building to be assigned to a lower SDC. As noted above, a lower SDC generally means less stringent design and detailing requirements, which translates into cost savings.

The SDC is a trigger mechanism for many seismic requirements, including the following:

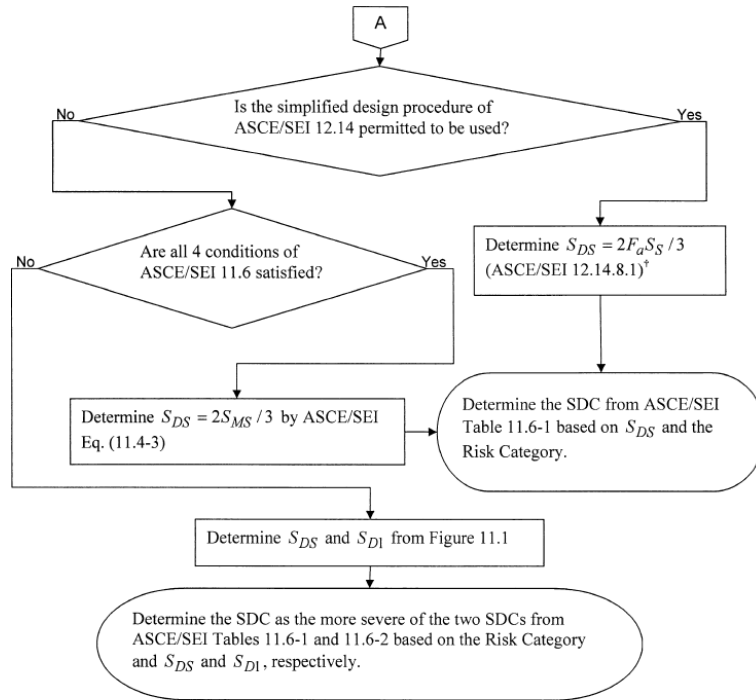
- Permissible SFRS
- Limitations on building height
- Consideration of structural irregularities
- The need for additional special inspections, structural testing, and structural observation for seismic resistance

Figure 11.2 can be used to determine the SDC of a building or structure.

*Figure 11.2* Seismic design category.



\*A structure assigned to SDC E or F shall not be located where there is a known potential for an active fault to cause rupture of the ground surface at the structure (ASCE/SEI 11.8).



\*Short-period site coefficient  $F_a$  is permitted to be taken as 1.0 for rock sites, 1.4 for soil sites, or may be determined in accordance with ASCE/SEI 11.4.3. Rock sites have no more than 10 ft (3 m) of soil between the rock surface and the bottom of spread footing or mat foundation. Mapped spectral response acceleration  $S_S$  is determined in accordance with ASCE/SEI 11.4.1 and need not be taken larger than 1.5 (ASCE/SEI 12.14.8.1).

**Example 11.1** A proposed 10-story office building is to be located at the following site in Los Angeles: latitude =  $34.04^\circ$ , longitude =  $-118.26^\circ$ . The geotechnical report for the site has classified the soil as Site Class C. Determine the SDC for the building assuming the simplified design procedure of ASCE/SEI 12.14 will not be used.

**Solution** The flowchart in Fig. 11.2 will be utilized to determine the SDC. For this building, it is unlikely that the first of the four conditions of ASCE/SEI 11.6 will be met, so the SDC cannot be determined based on the short-period design acceleration alone.

**Step 1. Determine  $S_S$  and  $S_1$ .** In lieu of the maps in the IBC or in ASCE/SEI 7, the mapped accelerations are determined by inputting the latitude and longitude of the site into the USGS web application. The output obtained from the website is as follows:  $S_S = 2.40$  and  $S_1 = 0.85$ .

Because  $S_S > 0.15$  and  $S_1 > 0.04$ , the building cannot be assigned to SDC A.

**Step 2. Determine the risk category.** The risk category is determined from IBC Table 1604.5. For a business occupancy where fewer than 300 people congregate in one area, the risk category is II.

**Step 3. Determine if the building is assigned to SDC E or F.** Because  $S_1 > 0.75$  with a risk category of II, the building is assigned to SDC E.

Therefore, the SDC is E for this building.

**Comments** According to ASCE/SEI 11.8, a structure assigned to SDC E or F shall not be located where there is a known potential for an active fault to cause rupture of the ground surface at the structure. The location of this building with respect to such active faults must be investigated.

**Example 11.2** A proposed hospital is to be located at the following site in New York City: latitude = 40.82°, longitude = -73.92°. No geotechnical report for the site is available and the building official has allowed for the soil to be classified as Site Class D. Determine the SDC for the building assuming the simplified design procedure of ASCE/SEI 12.14 will not be used.

**Solution** The flowchart in Fig. 11.2 will be utilized to determine the SDC. Assume that at least one of the four conditions of ASCE/SEI 11.6 will not be met, so the SDC will not be determined based on the short-period design acceleration alone.

**Step 1. Determine  $S_S$  and  $S_1$ .** In lieu of the maps in the IBC or in ASCE/SEI 7, the mapped accelerations are determined by inputting the latitude and longitude of the site into the USGS web application. The output from the website is as follows:  $S_S = 0.28$  and  $S_1 = 0.07$ .

Because  $S_S > 0.15$  and  $S_1 > 0.04$ , the building cannot be assigned to SDC A.

**Step 2. Determine the risk category.** The risk category is determined from IBC Table 1604.5. For an essential occupancy, the risk category is IV.

**Step 3. Determine if the building is assigned to SDC E or F.** Because  $S_1 < 0.75$ , the building is not assigned to SDC E or F.

**Step 4. Determine  $S_{DS}$  and  $S_{D1}$ .** Figure 11.1 is used to determine the design accelerations.

1. Determine  $S_S$  and  $S_1$ . The mapped accelerations have been determined in Step 1.
2. Determine if the building can be assigned to SDC A. It has been determined in Step 1 that the building cannot be assigned to SDC A.
3. Determine if a ground motion hazard analysis is required. The building will not be seismically isolated and will not have damping systems; thus a ground motion hazard analysis is not required.
4. Determine the site class of the soil. The site class has been determined to be Site Class D.
5. Determine  $S_{MS}$  and  $S_{M1}$ . The design accelerations adjusted for site class effects are determined by ASCE/SEI Eqs. (11.4-1) and (11.4-2):

$$S_{MS} = F_a S_S$$

$$S_{M1} = F_v S_1$$

The site coefficient  $F_a$  is determined from ASCE/SEI Table 11.4-1:

For Site Class D with  $0.25 < S_S < 0.5$ ,  $F_a = 1.58$  from linear interpolation (see Table 11.2).

Similarly, the site coefficient  $F_v$  is determined from ASCE/SEI Table 11.4-2:

For Site Class D with  $S_1 < 0.1$ ,  $F_v = 2.4$  (see Table 11.3).

Thus,

$$S_{MS} = 1.58 \times 0.28 = 0.44$$

$$S_{M1} = 2.4 \times 0.07 = 0.17$$

6. Determine  $S_{DS}$  and  $S_{D1}$ . The design accelerations are determined by ASCE/SEI Eqs. (11.4-3) and (11.4-4):

$$S_{DS} = \frac{2S_{MS}}{3} = 0.29$$

$$S_{D1} = \frac{2S_{M1}}{3} = 0.11$$

**Step 5. Determine the SDC.** The SDC is determined from ASCE/SEI Tables 11.6-1 and 11.6-2:

From ASCE/SEI Table 11.6-1, with  $0.167 < S_{DS} < 0.33$  and risk category IV, the SDC is C.

From ASCE/SEI Table 11.6-2, with  $0.067 < S_{D1} < 0.133$  and risk category IV, the SDC is C.

Therefore, the SDC is C for this building.

## 11.3. Seismic-Force-Resisting Systems

### 11.3.1. Overview

Numerous types of reinforced concrete systems are available to resist the effects from earthquakes. This section describes these systems and identifies their applicability and limitations with respect to the applicable SDC.

ASCE/SEI 12.2.1 requires that SFRSs conform to those indicated in ASCE/SEI Table 12.2-1. Structural system limitations and building height limits, along with other important quantities, are given in the table based on SDC.

There are no system limitations for structures assigned to SDC A. Such structures need only comply with the requirements in ASCE/SEI 11.7.

Bearing wall systems, building frame systems, moment-resisting frame systems, dual systems, shear wall-frame interactive systems, and cantilevered column systems are the general types of SFRSs given in ASCE/SEI Table 12.2-1. Each general category has systems of reinforced concrete. Table 11.6 contains a summary of the information provided in ASCE/SEI Table 12.2-1 for these systems (the number preceding each system is the number taken directly from the table in ASCE/SEI 7). Descriptions of each system are given in the following sections (Note: Cantilevered column systems are not covered in this publication). Information on the parameters contained in this table follows the system descriptions.

**Table 11.6** Seismic Force-resisting Systems of Reinforced Concrete

Seismic Force-resisting System	Structural System Limitations Including Structural Height Limits (ft)							
	Seismic Design Category							
	R	$\Omega_o$	$C_d$	B	C	D	E	F
<b>A. Bearing Wall Systems</b>								
1. Special reinforced concrete shear walls	5	2½	5	NL	NL	160	160	100
2. Ordinary reinforced concrete shear walls	4	2½	4	NL	NL	NP	NP	NP
3. Detailed plain concrete shear walls	2	2½	2	NL	NP	NP	NP	NP
4. Ordinary plain concrete shear walls	1½	2½	1½	NL	NP	NP	NP	NP
<b>B. Building Frame Systems</b>								
4. Special reinforced concrete shear walls	6	2½	5	NL	NL	160	160	100
5. Ordinary reinforced concrete shear walls	5	2½	4½	NL	NL	NP	NP	NP
6. Detailed plain concrete shear walls	2	2½	2	NL	NP	NP	NP	NP
7. Ordinary plain concrete shear walls	1½	2½	1½	NL	NP	NP	NP	NP
<b>C. Moment-resisting Frame Systems</b>								
5. Special reinforced concrete moment frames	8	3	5½	NL	NL	NL	NL	NL
6. Intermediate reinforced concrete moment frames	5	3	4½	NL	NL	NP	NP	NP
7. Ordinary reinforced concrete moment frames	3	3	2½	NL	NP	NP	NP	NP
<b>D. Dual Systems with Special Moment Frames Capable of Resisting at Least 25% of Prescribed Seismic Forces</b>								

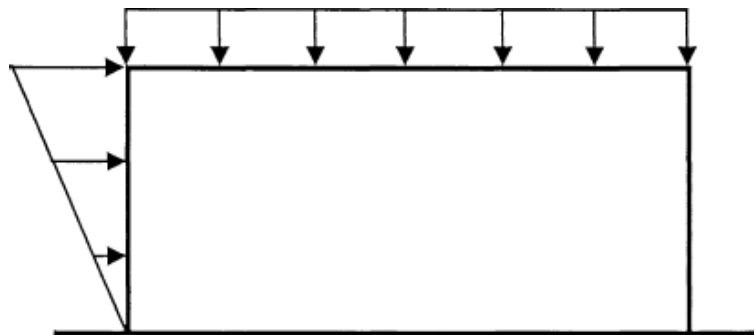


Seismic Force-resisting System	Structural System Limitations Including Structural Height Limits (ft)							
	Seismic Design Category							
	R	$\Omega$	C	B	C	D	E	F
3. Special reinforced concrete shear walls	7	2½	5½	NL	NL	NL	NL	NL
4. Ordinary reinforced concrete shear walls	6	2½	5	NL	NL	NP	NP	NP
<b>E. Dual Systems with Intermediate Moment Frames Capable of Resisting at Least 25% of Prescribed Seismic Forces</b>								
2. Special reinforced concrete shear walls	6½	2½	5	NL	NL	160	100	100
8. Ordinary reinforced concrete shear walls	5½	2½	4½	NL	NL	NP	NP	NP
<b>F. Shear wall-frame Interactive System with Ordinary Reinforced Concrete Moment Frames and Ordinary Reinforced Concrete Shear Walls</b>								
–	4½	2½	4	NL	NP	NP	NP	NP
NL = No height limitation								
NP = Not permitted								
In SI: 1 ft = 0.3 m								

### 11.3.2. Bearing Wall Systems

In a bearing wall system, bearing walls provide support for most or all of the gravity loads, and resistance to lateral loads is provided by the same bearing walls acting as shear walls (see Fig. 11.3). These systems do not have an essentially complete space frame that provides support for gravity loads.

Figure 11.3 Bearing wall system.



**SDC B.** Ordinary reinforced concrete shear walls are permitted to be used in buildings assigned to SDC B without any limitations. Such walls must satisfy the applicable requirements of ACI Chaps. 1 to 17 and 19 to 26; the provisions of Chap. 18 need not be satisfied. Detailed plain and ordinary plain concrete shear walls may also be used without limitations. Detailed plain concrete shear walls are walls complying with the requirements of ACI Chap. 14 and the additional reinforcement requirements of ASCE/SEI 14.2.2.8, and ordinary plain concrete shear walls are walls complying with the requirements of ACI Chap. 14 only.

Note that the systems noted above are the minimum system types that need to be provided for SDC B; a system listed for a

higher SDC can always be provided if desired, as long as all of the design and detailing requirements are satisfied (this is true for any type of system covered in the following sections).

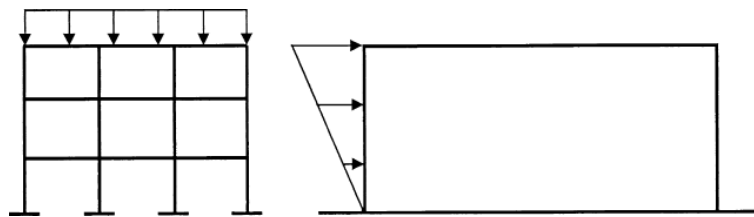
**SDC C.** Ordinary reinforced concrete shear walls are permitted to be used in buildings assigned to SDC C. It is assumed that the design and detailing requirements in ACI Chap. 11 are compatible with the anticipated level of inelastic response when the walls are subjected to the effects from moderately strong ground motion.

**SDC D, E, and F.** Special reinforced concrete shear walls are required in buildings assigned to SDC D, E, or F. The height of a building is limited to 160 ft (49 m) for SDC D and E and is limited to 100 ft (30 m) for SDC F. The design and detailing requirements of ACI 18.2.3 through 18.2.8 and 18.10 must be satisfied for walls in buildings assigned to these SDCs.

### 11.3.3. Building Frame Systems

A building frame system is a structural system with an essentially complete space frame that supports the gravity loads and shear walls that resist the lateral forces (see Fig. 11.4). It is assumed that all of the lateral forces are allocated to the shear walls; no interaction is considered between the shear walls and the frames.

*Figure 11.4 Building frame system.*



**SDC B.** Ordinary plain, detailed plain, and ordinary reinforced concrete shear walls are permitted to be used in buildings assigned to SDC B with no limitations. Building frame systems are generally not used in buildings assigned to SDC B because there is little to be gained from assigning the entire lateral resistance to the shear walls in absence of any special detailing requirements for the frames. A shear wall–frame interactive system, which is discussed in Section 11.3.6 of this publication, is usually more practical and economical in such cases.

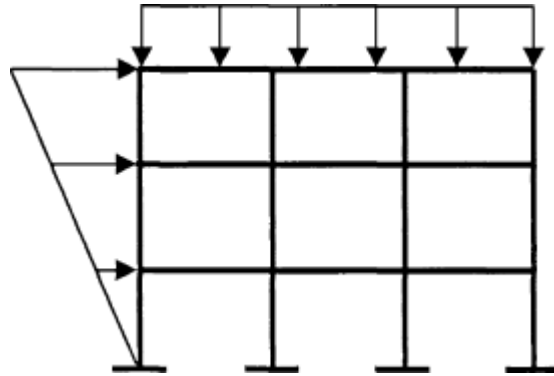
**SDC C.** Buildings assigned to SDC C are permitted to utilize ordinary reinforced concrete shear walls with no limitations. Like in the case of bearing wall systems, it is assumed that the design and detailing requirements in ACI Chap. 11 are compatible with the anticipated level of inelastic response.

**SDC D, E, and F.** Special reinforced concrete shear walls are required to be used in a building frame system (along with the applicable building height limits) in structures assigned to SDC D and higher. It is important to note that for these SDCs, the deformational compatibility requirements of ACI 18.14 must be satisfied. The beam–column frames must be designed to resist the effects caused by the lateral deflections due to the earthquake effects because they are connected to the walls by the diaphragm at each level. The frame members, which are not designated as part of the SFRS, must be capable of supporting their gravity loads when subjected to the design displacements caused by the seismic forces.

### 11.3.4. Moment-Resisting Frame Systems

In a moment-resisting frame system, gravity loads are supported by an essentially complete space frame and lateral forces are resisted primarily by flexural action of designated frame members (the entire space frame or selected portions of the space frame may be designated as the SFRS). A typical moment-resisting frame system is illustrated in Fig. 11.5.

Figure 11.5 Moment-resisting frame system.



**SDC B.** An ordinary reinforced concrete moment can be used in buildings assigned to SDC B with no limitations. In addition to the requirements of ACI Chaps. 1 to 17 and 19 to 26, the requirements of ACI 18.3 for ordinary moment frames must also be satisfied.

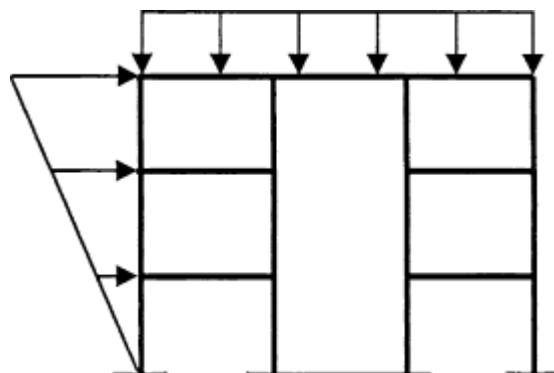
**SDC C.** Buildings assigned to SDC C are permitted to utilize intermediate reinforced concrete moment frames with no limitations. Such frames are to be designed and detailed in accordance with ACI 18.4.

**SDC D, E, or F.** Special reinforced concrete moment frames are required in buildings assigned to SDC D, E, or F. These frames can be used without any limitations and must be designed and detailed in accordance with ACI 18.2.3 through 18.2.8 and 18.6 through 18.8.

### 11.3.5. Dual Systems

In a dual system, an essentially complete space frame provides support for gravity loads and resistance to lateral forces is provided by moment-resisting frames and by shear walls (see Fig. 11.6). The frames and shear walls are designed to resist lateral forces in proportion to their relative rigidities. An additional requirement is that the moment frames alone must be capable of resisting at least 25% of the seismic forces (i.e., the frames act as a backup to the shear walls).

Figure 11.6 Dual system.



**SDC B.** Any of the dual systems listed in ASCE/SEI Table 12.2-1 are permitted to be used in structures assigned to SDC B. It is common for shear wall-frame interactive systems to be used in such cases (see Section 11.3.6).

**SDC C.** A dual system with intermediate moment frames and ordinary reinforced concrete shear walls is permitted as a minimum. The moment frames, which must be designed to independently resist 25% of the code-prescribed seismic forces,

must satisfy the requirements of ACI 18.4. Only the provisions of ACI Chaps. 1 to 17 and 19 to 26 need to be satisfied for the walls.

**SDC D, E, or F.** Dual systems with special moment frames and special reinforced concrete shear walls are required for structures assigned to SDC D, E, or F without limitations. Each component must be designed and detailed in accordance with the applicable provisions of ACI 18.2.1.6. A dual system with intermediate moment frames and special reinforced concrete shear walls is also permitted with the following limitations: (1) For SDC D, the building height is limited to 160 ft (49 m) and (2) for SDC E and F, the building height is limited to 100 ft (30 m). Note that the use of intermediate moment frames as part of a dual system in SDC D, E, or F is not recommended (see ACI R18.2).

## 11.3.6. Shear Wall-Frame Interactive Systems

Shear wall-frame interactive systems are similar to dual systems in that an essentially complete space frame provides support for gravity loads and resistance to lateral forces is provided by moment-resisting frames and by shear walls, which are designed to resist lateral forces in proportion to their relative rigidities. The main difference is that the moment-resisting frames do not have to be designed to resist 25% of the lateral forces.

Shear wall-frame interactive systems with ordinary reinforced concrete moment frames and ordinary reinforced concrete shear walls are permitted to be used in buildings assigned to SDC B with no limitations. Such systems are not permitted in buildings assigned to higher SDCs; a dual system would be required in such cases. Other than ACI 18.3, no special seismic design and detailing requirements are prescribed for this system.

## 11.3.7. System Properties

In addition to the information covered above, ASCE/SEI Table 12.2-1 contains important properties for each of the SFRSs in the table. A discussion of these properties follows.

### 11.3.7.1. Response Modification Coefficient $R$

In general, this coefficient accounts for the ability of a SFRS to respond to ground shaking in a ductile manner without loss of load-carrying capacity. In other words,  $R$  is an approximate way of accounting for the effective damping and energy dissipation that can be mobilized during inelastic response to ground shaking. It represents the ratio of the forces that would develop under the ground motion specified in ASCE/SEI 7-10 if the structure had responded to the ground motion in a linear-elastic manner.

A system that has no ability to respond in a ductile manner has an  $R$ -value equal to 1; from ASCE/SEI Table 12.2-1, the only such system is a cantilevered column system consisting of ordinary reinforced concrete moment frames (in such systems, the vertical forces and the seismic forces are resisted entirely by columns acting as cantilevers from their base). Systems that are capable of highly ductile response have an  $R$ -value equal to 8. A wide variety of systems possess this maximum value of  $R$ .

It can be seen from the methods provided in ASCE/SEI 12.8 that this coefficient essentially reduces the seismic load effects on a structure. Although the required design strength decreases as  $R$  increases, the level of design and detailing substantially increase with increasing  $R$ .

### 11.3.7.2. Overstrength Factor $\Omega_o$

This factor accounts for the fact that the actual seismic forces on some members of a structure can be significantly larger than those indicated by analysis using the prescribed design seismic forces. In general, these members cannot provide

reliable inelastic response or energy dissipation. For most of the structural systems in ASCE/SEI Table 12.2-1,  $\Omega_o$  ranges from 2 to 3.

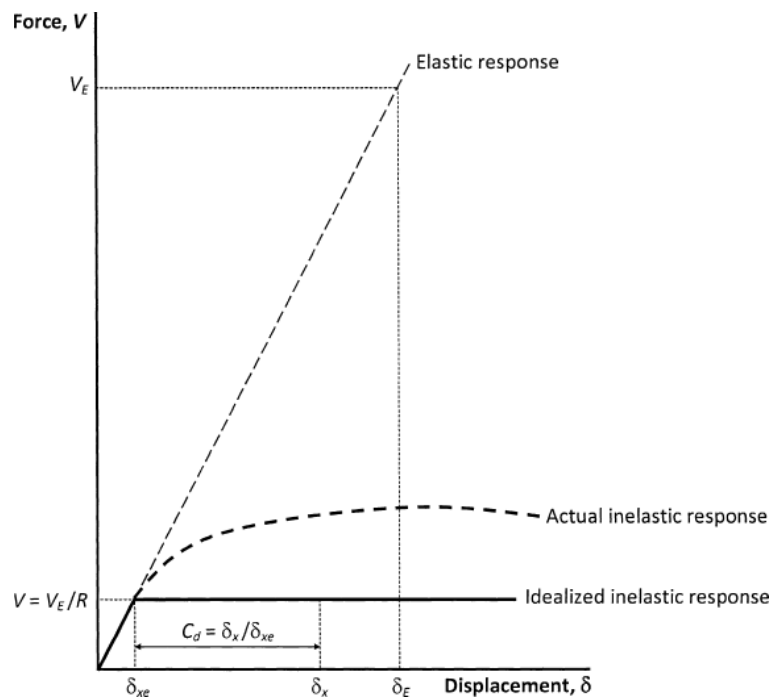
### 11.3.7.3. Deflection Amplification Factor $C_d$

As noted previously, the seismic forces prescribed in ASCE/SEI 7 are smaller than those anticipated during the actual design earthquake event. The deflection amplification factor is used to adjust the lateral displacements that are determined for a structure using the prescribed design seismic forces to the actual anticipated lateral displacements during the design earthquake.

It can be seen from the table that values of  $C_d$  are equal to or slightly less than the corresponding  $R$ -values for a given SFRS. The more ductile a system is (i.e., the greater the  $R$ -value), the greater the difference is between the values of  $R$  and  $C_d$ .

In order to acquire a better understanding of these properties, consider the relationship between horizontal seismic force and displacements illustrated in Fig 11.7. If a structure were to respond elastically to the ground motion, the corresponding deflection would be  $\delta_E$  and the corresponding seismic force on the structure would be  $V_E$ , which is many times greater than the design force  $V$  that is determined by the provisions of ASCE/SEI 7. In particular,  $V_E = RV$ .

Figure 11.7 Force-displacement curves.



The actual inelastic response of the structure to ground motion is also depicted in the figure. When designed and detailed properly, the SFRS is expected to reach significant yield when subjected to seismic forces greater than the prescribed design seismic forces. Significant yield is defined as the point where the first plastic hinge forms in the structure. Additional plastic hinges form as the seismic force increases until a maximum displacement is reached. It can be shown that full yielding of properly designed and detailed regular structures that are redundant can occur at force levels that are two to four times the code-prescribed design force levels.

For structures that are idealized with a bilinear inelastic response with a fundamental period greater than  $T_S = S_{D1}/S_{DS}$ , it has been observed that maximum displacement is approximately equal to that of an elastic system with the same initial stiffness (see Fig. 11.7). The maximum displacement of an inelastic system with a period smaller than  $T_S$  is likely to exceed that of the corresponding elastic system. The code-prescribed design forces based on a base shear equal to  $V$  produce essentially

fictitious displacements  $\delta_{xe}$ , which are smaller than the actual displacements because the code-prescribed design forces are reduced by  $R$ .

As can be seen from [Fig 11.7](#), the actual inelastic response is different than the idealized response mainly due to overstrength and the related increase in stiffness. Thus, the actual displacement of the system may be less than  $R\delta_{xe}$ . This difference is accounted for by multiplying the design elastic displacements  $\delta_{xe}$  corresponding to the code-prescribed forces by the deflection amplification factor  $C_d$ . The result is displacements that are closer in magnitude to those expected during the design-level earthquake.

This design philosophy builds into the structure sufficient inelastic deformability through ductile detailing of critical members, thereby enabling it to survive without collapse when subjected to several cycles of deformation into the inelastic range. The detailing requirements of Chap. 18 to achieve this are covered in the next section.

## 11.4. Design and Detailing Requirements

### 11.4.1. Overview

Chapter 18 of the ACI Code contains the seismic design and detailing requirements for structural members in structures assigned to SDC B, C, D, E, and F (ACI 18.1.1). The provisions in this chapter do not apply to structures assigned to SDC A. For structures assigned to SDC B and C, the requirements are applicable to members that have been designated to be part of the SFRS. For structures assigned to SDC D through F, requirements are given for members designated to be part of the SFRS and members that are not. The later provisions are commonly referred to as deformation compatibility requirements and are covered later in this section. In addition to the requirements in ACI Chap. 18, members in structures assigned to SDC B through F must also satisfy the applicable requirements in ACI Chaps. 1 to 17 and 19 to 26 (ACI 18.2.1.2).

As noted in the previous section, the philosophy behind the minimum design and detailing requirements in ACI Chap. 18 is that cast-in-place reinforced concrete structures are to respond in the nonlinear range when subjected to the effects from design-level earthquakes without critical loss of strength. During a design-level event, energy dissipation is to occur through appropriate design and detailing of the structural members in the system, and the stiffness of the structure decreases as it undergoes cycles of displacement reversals in the inelastic range. This combination of increased energy dissipation and reduced stiffness usually results in response accelerations and horizontal inertial forces that are smaller than those that would occur if the structure were to remain linearly elastic and lightly damped.<sup>3</sup> Therefore, by using design earthquake forces that are less than those expected by a design-level event, such as those prescribed in ASCE/SEI 7, the SFRS must be designed and detailed so that it dissipates the energy from the event while retaining a substantial portion of its strength into the inelastic range.

The design and detailing requirements in ACI Chap. 18 are related to the SDC that the structure has been assigned to. Methods to determine the SDC are given in [Section 11.2.3](#) of this book. In general, the level of design and detailing is compatible with the level of inelastic response assumed in the calculation of the design earthquake forces. As noted in [Section 11.3.7](#), the response modification coefficient  $R$  is an approximate way of accounting for the effective damping and energy dissipation that can be mobilized during inelastic response to ground shaking. In essence, larger values of  $R$  mean greater energy dissipation and smaller design earthquake forces. This also means that there are more design and detailing requirements that need to be satisfied than those for a SFRS with a smaller  $R$ -value.

Throughout ACI Chap. 18, the terms ordinary, intermediate, and special are used to assist in describing the compatibility between design and detailing requirements and the required level of inelastic response and energy dissipation. The design and detailing requirements are far less for an ordinary moment frame, for example, compared to those in an intermediate moment frame and a special moment frame, the latter of which having the most stringent set of requirements.

[Table 11.7](#) contains the sections in ACI Chap. 18 that must be satisfied for different structural components in addition to

those in ACI Chaps. 1 through 17 and 19 through 26. These requirements are organized with respect to SDC. As noted previously, there are no sections in Chap. 18 that are applicable to components in structures assigned to SDC A. In-depth information of the design and detailing requirements for the components in this table is presented in the following sections.

**Table 11.7** Sections of ACI Chap. 18 to be Satisfied Based on SDC

Component	SDC		
	B (ACI 18.2.1.3)	C (ACI 18.2.1.4)	D, E, F (ACI 18.2.1.5)
Frame members	ACI 18.3	ACI 18.4	ACI 18.6 – 18.9
Structural walls and coupling beams	None	None	ACI 18.10
Diaphragms and trusses	None	None	ACI 18.12
Foundations	None	None	ACI 18.13
Frame members not designated as part of the SFRS	None	None	ACI 18.14

## 11.4.2. Strength Reduction Factors

ACI 18.2.4 points the reader to ACI Chap. 21 that contains the strength reduction factors  $\phi$  that are applicable to structures that utilize special moment frames and special structural walls as the SFRS.

For any member in the SFRS that is designed to resist  $E$ , the strength reduction factor  $\phi$  for shear is 0.60 in cases where the nominal shear strength of the member  $V_n$  is less than the shear corresponding to the development of the nominal flexural strength of the member  $M_n$ , where  $M_n$  is calculated considering the most critical factored axial loads that include the effects from  $E$  (ACI 21.2.4.1). The use of this  $\phi$ -factor would be applicable to members that are controlled by shear, such a low-rise shear walls, portions of shear walls between openings in a wall, and diaphragms. In most cases, it would be impractical to increase  $V_n$  by providing additional reinforcement, for example, in order to make it greater than or equal to the shear corresponding to the development of  $M_n$ .

In the case of shear design of diaphragms, ACI 21.2.4.2 requires that  $\phi$  shall be less than or equal to the minimum  $\phi$ -factor for the vertical components of the SFRS.

For the shear design of joints in special moment frames and for the shear design of diagonally reinforced coupling beams,  $\phi$  is to be taken equal to 0.85 (ACI 21.2.4.3).

## 11.4.3. Material Properties

### 11.4.3.1. Concrete

The compressive strength of concrete used in special moment frames and special structural walls must comply with the provisions in ACI Table 19.2.1.1. The minimum value of  $f'_c$  to be specified in such systems is 3,000 psi (21 MPa) for normal-weight concrete. No upper limit is specified for normal-weight concrete. In the case of lightweight concrete, the lower limit is also 3,000 psi (21 MPa), and an upper limit of 5,000 psi (35 MPa) is prescribed. The maximum limit on  $f'_c$  is mainly due to the fact that experimental and field data on the behavior and performance of members made with lightweight concrete are very limited. Values of  $f'_c$  greater than 5,000 psi (35 MPa) for lightweight mixtures are permitted to be used if it can be



demonstrated that members made from the lightweight mixture provide strength and toughness levels that equal or exceed those of comparable members made with normal-weight concrete of the same compressive strength.

Concrete used in all other systems must comply with the general application provisions in ACI Table 19.2.1.1.

### 11.4.3.2. Reinforcement

Reinforcement used in any SFRS must comply with the provisions in ACI 20.2.2. For special moment frames and special structural walls, additional requirements are provided in ACI Table 20.2.2.4a, 20.2.2.4b, and ACI 20.2.2.5.

ACI Table 20.2.2.4a contains maximum values of yield stress and permissible bar types for deformed bars, deformed wires, welded wire, and welded bar mats. For flexure, axial force and temperature and shrinkage reinforcement in special moment frames and special structural walls,  $f_y$  is limited to 60,000 psi (420 MPa). Only deformed bars are permitted to be used in such systems and the bars must conform to the provisions of ACI 20.2.2.5:

- a. ASTM A706, Grade 60 (Grade 420)
- b. ASTM A615, Grade 40 (Grade 280) provided the requirements in (i) and (ii) below are satisfied; and ASTM A615, Grade 60 (Grade 420) provided the requirements in (i) through (iii) below are satisfied.
  - i. Actual yield strength based on mill tests does not exceed  $f_y$  by more than 18,000 psi (125 MPa).
  - ii. Ratio of the actual tensile strength to the actual yield strength is at least 1.25.
  - iii. Minimum elongation in 8 in (200 mm) shall be at least 14% for bar sizes Nos. 3 through 6 (Nos. 10 through 19); at least 12% for bar sizes Nos. 7 through 11 (Nos. 22 through 36); and at least 10% for bar sizes Nos. 14 and 18 (Nos. 43 and 57).

An upper limit is placed on the actual yield strength of the steel used as longitudinal reinforcement in special systems because brittle failures in shear or bond could occur if the strength of the reinforcement is substantially higher than that assumed in the design (higher strength reinforcement leads to higher shear and bond stresses). The requirement for the tensile strength of the reinforcement to be at least 1.25 times the yield strength is based on the assumption that the capability of a structural member to develop inelastic rotation capacity is a function of the length of the yield region along the axis of the member. It has been shown that the length of the yield region is related to the relative magnitudes of nominal and yield moments: the greater the ratio of nominal to yield moment, the longer the yield region.<sup>4</sup> Inelastic rotation can be developed in members that do not satisfy this condition, but they behave in a significantly different manner than members that are reinforced with strain-hardened steel whose behavior forms the basis of the provisions. The requirements for minimum elongation for ASTM A615, Grade 60 (Grade 420) steel are the same as those for ASTM 706, Grade 60 (Grade 420) steel.

Deformed and plain reinforcement utilized as lateral support for longitudinal bars or for concrete confinement in special seismic systems is limited to a yield stress of 100,000 psi (690 MPa). Welded deformed bar mats are not permitted to be used in such cases. Research has shown that reinforcement with higher yield strengths can be used effectively as confinement reinforcement.<sup>5-7</sup> Note that the restrictions on the values of the yield strength for calculating nominal shear strength  $V_n$  given in other chapters of the Code are intended to limit the width of shear cracks.

### 11.4.4. Mechanical and Welded Splices

ACI 18.2.7 and 18.2.8 contain provisions for mechanical and welded splices for use in special moment frames and special structural walls, respectively. In the case of mechanical splices, both Type 1 and Type 2 splices conforming to ACI 25.5.7 are permitted to be used. Additionally, a Type 2 splice must be able to develop the full specified tensile strength of the spliced bar because it is permitted to be located at any location in a member, including anticipated plastic hinge zones. Type 1 splices are not permitted to be located within the following plastic hinge zones: a distance equal to twice the member depth from (1) a



column or beam face in a special moment frame or (2) critical sections where yielding is likely to occur as a result of inelastic lateral displacement (such as at the base of a special structural wall).

Welded splices in special moment frames and special structural walls must conform to ACI 25.5.7. The restriction on the location of such splices is the same as that for Type 1 mechanical splices, that is, they are not permitted to be located within potential plastic hinge zones. Welding of stirrups, ties, inserts, or other similar elements to longitudinal reinforcement in a member of a special seismic system is not permitted.

## 11.5. Ordinary Moment Frames

### 11.5.1. Overview

ACI 18.3 contains provisions for ordinary moment frames that are assigned to SDC B. The additional requirements that need to be satisfied for members that are part of the SFRS are covered below.

### 11.5.2. Beams

According to ACI 18.3.2, beams must have at least two continuous bars at both the top and bottom faces of the section. At least one-fourth of the total bottom reinforcement along the span must be continuous and anchored to develop the yield strength of the bars in tension at the face of the supports.

These provisions are essentially structural integrity requirements that are intended to improve continuity and lateral force resistance. Note that these provisions are not applicable to moment frames consisting of slabs and columns.

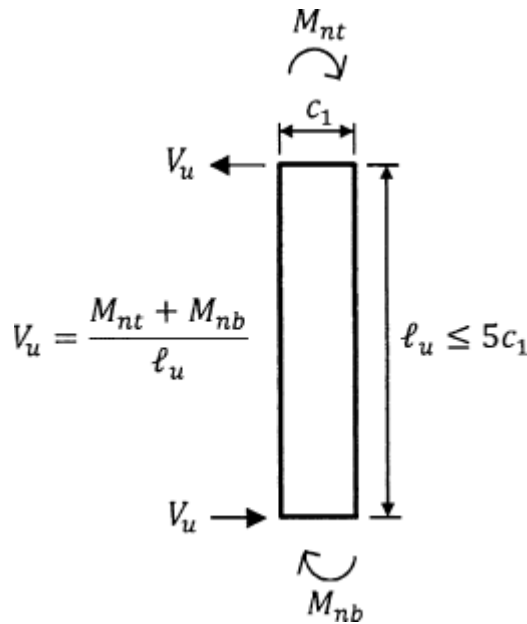
The design of these members for flexure, shear, and torsion are governed by the requirements in ACI Chap. 9.

### 11.5.3. Columns

In cases where the unsupported length of a column  $\ell_u$  is less than or equal to five times the plan dimension of the column in the direction of analysis  $c_1$ , the design shear strength of the column  $\phi V_n$  is to be taken as the lesser of the following (ACI 18.3.3):

- a. The shear  $V_u$  associated with the development of nominal moment strengths  $M_n$  of the column at each restrained end of the unsupported length due to reverse curvature bending (see Fig. 11.8). In the figure,  $M_{nt}$  and  $M_{nb}$  are the nominal flexural strengths at the top and bottom of the column, respectively. These flexural strengths are to be calculated for the factored axial force  $P_u$  that is consistent with the direction of analysis which results in the highest flexural strength. Sidesway to the right and sidesway to the left must both be considered.
- b. The maximum shear obtained from the design load combination of ACI Chap. 5 that includes the earthquake effect  $E$  with  $\Omega_o E$  substituted for  $E$  in the load combination. The quantity  $\Omega_o$  is the overstrength factor given in ASCE/SEI Table 12.2-1 for an ordinary moment frame of reinforced concrete, which is equal to 3.

Figure 11.8 Calculation of design shear strength of columns in ordinary moment frames.



The provisions for the design shear strength are intended to provide additional capacity to resist shear in columns that are relatively squat, which make them vulnerable to shear failure under earthquake loading.

## 11.6. Intermediate Moment Frames

### 11.6.1. Overview

Provisions for design and detailing of intermediate moment frames, which are permitted to be used in structures assigned to SDC C with no limitations (see ASCE/SEI Table 12.2-1), are given in ACI 18.4. Included are requirements for frames that consist of columns and two-way slabs without beams, which form the SFRS.

### 11.6.2. Beams

Table 11.8 contains a summary of the design and detailing requirements of ACI Chap. 9 and ACI 18.4.2 for beams in an intermediate moment frame with an axial force less than or equal to  $A_g f'_c / 10$  where  $A_g$  is the gross cross-sectional area of the beam. For beams with an axial force that exceeds this value, transverse reinforcement must be provided that conforms to the requirements of ACI 25.7.2.2 and either ACI 25.7.2.3 or 25.7.2.4, which pertain to rectangular or circular ties. Just like in a column, the ties are intended to provide lateral support for the longitudinal bars in the beam.

Table 11.8 Design and Detailing Requirements for Beams in Intermediate Moment Frames

	Requirement	ACI Section Number(s)
	Design beams as tension-controlled sections.	21.2.2, 22.3

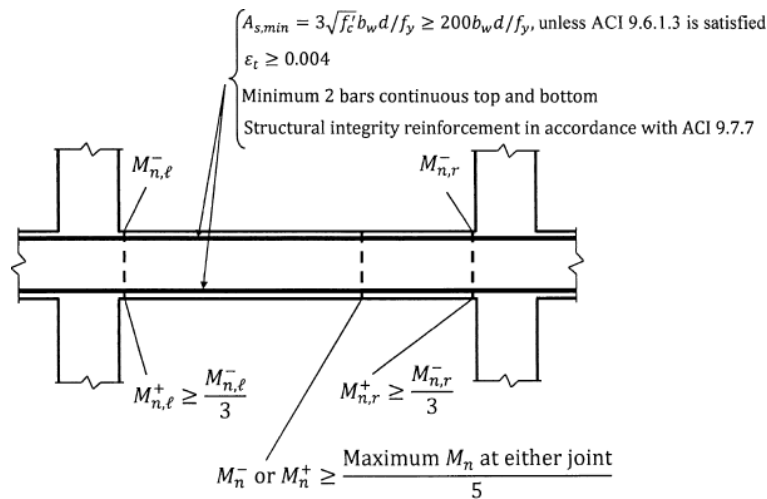
Requirement		ACI Section Number(s)
<b>Flexure</b>	<p>Flexural reinforcement shall not be less than:</p> $\frac{3\sqrt{f'_c}b_wd}{f_y} \text{ and } \frac{200b_wd}{f_y}$ <p>[ In SI: <math>\frac{0.25\sqrt{f'_c}b_wd}{f_y} \text{ and } \frac{1.4b_wd}{f_y}</math> ] at every section of a flexural member where tensile reinforcement is required by analysis, except as provided by ACI 9.6.1.3.</p>	9.6.1.2
	The net tensile strain $\epsilon_t$ at nominal strength shall be greater than or equal to 0.004.	9.3.3.1
	Positive moment strength at the face of a joint must be greater than or equal to one-third the negative moment strength at that face of the joint.	18.4.2.2
	Neither the negative nor the positive moment strength at any section along the length of the beam must be less than one-fifth the maximum moment strength at the face of either joint.	18.4.2.2
	Beams shall have at least two continuous bars at both the top and bottom face. Continuous bottom bars shall have an area not less than one-fourth the maximum area of the bottom bars along the span. Continuous bars must be anchored to develop $f_y$ at the face of the supports.	18.4.2.1
<b>Shear</b>	<p>Transverse reinforcement must be proportioned to resist the design shear force <math>\phi V_n</math>, which is the lesser of the following:</p> <ol style="list-style-type: none"> <li>The sum of (i) the shear associated with the development of nominal moment strengths of the beam at each restrained end of the clear span due to reverse curvature bending and (ii) the shear calculated for factored gravity loads.</li> <li>The maximum shear obtained from factored load combinations that include <math>E</math>, with <math>E</math> taken as twice that prescribed by the governing building code.</li> </ol>	18.4.2.3
	Hoops are required over a length equal to at least $2h$ from the face of the supporting member toward midspan at both ends of the beam.	18.4.2.4
	<p>Where hoops are required, the spacing shall not exceed the smallest of the following:</p> <ol style="list-style-type: none"> <li><math>d/4</math></li> <li><math>8 \times</math> diameter of smallest longitudinal bar</li> <li><math>24 \times</math> diameter of hoop bar</li> <li>12 in (300 mm)</li> </ol> <p>The first hoop shall be located not more than 2 in (50 mm) from the face of the supporting member.</p>	18.4.2.4
	Where hoops are not required, stirrups shall be spaced not more than $d/2$ throughout the length of the beam.	18.4.2.5

### 11.6.2.1. Design for Flexure

It is evident from [Table 11.8](#) that some of the requirements for flexure come from ACI Chap. 9, which must be satisfied for all beams. It is good practice to design the beams in an intermediate moment frame as tension-controlled sections. The material in [Chap. 6](#) of this book can be used to determine the required amount of flexural reinforcement based on the applicable load combinations in ACI Table 5.3.1.

The minimum moment capacity at any section of the beam given in ACI 18.4.2.2 is based on the nominal moment capacity at the face of either support. The minimum positive moment strength equal to at least 33% of the corresponding negative moment strength at the ends of the beam allows for the possibility that the positive moment caused by earthquake-induced lateral displacements exceeds the negative moment due to gravity loads. These requirements, together with those for confinement in ACI 18.4.2.4, are meant to provide a minimum threshold level of toughness for beams. A summary of the flexural requirements for beams in intermediate moment frames is given in Fig. 11.9.

Figure 11.9 Flexural requirements for beams in intermediate moment frames.



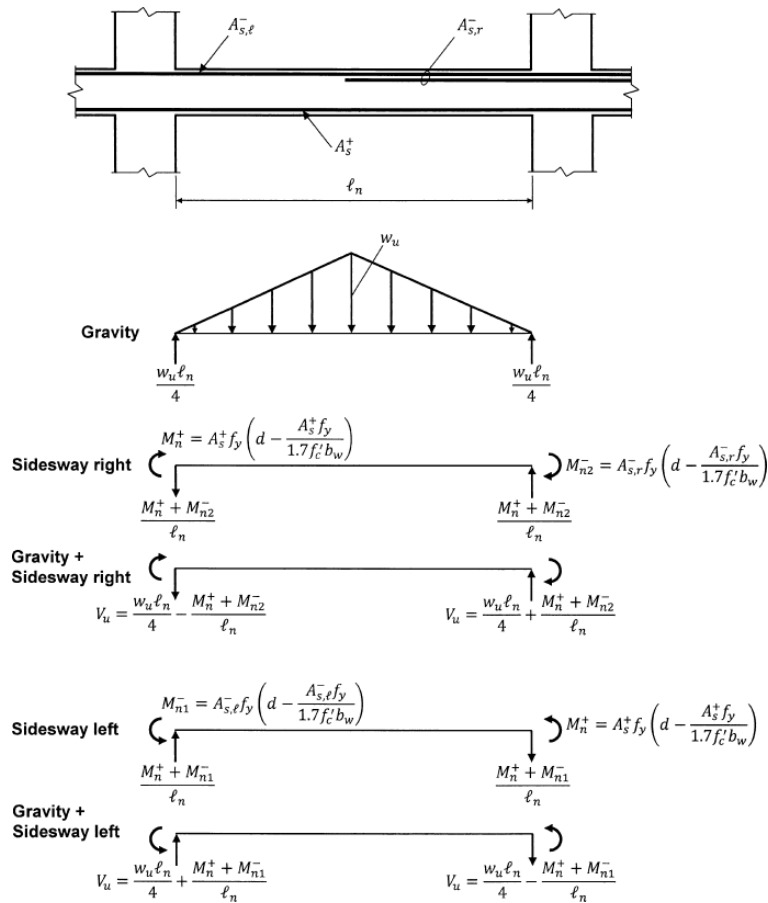
Note: Transverse reinforcement not shown for clarity

There are no restrictions on where lap splices of the flexural reinforcement can be located along the span. Generally, top reinforcement is spliced near midspan and bottom reinforcement is spliced near the ends. However, because the potential exists for plastic hinges to form at the ends of the beam, it would be appropriate to locate the splices of the bottom bars somewhere between the ends and midspan.

### 11.6.2.2. Design for Shear

Two methods are provided in ACI 18.4.2.3 to determine the design shear strength  $\phi V_n$  for beams (the maximum factored shear force  $V_u$  that the beam is designed for is set equal to  $\phi V_n$ ). According to the first method, the factored shear force  $V_u = \phi V_n$  is determined by adding the shear effects associated with the application of the nominal moment strengths  $M_n$  at each end of the member to those associated with the factored gravity loads. This method is illustrated in Fig. 11.10 for the cases of sidesway to the right and sidesway to the left for a beam with a factored triangular load due to gravity, which is common for framing systems with beam-supported slabs (see ACI 8.10.8.1). The total factored shear forces at the ends of the beam due to gravity and earthquake effects are determined from statics. Similar equations can be derived for other distributions of gravity load.

Figure 11.10 Design shear forces for beams in intermediate moment frames.



In the second method, the factored shear force  $V_u = \phi V_n$  is obtained from the design load combinations that include the earthquake load effects  $E$ , where  $E$  is assumed to be twice that prescribed by the building code. In this option, the governing load combination for shear design is given by ACI Eq. (5.3.1e):

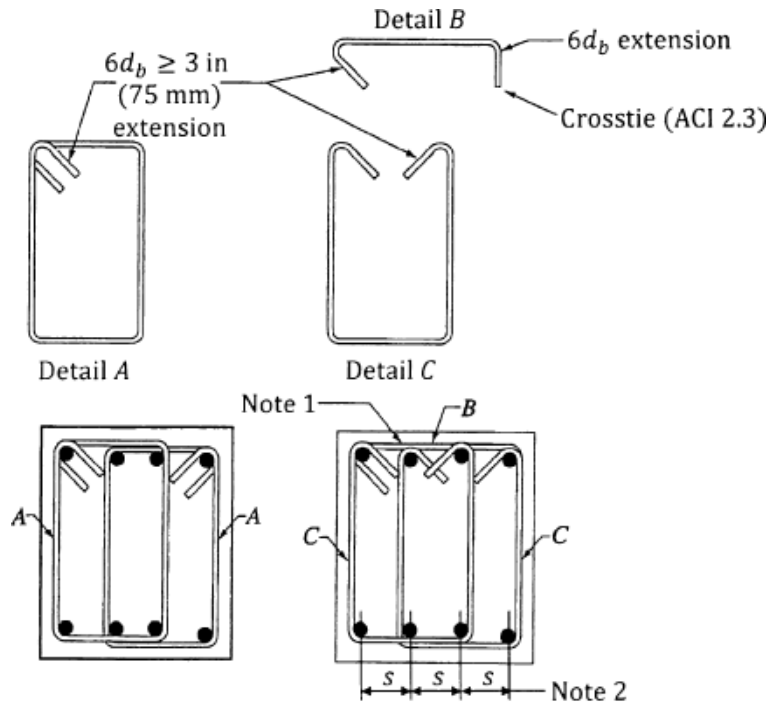
$$U = 1.2D + 2.0E + 1.0L + 0.2S$$

where  $E = Q_E + 0.2S_{DS} D$  (ASCE/SEI 12.4.2; Note: The redundancy factor  $\rho = 1.0$  for structures assigned to SDC C). The quantity  $Q_E$  represents the effects on a beam due to horizontal seismic forces applied to the structure and  $0.2S_{DS} D$  represents the effects due to vertical seismic forces.

Design shear strength  $\phi V_n$  shall not be less than the smaller of those obtained by these two methods.

The required amount of shear reinforcement to resist  $V_u = \phi V_n$  can be obtained using the methods given in Chap. 6 of this book. Hoops must be provided in the anticipated plastic hinge zone, the length of which is at least  $2h$  measured from the face of the supporting member toward the midspan of the beam at both ends of the beam. A hoop is defined as a closed tie or continuously wound tie that is made up of one or several reinforcement elements, each having seismic hooks that conform to ACI 25.3.4 at both ends (ACI 2.3 and 25.7.4.1). The ends of the reinforcement elements in hoops must engage a longitudinal bar in the beam (ACI 25.7.4.2). Examples of hoops and overlapping hoops are illustrated in Fig. 11.11.

Figure 11.11 Examples of hoops and overlapping hoops.

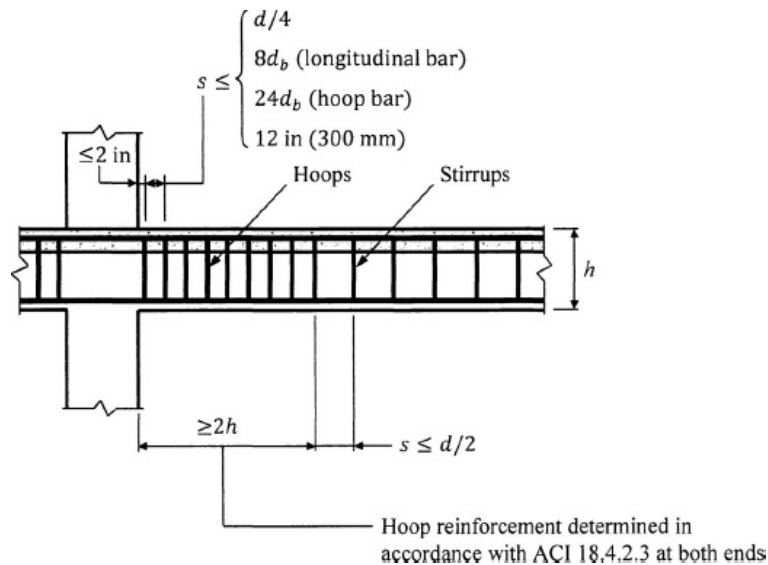


**Notes**

1. The 90-degree hooks of two consecutive crossties engaging the same longitudinal bars shall be alternated end for end.
2. Spacing  $s$  between longitudinal bars restrained by legs of crossties or hoops must not exceed 14 in (350 mm).

Once the required size and spacing of the hoops are determined, the spacing must be checked against the minimum spacing requirements of ACI 18.4.2.4. Figure 11.12 contains the transverse reinforcement requirements for beams in intermediate moment frames.

Figure 11.12 Transverse reinforcement requirements for beams in intermediate moment frames.



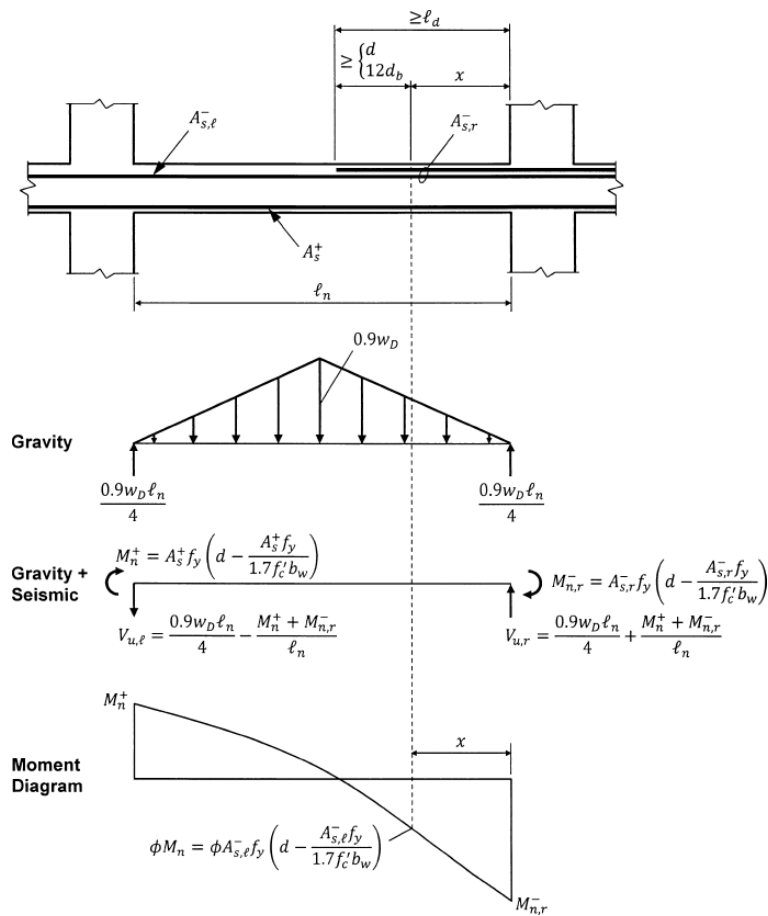
### 11.6.2.3. Development and Cutoff Points of Flexural Reinforcement

Flexural bars may be terminated along the span as long as the applicable requirements in ACI 9.7.3 are satisfied. The load combination used to find cutoff points is 0.9 times the dead load with the nominal moment strengths  $M_n$  at the ends of the member, because this combination produces the longest bar lengths.

The following equation can be utilized to determine the theoretical cutoff point  $x$  from the face of the support for negative reinforcing bars (see Fig. 11.13):

$$\frac{x}{2} \left( \frac{0.9w_D x}{\ell_n/2} \right) \frac{x}{3} - V_{u,r} x + M_{n,r}^- = \phi M_n$$

**Figure 11.13** Cutoff point of negative flexural reinforcement for beams in intermediate moment frames.



As shown in the figure,  $\phi M_n$  is the design strength of the beam based on the area of negative reinforcement  $A_{s,\ell}^-$  at the left support. The distance  $x$  is the theoretical location where the negative reinforcement  $A_{s,r}^-$  at the right support is no longer required for moment strength, that is,  $A_{s,\ell}^-$  alone is sufficient at this point. Similar equations can be derived for cases other than triangular gravity loads.

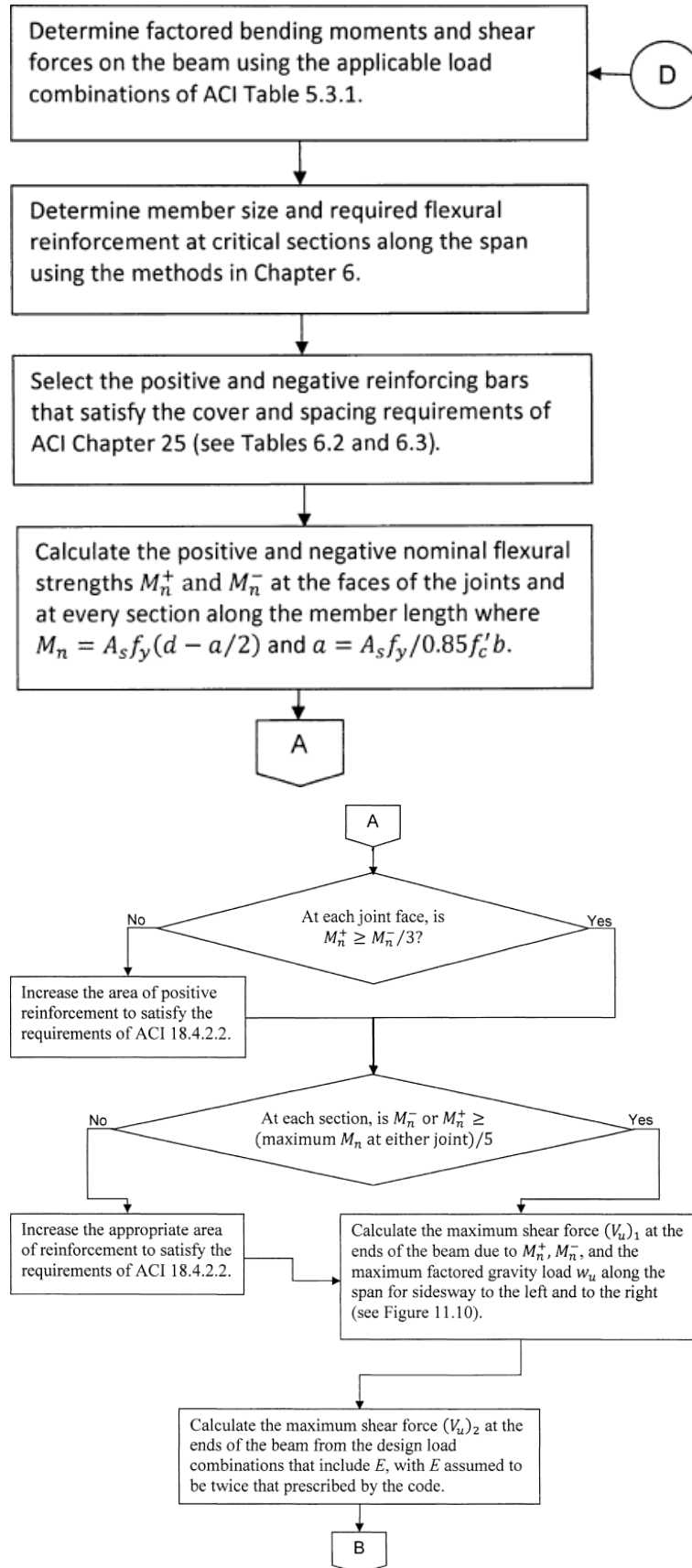
According to ACI 9.7.3.3, the negative bars at the right support must extend a distance equal to the larger of  $d$  or  $12d_b$  beyond  $x$ . Additionally, the total length of the bars from the face of the right support must be at least equal to the development length  $\ell_d$  determined by ACI Eq. (25.4.2.3a).

Flexural reinforcement shall not be terminated in a tension zone unless one or more of the conditions of ACI 9.7.3.5 are satisfied. Typically, ACI 9.7.3.5(a) is satisfied at the cutoff point.

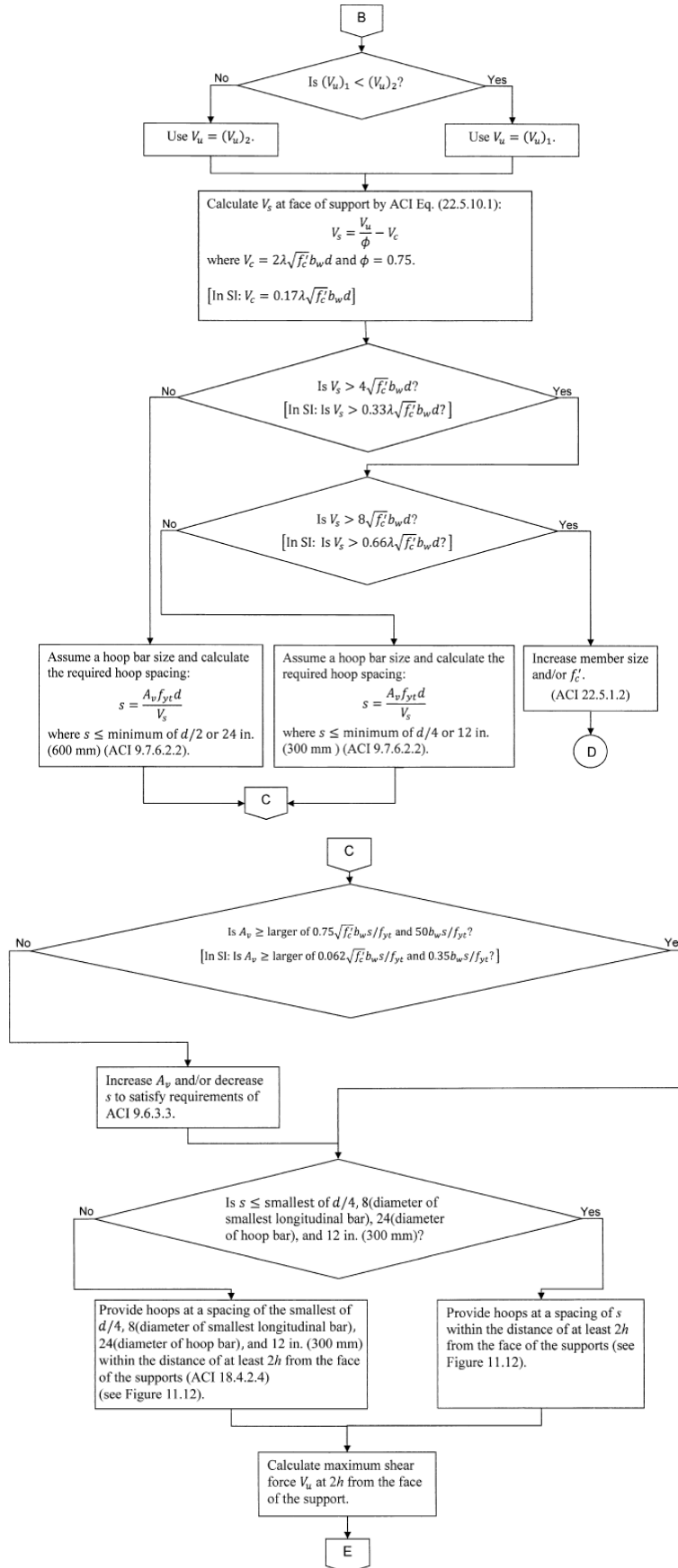
A summary of the overall design procedure for a beam in an intermediate moment frame is given in Fig. 11.14 for cases where

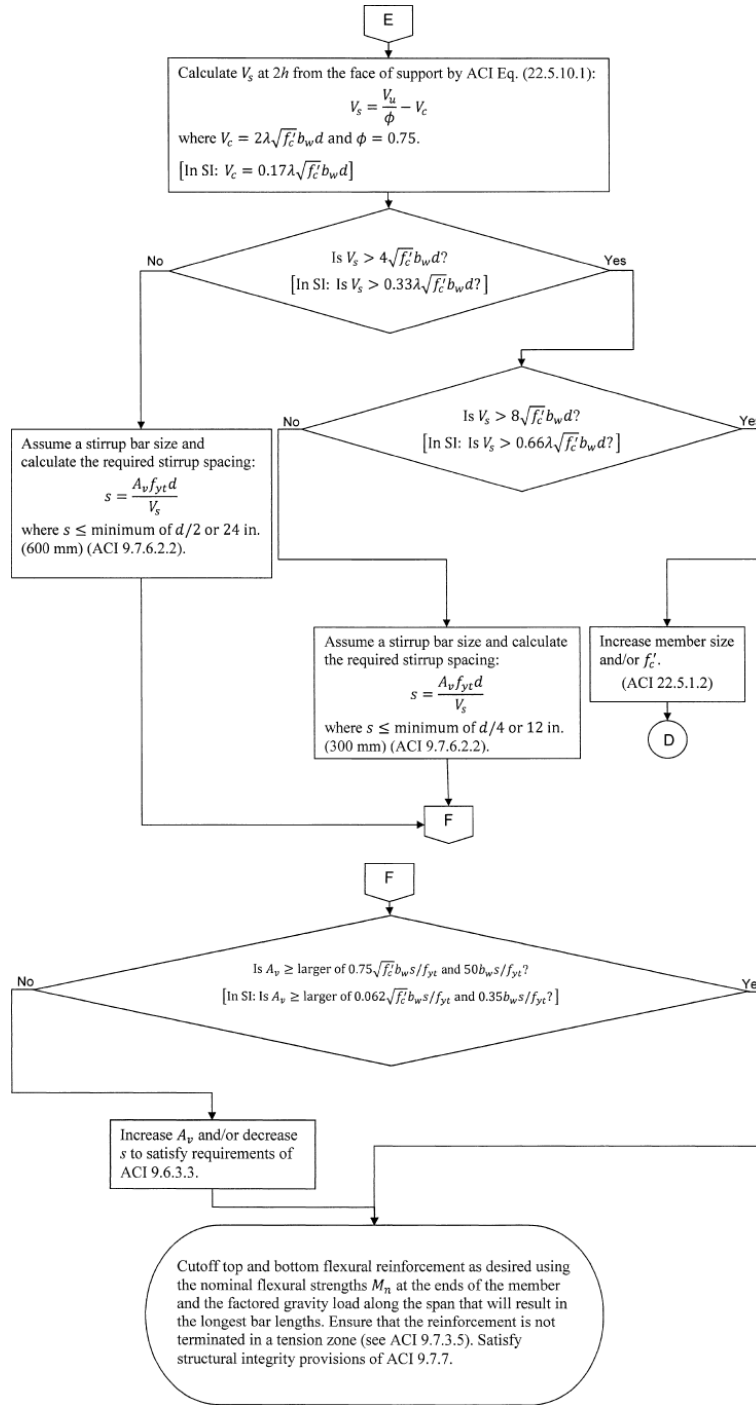
$$P_u \leq A_g f'_c / 10.$$

Figure 11.14 Design procedure for beams in intermediate moment frames.



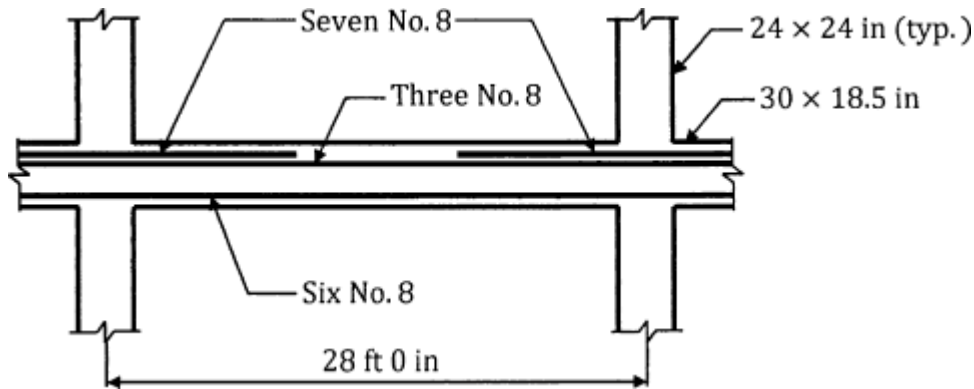






**Example 11.3** A beam that is part of an intermediate moment frame is illustrated in Fig. 11.15. Assuming normal-weight concrete with  $f'_c = 4,000$  psi and Grade 60 reinforcement, determine if the provisions of ACI 18.4.2.2 are satisfied.

Figure 11.15 The beam in Example 11.3.



**Solution** The nominal moment strengths are determined for both the negative reinforcement and the positive reinforcement at the supports using the appropriate methods in Chap. 5.

**Negative Reinforcement** At the supports, the total negative reinforcement is 10 No. 8 bars, which have a total area  $A_s = 10 \times 0.79 = 7.90 \text{ in}^2$ .

The flowchart in Fig. 5.10 will be used to determine  $M_n^-$ .

**Step 1. Check minimum reinforcement requirements.** Determine  $A_{s,min}$  in accordance with ACI 9.6.1.2 and compare it to the provided  $A_s^-$ . Because the compressive strength of the concrete is less than 4,400 psi, the minimum amount is determined by the lower limit given in that section:

$$A_{s,min} = \frac{200b_wd}{f_y} = \frac{200 \times 30 \times 16.0}{60,000} = 1.60 \text{ in}^2 < A_s^- = 7.90 \text{ in}^2$$

**Step 2. Determine  $\beta_1$ .** According to ACI 22.2.2.4.3,  $\beta_1 = 0.85$  for  $f'_c = 4,000$  psi.

**Step 3. Determine neutral axis depth  $c$ .**

$$c = \frac{A_s f_y}{0.85 f'_c b \beta_1} = \frac{7.90 \times 60}{0.85 \times 4 \times 30 \times 0.85} = 5.5 \text{ in}$$

**Step 4. Determine  $\epsilon_t$ .**

$$\epsilon_t = 0.003 \left( \frac{d}{c} - 1 \right) = 0.003 \left( \frac{16.0}{5.5} - 1 \right) = 0.006$$

Because  $\epsilon_t > 0.004$ , the maximum reinforcement requirement of ACI 9.3.3.1 is satisfied. Also, the section is tension-controlled because  $\epsilon_t > 0.005$  (ACI Table 21.2.2).

**Step 5. Determine the depth of the equivalent stress block  $a$ .**

$$a = \beta_1 c = 0.85 \times 5.5 = 4.7 \text{ in}$$

**Step 6. Determine the nominal moment strength  $M_n^-$ .**

$$M_n^- = A_s^- f_y \left( d - \frac{a}{2} \right) = 7.90 \times 60,000 \left( 16 - \frac{4.7}{2} \right) / 12,000 = 539.2 \text{ ft kips}$$

**Positive Reinforcement** At midspan, the total positive reinforcement is six No. 8 bars, which have a total area  $A_s = 6 \times 0.79 = 4.74 \text{ in}^2$ .

Using the flowchart in Fig. 5.10,  $M_n^+ = 346.2 \text{ ft kips}$ .

ACI 18.4.2.2 requires that the positive moment strength at the face of a joint be greater than or equal to one-third the negative moment strength provided at that location. Assuming all of the bottom No. 8 bars pass through the joints at both ends, this requirement is satisfied because  $M_n^+ = 346.2 \text{ ft kips} > M_n^- / 3 = 539.2 / 3 = 179.7 \text{ ft kips}$ .

Also, the negative or positive moment strength at any section of the beam must be greater than or equal to one-fifth the maximum moment strength provided at the face of either joint. Because the negative reinforcement produces the maximum moment strength at both joints, one-fifth of the maximum moment strength is equal to  $539.2 / 5 = 107.8 \text{ ft kips}$ . In order to satisfy this requirement, at least two No. 8 bars ( $M_n =$

122.7 ft kips > 107.8 ft kips) must be provided along the length of the beam at both the top and bottom faces. It is evident from the reinforcement layout in Fig. 11.15 that this requirement is satisfied.

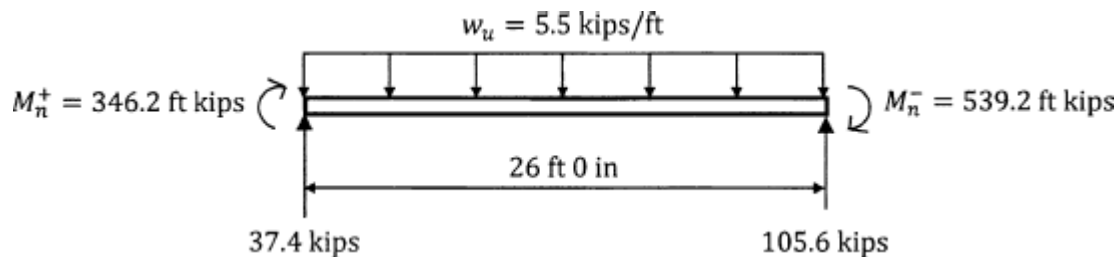
Thus, the provisions of ACI 18.4.2.2 are satisfied for this beam. Also, this reinforcement layout satisfies the provisions of ACI 18.4.2.1 because all of the bottom bars are continuous.

**Example 11.4** For the beam in Example 11.3, determine the required transverse reinforcement along the span length. The design spectral acceleration  $S_{DS}$  has been determined to be 0.4. The service uniformly distributed dead and live loads are as follows:  $w_D = 3.8$  kips/ft and  $w_L = 1.2$  kips/ft. The seismic shear force  $Q_E$  has been determined to be 10.0 kips from a lateral analysis of the building.

**Solution** Shear requirements for beams in intermediate moment frames are given in ACI 18.4.2.3. Design shear strength shall not be less than the smaller of the two values calculated by that section.

**Design Shear Strength by ACI 18.4.2.3(a)** Sidesway to the right and sidesway to the left must both be investigated to obtain the maximum shear force. The largest shear force associated with seismic effects is obtained from ACI Eq. (5.3.1e). Shear forces due to gravity loads plus nominal moment strengths for sidesway to the right are shown in Fig. 11.16. Because the same negative reinforcement is provided at each end of the beam, sidesway to the left gives the same maximum shear force.

Figure 11.16 Factored shear forces on the beam in Example 11.4.



The factored uniform load on the beam is determined as follows:

$$\text{ACI Eq. (5.3.1e): } U = 1.2D + 1.0E + 0.5L$$

ASCE/SEI 12.4.2:  $E = Q_E + 0.2S_{DS} D$  (redundancy factor  $\rho = 1.0$  for structures assigned to SDC C)

Substituting  $S_{DS}$  into the equation for  $E$  results in the following:

$$E = Q_E + (0.2 \times 0.40)D = Q_E + 0.08D$$

The factored load  $w_u$  is obtained by substituting  $E = Q_E + 0.08D$  into ACI Eq. (5.3.1e):

$$w_u = 1.2w_D + Q_E + 0.08w_D + 0.5w_L = (1.28 \times 3.8) + (0.5 \times 1.2) = 5.5 \text{ kips/ft}$$

The negative and positive nominal moment strengths shown in Fig. 11.16 were determined in Example 11.3. It is evident from the figure that the maximum shear force is equal to 105.6 kips.

**Design Shear Strength by ACI 18.4.2.3(b)** In this method,  $E$  is taken as equal to twice that prescribed from ASCE/SEI 7. Thus,

$$E = 2(Q_E + 0.2S_{DS}D) = 2Q_E + (0.4 \times 0.40)D = 2Q_E + 0.16D$$

The service-level shear forces from the dead and live loads are the following:

$$V_D = 3.8 \times (26/2) = 49.4 \text{ kips}$$

$$V_L = 1.2 \times (26/2) = 15.6 \text{ kips}$$

As in the first method, the largest shear force associated with seismic effects is obtained from ACI Eq. (5.3.1e):

$$V_u = 1.2V_D + 0.5V_L + 2V_E + 0.16V_D = 1.36V_D + 0.5V_L + 2V_E = 95.0 \text{ kips}$$

Therefore, design the transverse reinforcement for a maximum shear force of 95.0 kips.

The nominal shear strength provided by the concrete is determined by ACI Eq. (22.5.5.1):

$$V_c = 2\lambda\sqrt{f'_c}b_wd = 2 \times 1.0\sqrt{4,000} \times 30 \times 16/1,000 = 60.7 \text{ kips}$$

Because  $V_u = 95.0$  kips is greater than  $\phi V_c = 0.75 \times 60.7 = 45.5$  kips, provide shear reinforcement in accordance with ACI 22.5.10.5. Assuming No. 3 hoops with four legs, the required spacing  $s$  is determined by ACI Eq. (22.5.10.5.3):

$$s = \frac{\phi A_v f_{yt} d}{V_u - \phi V_c} = \frac{0.75 \times (4 \times 0.11) \times 60 \times 16}{95.0 - 45.5} = 6.4 \text{ in}$$

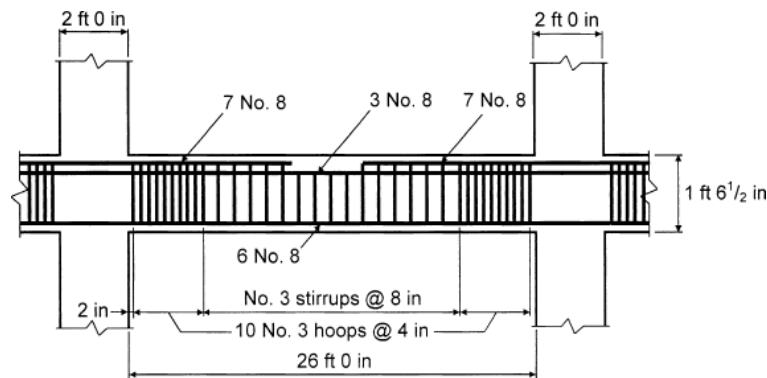
According to ACI 18.4.2.4, the maximum spacing of hoops over the length  $2h = 2 \times 18.5 = 37$  in from the face of the support at each end of the member is the smallest of the following:

- $d/4 = 16/4 = 4.0$  in (governs)
- $8(\text{diameter of smallest longitudinal bar}) = 8 \times 1.0 = 8.0$  in
- $24(\text{diameter of hoop bar}) = 24 \times 0.375 = 9.0$  in
- 12 in

Use 10 No. 3 hoops (four legs) at each end of the beam spaced at 4 in on center with the first hoop located 2 in from the face of the support. For the remainder of the beam, the maximum stirrup spacing is  $d/2 = 8$  in (ACI 18.4.2.5). Use No. 3 stirrups at 8 in on center for the remainder of the beam.

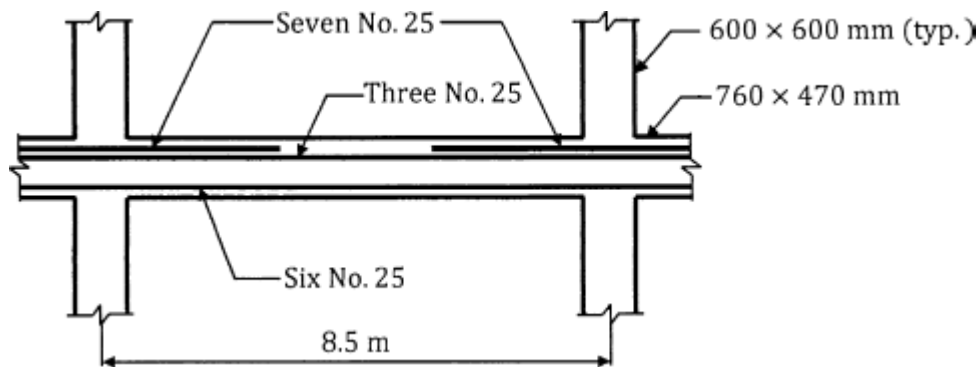
Reinforcement details for this beam are depicted in Fig. 11.17.

Figure 11.17 Reinforcement details for the beam in Example 11.4.



**Example 11.5** A beam that is part of an intermediate moment frame is illustrated in Fig. 11.18. Assuming normal-weight concrete with  $f'_c = 28 \text{ MPa}$  and Grade 420 reinforcement, determine if the provisions of ACI 18.4.2.2 are satisfied.

Figure 11.18 The beam in Example 11.5.



**Solution** The nominal moment strengths are determined for both the negative reinforcement and the positive reinforcement at the supports using the appropriate methods in Chap. 5.

**Negative Reinforcement** At the supports, the total reinforcement is 10 No. 25 bars, which have a total area  $A_s = 10 \times 510 = 5,100 \text{ mm}^2$ .

The flowchart in Fig. 5.10 will be used to determine  $M_n^-$ .

**Step 1. Check minimum reinforcement requirements.** Determine  $A_{s,min}$  in accordance with ACI 9.6.1.2 and compare it to the provided  $A_s^-$ . Because the compressive strength of the concrete is less than 30 MPa, the minimum amount is determined by the lower limit given in that section:

$$A_{s,min} = \frac{1.4b_w d}{f_y} = \frac{1.4 \times 760 \times 410}{420} = 1,039 \text{ mm}^2 < A_s^- = 5,100 \text{ mm}^2$$

**Step 2. Determine  $\beta_1$ .** According to ACI 22.2.2.4.3,  $\beta_1 = 0.85$  for  $M_n^-$ .

**Step 3. Determine neutral axis depth  $c$ .**

$$c = \frac{A_s f_y}{0.85 f_c' b \beta_1} = \frac{5,100 \times 420}{0.85 \times 28 \times 760 \times 0.85} = 140 \text{ mm}$$

**Step 4. Determine  $\epsilon_t$ .**

$$\epsilon_t = 0.003 \left( \frac{d}{c} - 1 \right) = 0.003 \left( \frac{410}{140} - 1 \right) = 0.006$$

Because  $\epsilon_t > 0.004$ , the maximum reinforcement requirement of ACI 9.3.3.1 is satisfied. Also, the section is tension-controlled because  $\epsilon_t > 0.005$  (ACI Table 21.2.2).

**Step 5. Determine the depth of the equivalent stress block  $a$ .**

$$a = \beta_1 c = 0.85 \times 140 = 120 \text{ mm}$$

**Step 6. Determine the nominal moment strength  $M_n^-$ .**

$$M_n^- = A_s^- f_y \left( d - \frac{a}{2} \right) = 5,100 \times 420 \left( 410 - \frac{120}{2} \right) / 1 \times 10^6 = 750 \text{ kN m}$$

**Positive Reinforcement** At midspan, the total reinforcement is six No. 25 bars, which have a total area  $A_s = 6 \times 510 = 3,060 \text{ mm}^2$ .

Using the flowchart in Fig. 5.10,  $M_n^+ = 481 \text{ kN m}$ .

ACI 18.4.2.2 requires that the positive moment strength at the face of a joint be greater than or equal to one-third the negative moment strength provided at that location. Assuming all of the bottom No. 25 bars pass through the joints at both ends, this requirement is satisfied because  $M_n^+ = 481 \text{ kN m} > M_n^- / 3 = 750 / 3 = 250 \text{ kN m}$ .

Also, the negative or positive moment strength at any section of the beam must be greater than or equal to one-fifth the maximum moment strength provided at the face of either joint. Because the negative reinforcement produces the maximum moment strength at both joints, one-fifth of the maximum moment strength is equal to  $750 / 5 = 150 \text{ kN m}$ . In order to satisfy this requirement, at least two No. 25 bars ( $M_n = 171 \text{ kN m} > 150 \text{ kN m}$ ) must be provided along the length of the beam at both the top and bottom faces. It is evident from the reinforcement layout in Fig. 11.18 that this requirement is satisfied.

Thus, the provisions of ACI 18.4.2.2 are satisfied for this beam. Also, this reinforcement layout satisfies the provisions of ACI 18.4.2.1 because all of the bottom bars are continuous.

## 11.6.3. Columns

A summary of the design and detailing requirements of ACI 18.4.3 for columns in an intermediate moment frame is given in Table 11.9. These requirements are all related to shear design and transverse reinforcement. The provisions in ACI Chap. 10 are to be used to design the column for the effects due to axial force and bending moment from the applicable load combinations, including slenderness effects (see Chap. 8). All factored load combinations must fall within the axial loadbending moment interaction diagram that is associated with the section.

**Table 11.9** Design and Detailing Requirements for Columns in Intermediate Moment Frames

Requirement	ACI Section Number(s)
<p>Transverse reinforcement must be proportioned to resist the design shear force <math>\phi V_n</math>, which is the lesser of the following:</p> <ol style="list-style-type: none"> <li>The shear associated with the development of nominal moment strengths of the column at each restrained end of the unsupported length due to reverse curvature bending. Column flexural strength shall be calculated for the factored axial force, consistent with the direction of the lateral forces considered, resulting in the highest flexural strength.</li> <li>The maximum shear obtained from the factored load combinations that include <math>E</math>, with <math>\Omega_o E</math> substituted for <math>E</math>.</li> </ol>	18.4.3.1
Columns shall be spirally reinforced in accordance with ACI Chap. 10 or shall conform to the provisions of ACI 18.4.3.3 through 18.4.3.5.	18.4.3.2
<p>Hoops at a spacing of no more than <math>s_o</math> shall be provided at both ends of a column over a length <math>\ell_o</math> measured from the joint face.</p> <p>The spacing <math>s_o</math> shall not exceed the smallest of:</p> <ol style="list-style-type: none"> <li><math>8 \times</math> diameter of smallest longitudinal bar enclosed</li> <li><math>24 \times</math> diameter of hoop bar</li> <li><math>0.5 \times</math> smallest cross-sectional dimension of the column</li> <li>12 in (300 mm)</li> </ol> <p>The length <math>\ell_o</math> shall not be less than the largest of:</p> <ol style="list-style-type: none"> <li>(Clear span of the column)/6</li> <li>Maximum cross-sectional dimension of the column</li> <li>18 in (450 mm)</li> </ol>	18.4.3.3
The first hoop shall be located not more than $s_o/2$ from the joint face.	18.4.3.4
Spacing of transverse reinforcement outside of the length $\ell_o$ shall conform to ACI 10.7.6.5.2.	18.4.3.5
Columns supporting reactions from discontinuous stiff members, such as walls, shall have transverse reinforcement spaced at $s_o$ (as defined in ACI 18.4.3.3) over their full length below the level at which the discontinuity occurs if the portion of the factored axial compressive force in these members related to earthquake effects exceeds $A_g f'_c/10$ .	18.4.3.6
The limit of $A_g f'_c/10$ shall be increased to $A_g f'_c/4$ where design forces have been magnified to account for the overstrength of the vertical elements of the SFRS.	18.4.3.6
Transverse reinforcement shall extend above and below the column in accordance with ACI 18.7.5.6(b).	18.4.3.6

Limits for longitudinal reinforcement are given in ACI 10.6.1.1. The minimum area of longitudinal reinforcement must not be less than  $0.01A_g$  and must not be greater than  $0.08A_g$ . Providing more than about 2% longitudinal reinforcement in a column is usually not practical or economical.

No restrictions are given on the location of splices of longitudinal reinforcement in columns in intermediate moment frames. However, as discussed in the following paragraphs, the plastic hinge regions are anticipated to form at the ends of the column. Thus, it is good practice to locate lap splices away from these regions.

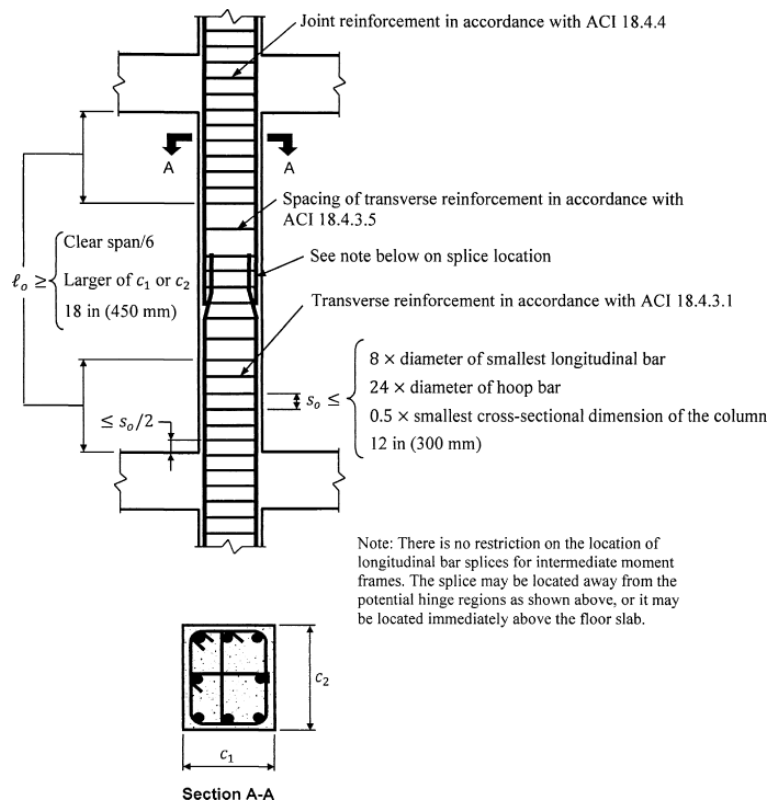
The provisions for determining the design shear strength  $\phi V_n$  for columns in intermediate moment frames is essentially the same as those for columns in ordinary moment frames where the unsupported length is less than or equal to five times the



cross-sectional column in the direction of analysis. Thus, the requirement in ACI 18.4.3.1(a) based on the development of nominal moment strengths is the same as that illustrated in Fig. 11.8 without the limitation related to the unsupported length and cross-sectional dimension of the column. The requirement in ACI 18.4.3.1(b) is similar to that for beams in ACI 18.4.2.3(b) except instead of increasing  $E$  by a factor of 2, it is increased by the overstrength factor  $\Omega_o$ , which for an intermediate moment frame, is equal to 3 from ASCE/SEI Table 12.2-1. The higher factor for columns recognizes their importance with respect to possible shear failure compared to that for beams.

Column ends require adequate confinement (in the form of spirals or hoops) to ensure column ductility in the event of hinge formation during a seismic event (the plastic hinge region is assumed to form over the length  $\ell_o$ ). The transverse reinforcement requirements for columns with hoops and ties are illustrated in Fig. 11.19.

Figure 11.19 Transverse reinforcement requirements for columns in intermediate moment frames.



Outside of the anticipated plastic hinge length  $\ell_o$ , the spacing of the transverse reinforcement must conform to the lateral reinforcement provisions of ACI 25.7.2 for ties and ACI 25.7.3 for spirals, and to the provisions for spacing limits for shear reinforcement of ACI 10.7.6.5.2. The smallest spacing obtained from these requirements is to be used in the center region of a column.

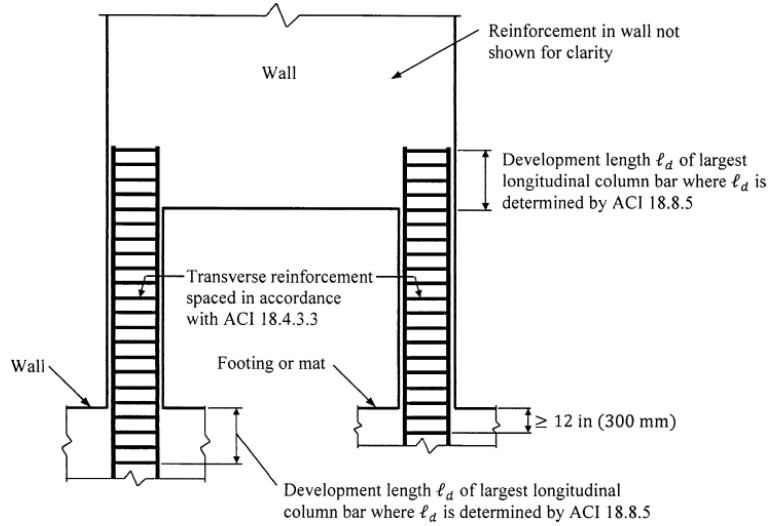
The transverse reinforcement requirements of ACI 18.4.3.6 must be satisfied for columns that support reactions from discontinuous stiff members, such as walls. In cases where the factored axial compressive force related to earthquake effects in such columns exceeds  $A_g f'_c / 4$ , transverse reinforcement must be provided over the entire length of the column at a spacing of  $s_o$ , which is defined in ACI 18.4.3.3. Note that the provisions of ASCE/SEI 12.3.3.3 apply to structural members that support discontinuous frames or shear walls systems where the discontinuity is severe enough to be deemed a structural irregularity. In such cases, the supporting members must be designed to resist the load combinations with overstrength factor  $\Omega_o$  of ASCE/SEI 12.4.3.2, and the limit on the axial compressive force in ACI 18.4.3.6 is  $A_g f'_c / 4$ . Otherwise, the limit is  $A_g f'_c / 10$ .

The transverse reinforcement must extend above into the discontinued member and below into the supporting element in



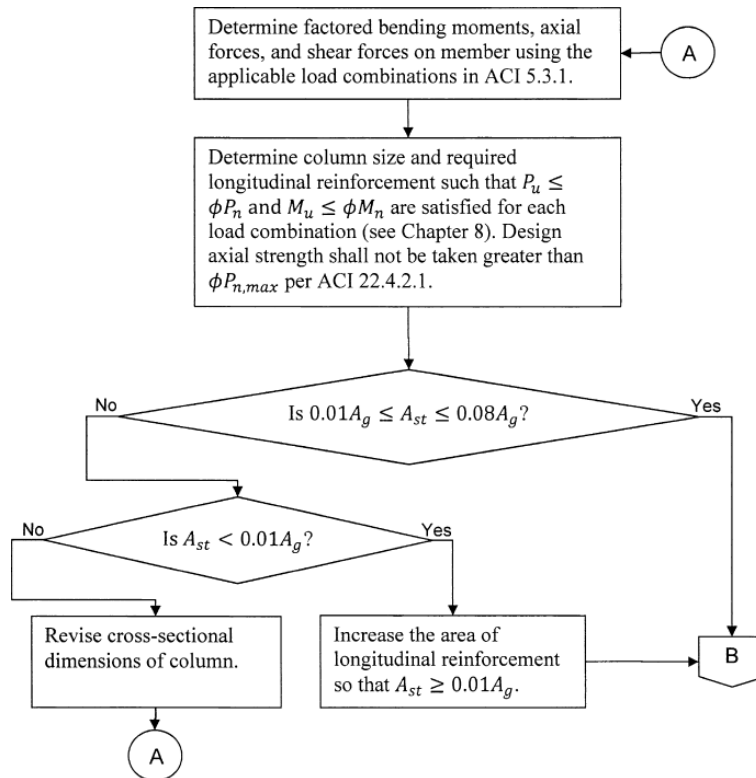
accordance with the provisions of ACI 18.7.5.6(b), which are applicable to similar columns in special moment frames (see Fig 11.20).

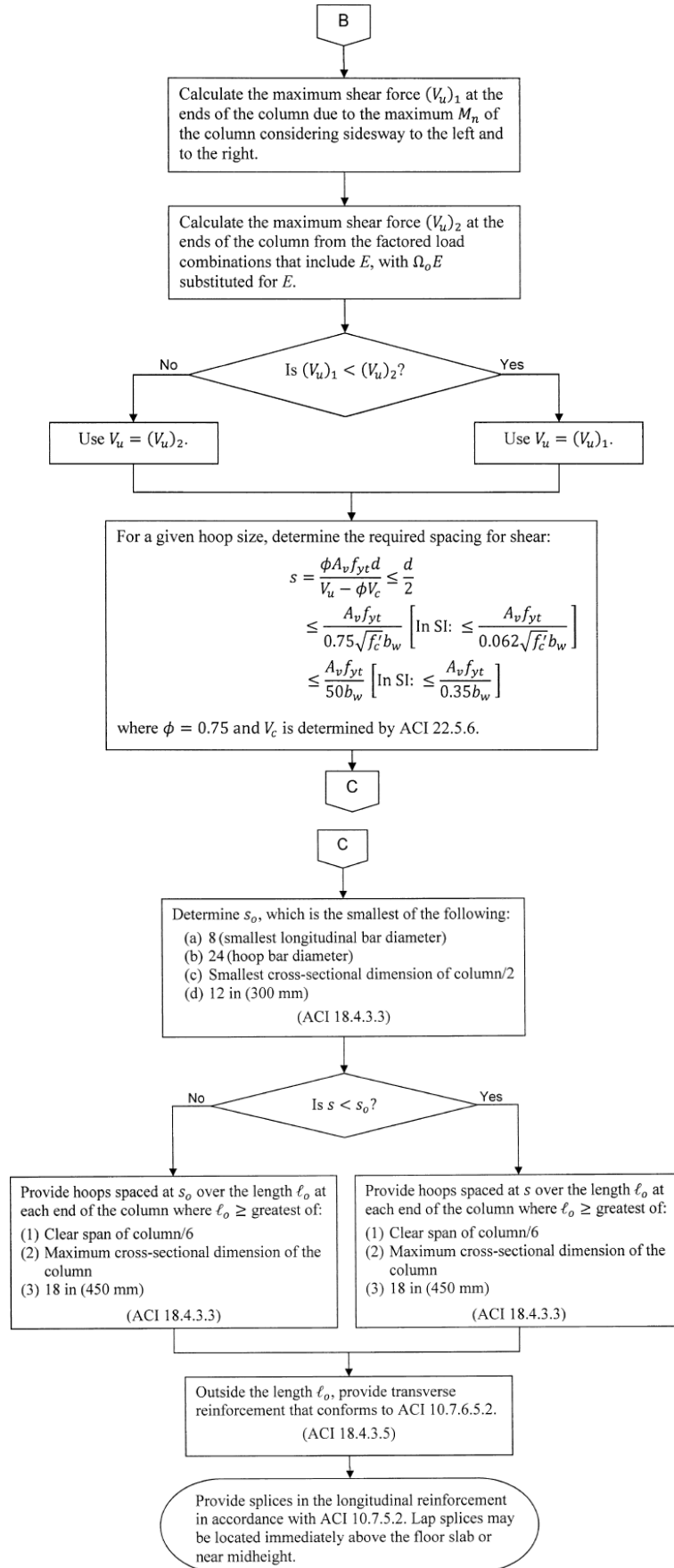
**Figure 11.20** Transverse reinforcement requirements for columns supporting discontinued stiff members in intermediate moment frames.



A summary of the overall design procedure for columns in intermediate moment frames is given in Fig. 11.21.

**Figure 11.21** Design procedure for columns in intermediate moment frames.





**Example 11.6** A 20 × 20 in column supports the first elevated floor level in an intermediate moment frame. The unsupported length of the column is 10 ft. It is supported by a spread footing, and it is assumed that its base is pinned. The design spectral acceleration  $S_{DS}$  has been determined to be 0.40, and the longitudinal reinforcement has been determined to be 12 No. 9 bars (see Fig. 11.22), which are adequate for the

governing load combinations in Table 11.10. Determine the required transverse reinforcement along the height of the column assuming normal-weight concrete with  $f'_c = 4,000 \text{ psi}$  and Grade 60 reinforcement.

Figure 11.22 The column in Example 11.6.

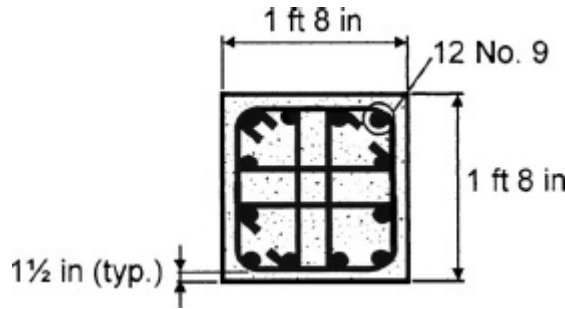


Table 11.10 Summary of Axial Forces, Bending Moments, and Shear Forces for the Column in Example 11.6

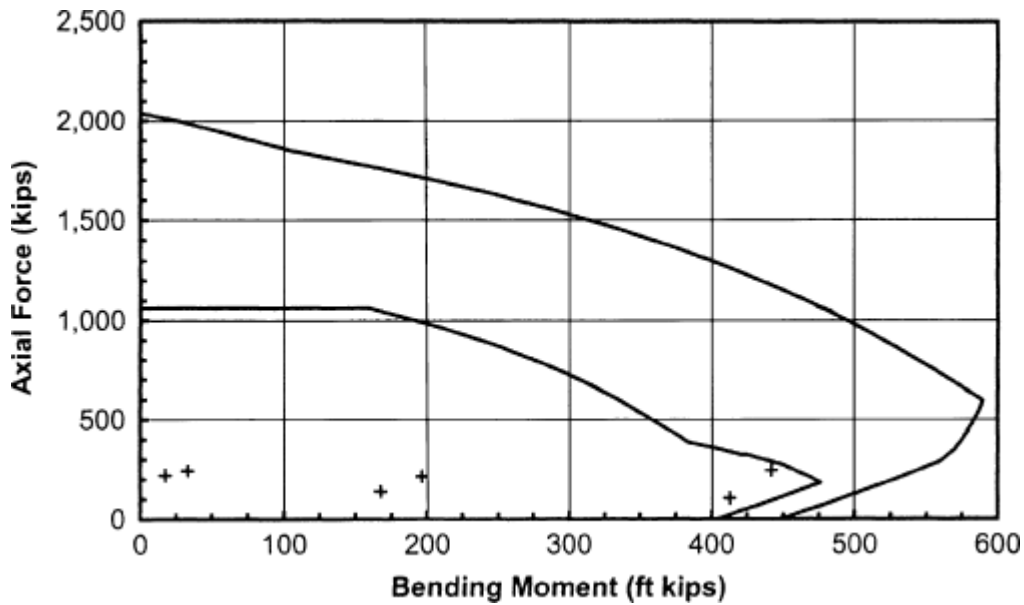
Load Case		Axial Force (kips)	Bending Moment (ft kips)	Shear Force (kips)
Dead ( $D$ )		157.9	11.3	$\pm 2.3$
Live ( $L$ )		31.5	11.0	$\pm 2.2$
Roof live ( $L_r$ )		4.9	—	—
Wind ( $W$ )		$\pm 5.8$	$\pm 177.6$	$\pm 17.8$
Seismic ( $Q_E$ )		$\pm 26.4$	$\pm 422.1$	$\pm 42.2$
<b>Load Combination</b>				
5.3.1a	$1.4D$	221.1	15.8	3.2
5.3.1b	$1.2D + 1.6L + 0.5L_r$	242.3	31.2	6.3
5.3.1d	$1.2D + 0.5(L + L_r) + 1.0W$	213.4	196.7	21.6
5.3.1e	$1.28D + 0.5L + Q_E$	244.3	442.1	46.2
5.3.1f	$0.9D - 1.0W$	136.4	167.4	15.7
5.3.1g	$0.82D - Q_E$	103.1	412.8	40.3

**Solution** The flowchart in Fig. 11.21 will be used to determine the required transverse reinforcement in this column.

**Step 1. Determine the factored bending moments, axial forces, and shear forces in accordance with ACI 5.3.1.** A summary of the applicable load combinations from ACI Table 5.3.1 is given in Table 11.10. ACI Eq. 5.3.1(c) was not included because it is evident that the load effects from this combination do not govern the design of the column.

**Step 2. Determine required column size and longitudinal reinforcement.** Chapter 8 was used to determine that 12 No. 9 bars are adequate for the load combinations in Table 11.10 (see the design interaction diagram for the column in Fig. 11.23; the crosses in the diagram represent the load combinations and all fall within the interior of the design strength interaction diagram).

Figure 11.23 Design and nominal strength interaction diagrams for the column in Example 11.6.



Step 3. Check the minimum and maximum area of longitudinal reinforcement.

$$0.01 \times 20^2 = 4.0 \text{ in}^2 < A_{st} = 12 \times 1.00 = 12.0 \text{ in}^2 < 0.08 \times 20^2 = 32.0 \text{ in}^2$$

Step 4. Calculate design shear strength by ACI 18.4.3.1(a). The largest nominal moment strength  $M_n$  corresponding to load combinations including  $E$  is obtained from ACI Eq. (5.3.1e) (see Table 11.10). For  $P_n = P_u/\phi = 244.3/0.817 = 299.0$  kips,  $M_n = 561.3$  ft kips (see the nominal strength interaction diagram in Fig. 11.23).

Because the column is pinned at its base, the shear force associated with the nominal moment strength is equal to  $(V_u)_1 = M_n/\ell_u = 561.3/10 = 56.1$  kips.

Step 5. Calculate design shear strength by ACI 18.4.3.1(b). As in the first method, the largest shear force is obtained from ACI Eq. (5.3.1e). In this equation,  $\Omega_0 E$  is substituted for  $E$ , which is equal to the following:

$$E = \Omega_0(Q_E + 0.2S_{DS}D) = 3(Q_E + 0.2S_{DS}D) = 3Q_E + (3 \times 0.2 \times 0.4)D = 3Q_E + 0.24D$$

where  $\Omega_0 = 3$  for an intermediate moment frame (see ASCE/SEI Table 12.2-1) and the redundancy coefficient  $\rho = 1.0$  for structures assigned to SDC C.

Therefore,

$$(V_u)_2 = 1.2V_D + 0.5V_L + (3Q_E + 0.24V_D) = (1.44 \times 2.3) + (0.5 \times 2.2) + (3 \times 42.2) = 131.0 \text{ kips}$$

In accordance with ACI 18.4.3.1,  $V_u = 56.1$  kips. Note that this shear force is greater than that obtained from analysis (see Table 11.10).

Step 6. Determine the required hoop spacing. Determine the nominal shear strength of the concrete  $V_c$  by ACI Eq. (22.5.6.1):

$$\begin{aligned} V_c &= 2 \left( 1 + \frac{N_u}{2,000A_g} \right) \lambda \sqrt{f'_c} b_w d = 2 \left( 1 + \frac{244,300}{2,000 \times 20^2} \right) \times 1.0 \sqrt{4,000} \times 20 \times 15.9/1,000 \\ &= 52.5 \text{ kips} \end{aligned}$$

In this equation,  $N_u = 244.3$  kips is the factored axial compressive force on the column corresponding to the maximum shear force and  $d = 15.9$  in was obtained from a strain compatibility analysis of the section using  $N_u = 244.3$  kips.

Because  $\phi V_c = 0.75 \times 52.5 = 39.4$  kips  $< V_u = 56.1$  kips, provide shear reinforcement in accordance with ACI 22.5.10.5.3. Assuming No. 3 hoops and crossties around all the longitudinal bars as shown in Fig. 11.22, the required spacing  $s$  is the following:

$$s = \frac{\phi A_v f_{yt} d}{V_u - \phi V_c} = \frac{0.75 \times (4 \times 0.11) \times 60 \times 15.9}{56.1 - 39.4} = 18.9 \text{ in}$$

Minimum spacing for shear reinforcement is the smaller of the following (ACI 10.6.2.2):

$$s = \begin{cases} \frac{A_v f_{yt}}{0.75 \sqrt{f'_c} b_w} = \frac{0.44 \times 60,000}{0.75 \sqrt{4,000} \times 20} = 27.8 \text{ in} \\ \frac{A_v f_{yt}}{50 b_w} = \frac{0.44 \times 60,000}{50 \times 20} = 26.4 \text{ in (governs)} \end{cases}$$

Also, shear reinforcement must not exceed  $d/2 = 8$  in (ACI 10.7.6.5.2).

The transverse reinforcement requirements of ACI 18.4.3.3 must also be satisfied for intermediate moment frames. The vertical spacing  $s_o$  of No. 3 hoops must not exceed the smallest of the following:

- 8 (smallest longitudinal bar diameter) =  $8 \times 1.128 = 9.0$  in (governs)
- 24 (hoop bar diameter) =  $24 \times 0.375 = 9.0$  in (governs)
- Least cross-sectional dimension of the column/2 = 10 in
- 12 in

where  $\ell_o$  is the greatest of the following:

- Clear span of the column/6 =  $10 \times 12/6 = 20$  in (governs)
- Maximum cross-sectional dimension of the column = 20 in (governs)
- 18 in

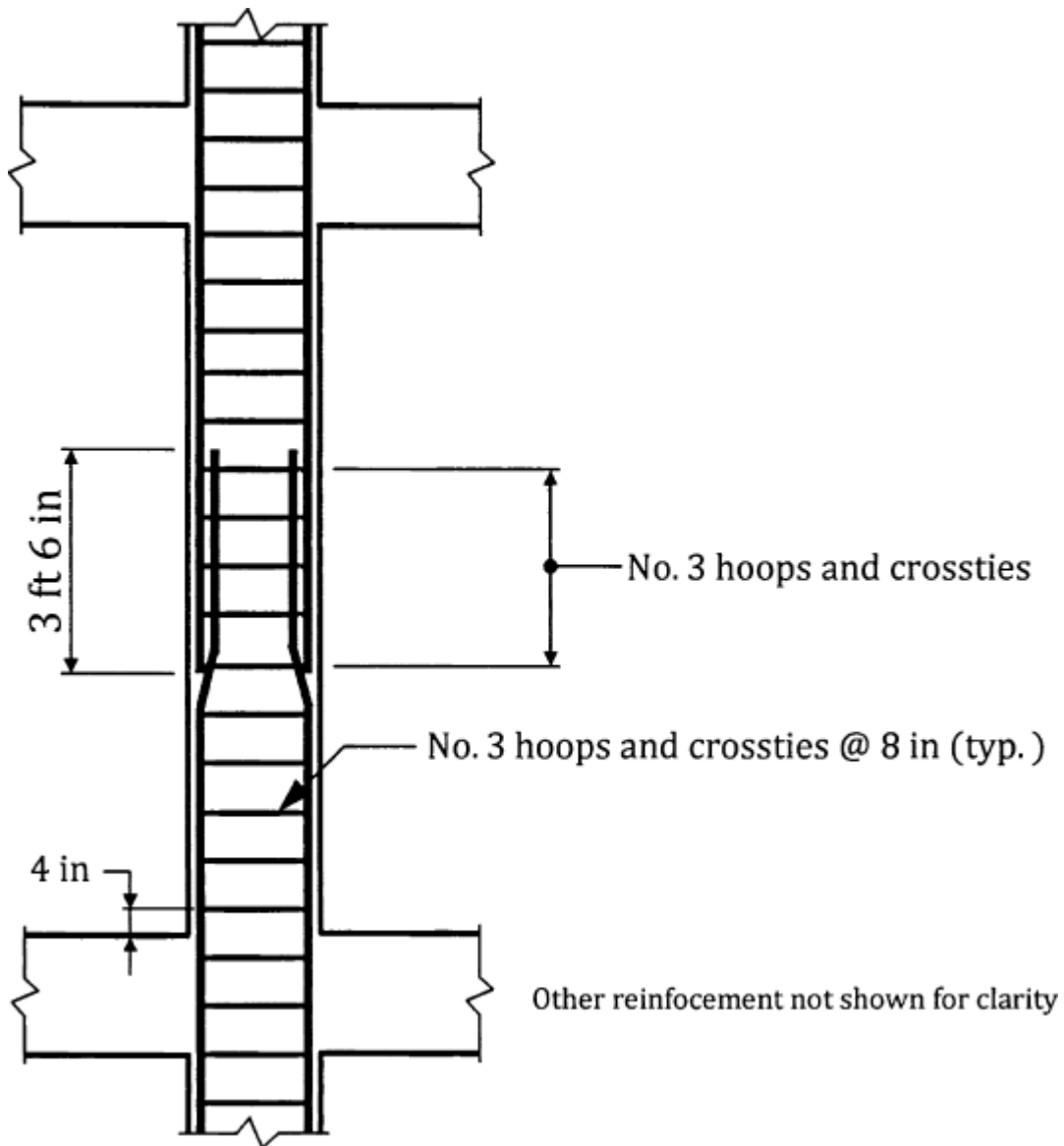
Therefore, use three sets of No. 3 hoops and crossties spaced at 8 in on center with the first set located vertically not more than  $s_o/2 = 8/2 = 4$  in from the face of the joint at each end of the column.

For the remainder of the column, tie spacing shall conform to ACI 10.7.6.5.2. Use No. 3 ties spaced at  $d/2 = 8$  in within this region.

In this example, the lap splices of the longitudinal bars are provided at mid height of the column. From the design strength interaction diagram in Fig. 11.23, it is evident that the bar stress corresponding to at least one factored load combination is greater than  $0.5 f_y$ . Thus, a Class B lap splice must be provided in accordance with ACI 25.5. It can be determined that the required lap splice length is equal to 3 ft 6 in. Similar to the anticipated plastic hinge regions, No. 3 hoops and crossties are provided over the entire lap length to adequately confine the splice.

The reinforcement details for the column in this intermediate moment frame are shown in Fig. 11.24.

Figure 11.24 Reinforcement details for the column in Example 11.6.



## 11.6.4. Joints

Transverse reinforcement conforming to ACI Chap. 15 must be provided within the joints of intermediate moment frames. Such confinement is essential to ensure that the flexural strength of the beams and columns can be developed without deterioration of the joint under loading cycles from an earthquake.

Detailing provisions are given in ACI 15.4. For beam-column joints that are not laterally supported by beams of approximately equal depth on all four sides of the joint and for joints which are part of a SFRS, the total area of transverse reinforcement in a joint must be at least the greater of the following (ACI 15.4.2):

$$\frac{0.75\sqrt{f'_c}bs}{f_{yt}} \left[ \text{In SI: } \frac{0.062\sqrt{f'_c}bs}{f_{yt}} \right]$$

$$\frac{50bs}{f_{yt}} \left[ \text{In SI: } \frac{0.35\sqrt{f'_c}bs}{f_{yt}} \right]$$

In these equations,  $b$  is the dimension of the column section perpendicular to the direction of analysis and  $s$  is the spacing of the transverse reinforcement in the joint, which must be less than or equal to one-half the depth of the shallowest beam that frames into the joint.

The equations above are the same for minimum shear reinforcement in columns (ACI 10.6.2.2). To simplify detailing, it is common for the transverse reinforcement that is required at the ends of a column to be carried through the joint as well. In [Example 11.6](#), the No. 3 hoops and crossties spaced at 8 in could be provided through the joint for simplicity instead of increasing the spacing based on the equations above.

Where longitudinal beam or column reinforcement is spliced or terminated in a joint that is not restrained by a beam or slab in accordance with ACI 15.2.4 or 15.2.5, respectively, closed transverse reinforcement in accordance with ACI 10.7.6 must be provided in the joint (i.e., the transverse reinforcement required in the column must be provided in the joint). This helps in ensuring that the flexural strength of the member can be developed and maintained under repeated loads. The provisions of ACI 25.4 related to the development of reinforcement are to be satisfied for longitudinal bars that terminate in a joint.

## 11.6.5. Two-Way Slabs Without Beams

Two-way slabs without beams may be considered as part of the SFRS in intermediate moment frames in buildings assigned to SDC B and C, but are not permitted in SDC D or above. A summary of the requirements is given in [Table 11.11](#).

**Table 11.11** Design and Detailing Requirements for Two-Way Slabs Without Beams in Intermediate Moment Frames

Requirement	ACI Section Number(s)
Load combinations defined by ACI Eqs. (5.3.1e) and (5.3.1g) must be included in the design of the slab. All reinforcement provided to resist $M_{sc}$ , the portion of the factored slab moment balanced by the supporting members at a joint, must be placed within the column strip defined in ACI 8.4.1.5.	18.4.5.1
Reinforcement placed within an effective slab width $b_{slab}$ defined in ACI 8.4.2.3.3 must be proportioned to resist the moment $\gamma_f M_{sc}$ where $\gamma_f$ is determined by ACI Eq. (8.4.2.3.2). Effective slab widths for exterior and corner connections must not extend beyond the column face a distance greater than $c_f$ measured perpendicular to the slab span.	18.4.5.2
At least one-half of the reinforcement in the column strip at the support shall be placed within the effective slab width $b_{slab}$ defined in ACI 8.4.2.3.3.	18.4.5.3
At least one-quarter of the top reinforcement at the support in the column strip shall be continuous throughout the span.	18.4.5.4
Continuous bottom reinforcement in the column strip shall be at least one-third of the top reinforcement at the support in the column strip.	18.4.5.5
At least one-half of all bottom middle strip reinforcement and all bottom column strip reinforcement at midspan shall be continuous and shall develop $f_y$ at the face of the support as defined in ACI 8.10.3.2.1.	18.4.5.6
At discontinuous edges of the slab, all top and bottom reinforcement at the support shall be developed at the face of the support as defined in ACI 8.10.3.2.1.	18.4.5.7
At the critical sections for columns defined in ACI 22.6.4.1, two-way shear caused by factored gravity loads shall not exceed $0.4\phi V_c$ where $V_c$ is calculated by ACI 22.6.5. This requirement need not be satisfied if the slab design satisfies the provisions of ACI 18.14.5.	18.4.5.8

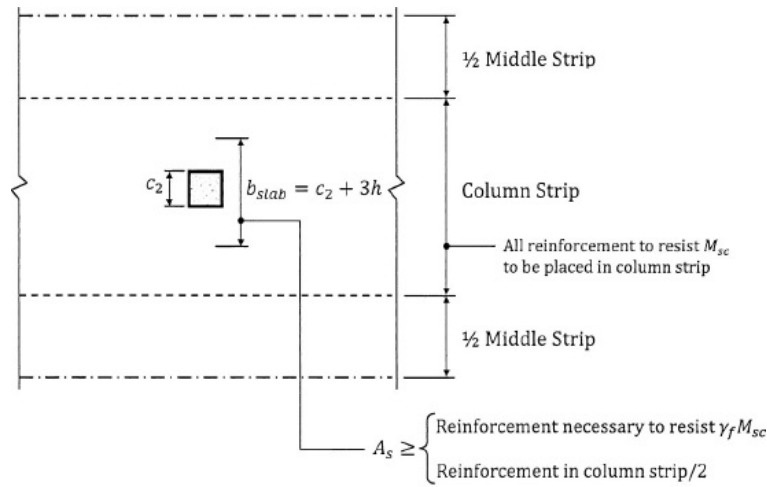
Two-way slabs are to be designed for flexure in accordance with ACI Chap. 8 using the appropriate load combinations in ACI Table 5.3.1. Like beams, it is good practice to design two-way slabs in an intermediate moment frame as tension-controlled sections.

Load combinations defined by ACI Eqs. (5.3.1e) and (5.3.1g) include earthquake effects  $E$ . Because sidesway to the right and sidesway to the left must be taken into account, it is possible a net positive moment may be introduced at the support. Depending on the magnitude of the net positive moment, more bottom reinforcement may be needed than that which would otherwise be provided at the support.

In regards to moment transfer by flexure at supports, the factored bending moment  $M_{sc}$  refers to that portion of the factored slab moment that is balanced by the supporting members at a joint for a given design load combination that includes seismic load effect  $E$  acting in a particular horizontal direction; it is not necessarily equal to the total design moment at the support. Only a fraction of this moment  $\gamma_f M_{sc}$  is assigned to the effective slab width  $b_{slab}$  where  $\gamma_f$  is the factor determined by ACI Eq. (8.4.2.3.2) that is used to determine the fraction of  $M_{sc}$  transferred by flexure at slab-column connections (see Fig. 11.25).



Figure 11.25 Reinforcement details at supports of two-way slabs without beams.

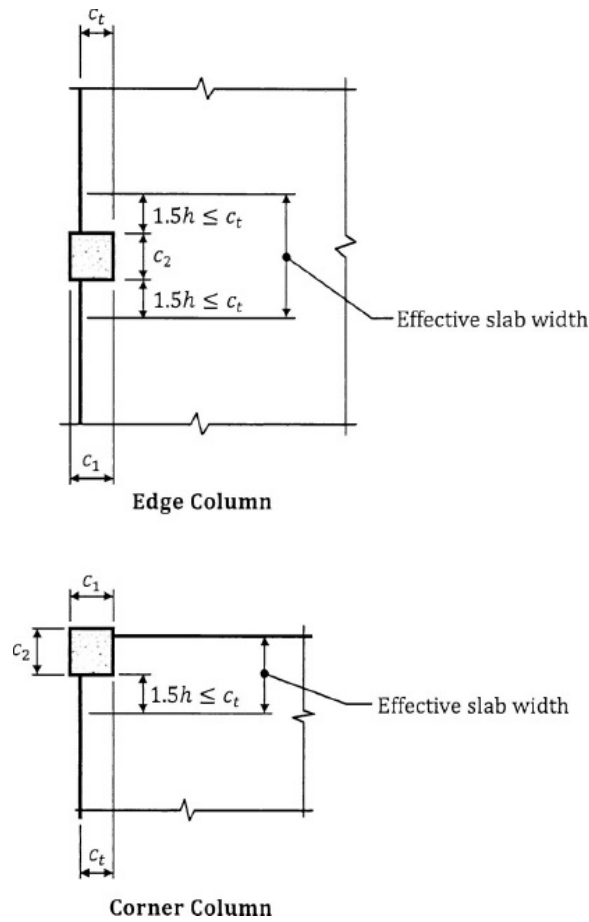


Note: Applies to both top and bottom reinforcement

The reinforcement to be placed in  $b_{slab}$  for moment transfer is the greater of that which is required to resist  $\gamma_f M_{sc}$  and one-half the reinforcement required in the entire column strip. When two-way slab systems are part of the SFRS system, distribution of moment transfer reinforcement at interior columns and at edge columns bending parallel to an edge depends on the ratio of the factored moments from gravity loads to factored moments from seismic loads in the slab. For ratios greater than 1, the combined moment in the slab on each face of the support is negative, and all of the moment transfer reinforcement should be placed at the top of the slab. However, for ratios less than 1, the combined moment is positive on one face of the support and negative on the other face. In this situation, it would be prudent to divide moment transfer reinforcement between the top and bottom of the slab, with the top and bottom reinforcement continuous over the column to account for moment reversals.

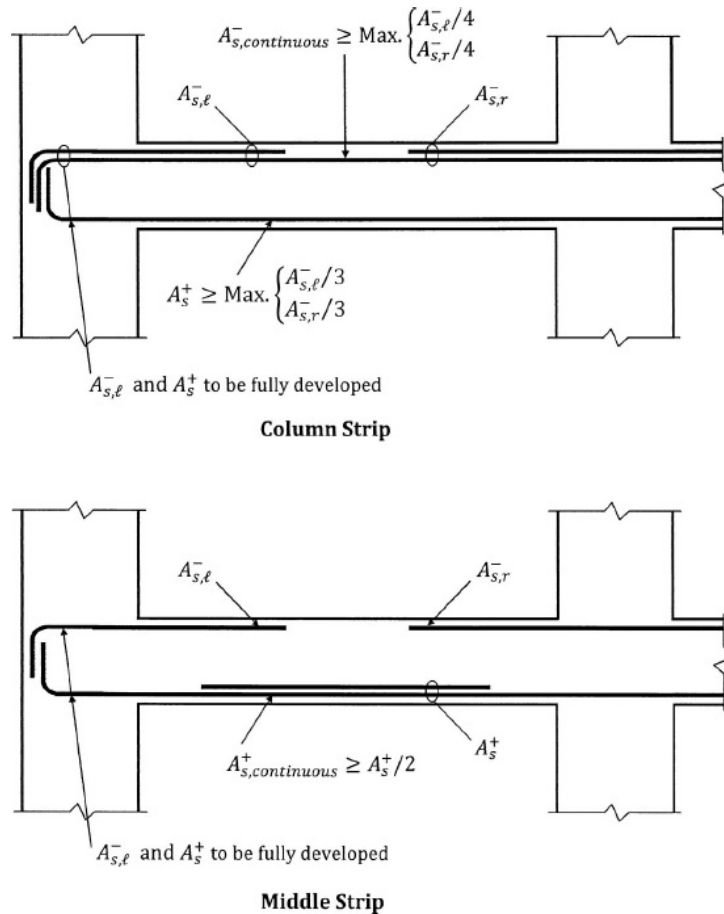
The provisions for effective slab width for flexural reinforcement placement in edge and corner columns are illustrated in Fig. 11.26. Flexural reinforcement that is perpendicular to the edge is not considered to be fully effective unless it is placed within the effective slab width. The portion of the effective slab width that extends beyond the face of the column is the smaller of  $1.5h$  or  $c_t$ , where  $c_t$  is the distance from the interior face of the column to the slab edge in the direction of analysis.

Figure 11.26 Effective slab width for reinforcement placement in edge and corner connections.



Reinforcement details in accordance with ACI 18.4.5.4 through 18.4.5.7 for the column strips and middle strips in two-way slabs without beams in intermediate moment frames are given in Fig. 11.27.

Figure 11.27 Reinforcement details in two-way slabs without beams.



It has been demonstrated that slab–column frames are susceptible to punching shear failures and reduced lateral displacement ductility during seismic events where shear stresses due to gravity loads are relatively large.<sup>8</sup> Thus, a limit of  $0.4\phi V_c$  is prescribed in ACI 18.4.5.8 on the shear caused by factored gravity loads. This limit helps to enable the slab–column connection to have adequate toughness to withstand the anticipated inelastic moment transfer. The requirements of ACI 18.4.5.8 are permitted to be waived where the slab design satisfies the requirements of ACI 18.14.5, which are applicable to slab–column connections of two-way slabs without beams that are not designated as part of the SFRS. Note that the slab–column connections must also satisfy the shear and moment strength requirements of ACI Chap. 8 (see [Chap. 7](#)).

## 11.7. Special Moment Frames

### 11.7.1. Overview

Provisions for design and detailing of special moment frames, which are permitted to be used in structures assigned to SDC D, E, and F with no limitations (see ASCE/SEI Table 12.2-1), are given in ACI 18.6 (beams), 18.7 (columns), and 18.8 (joints). The provisions for strength reduction factors, material properties, and mechanical and welded splices in ACI 18.2.4 through 18.2.8 are also applicable to special moment frames (see [Sections 11.4.2, 11.4.3, and 11.4.4](#)).

One of the main challenges in the design of special moment frames occurs at the joints of the frame. It will become evident from the discussions in [Section 11.7.4](#) that the provisions of ACI 18.8 have a major impact on proportioning and detailing the members in a special moment frame.

### 11.7.2. Beams

## 11.7.2. DETAILS

The provisions for beams that are part of the SFRS in a special moment frame are given in ACI 18.6. These provisions have been developed on the assumption that the beams of the special moment frame are horizontal and that the columns are vertical. It is acceptable for the beams and columns to be inclined from the horizontal and vertical, respectively, provided the resulting system behaves as a frame, that is, lateral resistance is provided primarily by moment transfer between beams and columns rather than by other means (such as strut or brace action). It is also acceptable for beams of special moment frames to frame into a boundary of a wall provided the boundary is reinforced the same as a column in a special moment frame in accordance with ACI 18.7. Note that beams that cantilever off of the columns of special moment frames are not considered to be part of the SFRS.

Table 11.12 contains a summary of the requirements in ACI 18.6. Note that these provisions are applicable to all beams in special moment frames regardless of the magnitude of the axial compressive force on the beam.

**Table 11.12** Design and Detailing Requirements for Beams in Special Moment Frames

Requirement		ACI Section Number(s)
Dimensional Limits	Clear span $\ell_n \geq 4d$	18.6.2.1(a)
	Width of member $b_w \geq$ lesser of $0.3h$ and 10 in (250 mm)	18.6.2.1(b)
	Projection of the beam width beyond the width of the supporting column $\leq$ lesser of $c_2$ and $0.75c_1$	18.6.2.1(c)
Flexure	Design flexural members as tension-controlled sections.	21.2.2, 22.3
	At least two bars shall be provided continuously at both the top and bottom of a section.	18.6.3.1
	At any top and bottom section, the provided amount of reinforcement must be greater than or equal to the minimum reinforcement per ACI 9.6.1.2: $\frac{3\sqrt{F'_c b_w d}}{f_y} \text{ and } \frac{200b_w d}{f_y}$ $\left[ \text{In SI: } \frac{0.25\sqrt{F'_c b_w d}}{f_y} \text{ and } \frac{1.4b_w d}{f_y} \right]$ Also, the reinforcement ratio $\rho$ shall not exceed 0.025.	18.6.3.1
	Positive moment strength at the face of a joint must be greater than or equal to one-half the negative moment strength at that face of the joint.	18.6.3.2
	Neither the negative nor the positive moment strength at any section along the length of the beam must be less than one-fourth the maximum moment strength at the face of either joint.	18.6.3.2

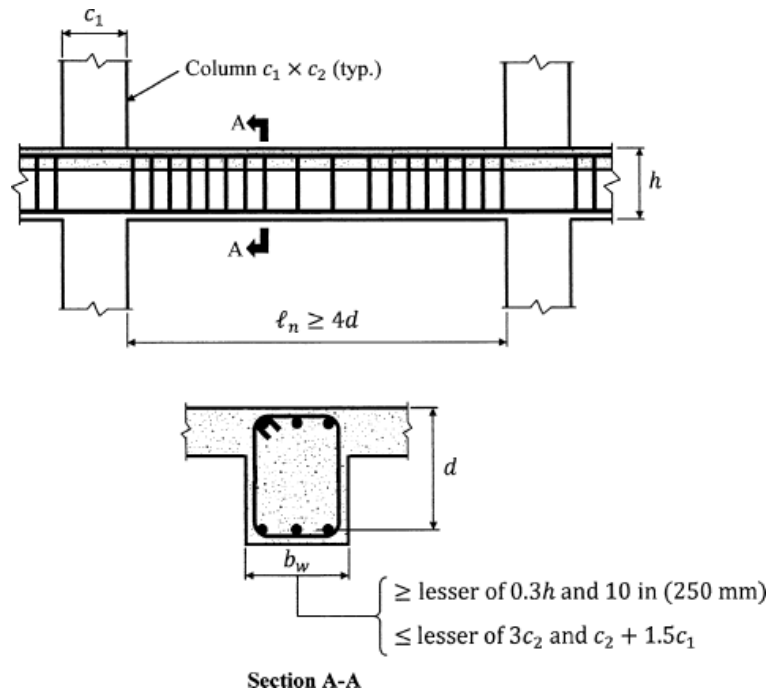
Requirement		ACI Section Number(s)
Splices	<p>Lap splices of flexural reinforcement are permitted only if hoop or spiral reinforcement is provided over the lap length. Hoop or spiral reinforcement spacing shall not exceed the smaller of <math>d/4</math> or 4 in (100 mm).</p> <p>Lap splices shall not be used:</p> <ol style="list-style-type: none"> <li>Within joints</li> <li>Within a distance of <math>2h</math> from the face of the joint</li> <li>Within a distance of <math>2h</math> from critical sections where analysis indicates flexural yielding is likely to occur as a result of lateral displacements beyond the elastic range of behavior.</li> </ol>	18.6.3.3
	Mechanical splices shall conform to ACI 18.2.7 and welded splices shall conform to ACI 18.2.8.	18.6.3.4
Transverse Reinforcement	<p>Hoops are required in the following regions:</p> <ol style="list-style-type: none"> <li>Over a length equal to at least <math>2h</math> from the face of the supporting column toward midspan at both ends of the beam.</li> <li>Over lengths equal to at least <math>2h</math> on both sides of a section where flexural yielding is likely to occur as a result of lateral displacements beyond the elastic range of behavior.</li> </ol>	18.6.4.1
	Where hoops are required, primary longitudinal reinforcing bars closest to the tension and compression faces shall have lateral support in accordance with ACI 25.7.2.3 and 25.7.2.4. The spacing of transversely supported flexural reinforcing bars shall not exceed 14 in (350 mm). Skin reinforcement required by ACI 9.7.2.3 need not be laterally supported.	18.6.4.2
	<p>Hoops in flexural members may be made up of two pieces of reinforcement: (1) a stirrup having seismic hooks at both ends and (2) a crosstie as defined in ACI 2.3.</p> <p>Consecutive crossties engaging the same longitudinal bar shall have the 90-degree hooks at opposite sides of the flexural member.</p> <p>In cases where the longitudinal reinforcement secured by crossties is confined by a slab on only one side of the flexural member, the 90-degree hooks of the crossties shall be placed on that side.</p>	18.6.4.3
	<p>Where hoops are required, the spacing shall not exceed the least of the following:</p> <ol style="list-style-type: none"> <li><math>d/4</math></li> <li><math>6 \times</math> diameter of the smallest primary flexural reinforcing bars excluding longitudinal skin reinforcement required by ACI 9.7.2.3</li> <li>6 in (150 mm)</li> </ol> <p>The first hoop shall be located not more than 2 in (50 mm) from the face of the supporting column.</p>	18.6.4.4
	Where hoops are required, they shall be designed to resist the shear determined in accordance with ACI 18.6.5.	18.6.4.5
	Where hoops are not required, stirrups with seismic hooks at both ends shall be spaced not more than $d/2$ throughout the length of the beam.	18.6.4.6

Requirement		ACI Section Number(s)
	<p>Where the factored axial compressive force on a beam exceeds <math>A_g f'_c/10</math>, hoops satisfying ACI 18.7.5.2 through 18.7.5.4 shall be provided along the lengths given in ACI 18.6.4.1.</p> <p>Along the remaining length of the beam, hoops satisfying ACI 18.7.5.2 shall be spaced at a distance not exceeding the lesser of the following:</p> <ol style="list-style-type: none"> <li>6 × diameter of the smallest longitudinal beam bars</li> <li>6 in (150 mm)</li> </ol> <p>Where concrete cover over transverse reinforcement exceeds 4 in (100 mm), additional transverse reinforcement shall be provided with cover less than or equal to 4 in (100 mm) and spacing less than or equal to 12 in (300 mm).</p>	18.6.4.7
Shear	<p>Design shear force <math>V_e</math> is to be calculated from consideration of the forces on the portion of the beam between faces of the joints. It is assumed that moments of opposite sign corresponding to the probable flexural strength <math>M_{pr}</math> act at the joint faces and that the beam is loaded with the factored tributary gravity load along its span.</p>	18.6.5.1
Shear	<p>Transverse reinforcement over the lengths defined in ACI 18.6.4.1 shall be designed to resist shear forces assuming <math>V_c = 0</math> when both (a) and (b) occur:</p> <ol style="list-style-type: none"> <li>The earthquake-induced shear force calculated by ACI 18.6.5.1 represents one-half or more of the maximum required shear strength within those lengths.</li> <li>The factored axial compressive force <math>P_u</math>, including earthquake effects, is less than <math>A_g f'_c/20</math>.</li> </ol>	18.6.5.2

### 11.7.2.1. Dimensional Limits

The dimensional limits of ACI 18.6.2.1, which are summarized in Fig. 11.28, have been guided by experimental evidence and observations of reinforced concrete frames that have performed well in the past. In particular, experiments have shown that when subjected to reversal of displacement into the nonlinear range, continuous beams with length-to-depth ratios less than 4 behave significantly different than beams with ratios greater than or equal to 4, especially in regard to shear strength. Therefore, it is important to ensure the dimensional limits of this section are satisfied.

Figure 11.28 Dimensional limits of beams in special moment frames.



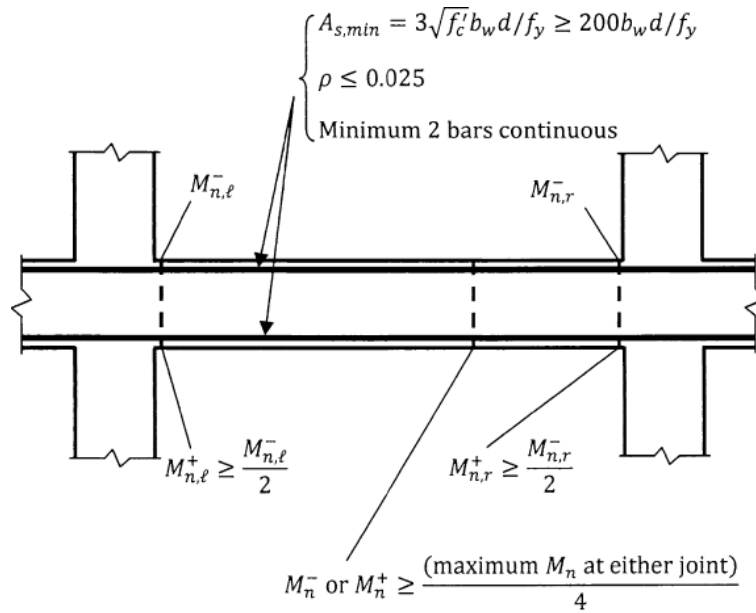
The dimensional limit in ACI 18.6.2.1(c) gives the maximum beam width that can effectively transfer forces into the beam-column joint. ACI Fig. R18.6.2 illustrates an example of maximum effective beam width where the beam is wider than the supporting column. Included is the transverse reinforcement required in the portions of the beam outside of the column.

## 11.7.2.2. Design for Flexure

Table 11.12 includes some of the requirements for flexure in ACI Chap. 9, which must be satisfied for all beams. As usual, it is good practice to design the beams in a special moment frame as tension-controlled sections. Chap. 6 of this book can be used to determine the required amount of flexural reinforcement for the applicable load combinations in ACI Table 5.3.1. Note that the upper limit for the amount of flexural reinforcement given in ACI 18.6.3.1, which is  $\rho = A_s/b_w d = 0.025$ , is different than that in ACI 9.3.3.1, which is based on the strain limit in the reinforcement (for comparison purposes, the to 366pt maximum reinforcement ratio corresponding to a strain of 0.004 in the extreme tension reinforcement for a beam with 4,000 psi concrete and Grade 60 reinforcement is 0.021). The purpose of the maximum reinforcement ratio is primarily to reduce steel congestion and to limit shear stresses in beams of typical proportions. It is shown in Section 11.7.4 that the magnitude of the shear force that needs to be resisted in a beam-column joint is directly proportional to the amount of flexural reinforcement that passes through the joint from the beams. For this reason, as well as for the others stated previously, the beam flexural reinforcement ratio should be kept as small as practical.

ACI 18.6.3.2 requires minimum positive moment strength at the ends of the beam equal to at least 50% of the corresponding negative moment strength at that face of the joint. Like for intermediate moment frames, this allows for the possibility that the positive moment due to earthquake-induced lateral displacements may exceed the negative moment due to gravity loads. Also, minimum moment capacity at any section must be at least 25% of the maximum moment strength at the face of either joint. These requirements ensure strength and ductility under large lateral displacements. A summary of the flexural requirements for beams in special moment frames is given in Fig. 11.29.

Figure 11.29 Flexural requirements for beams in special moment frames.

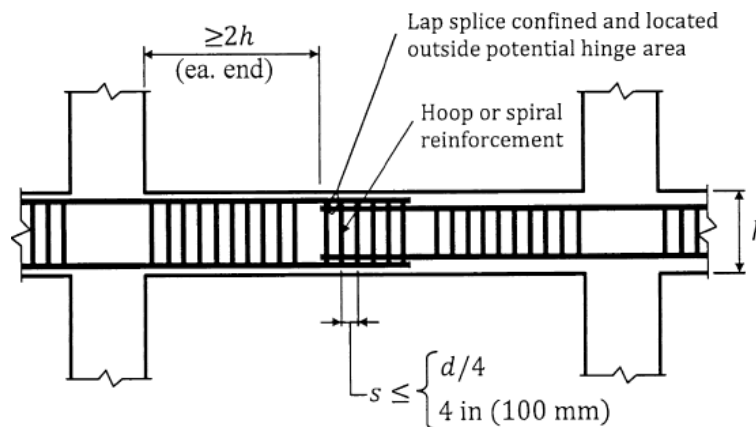


Note: transverse reinforcement not shown for clarity

### 11.7.2.3. Splices of Longitudinal Reinforcement

Unlike intermediate moment frames where there are essentially no restrictions on the location of lap splices of the longitudinal reinforcement in a beam, ACI 18.6.3.3 contains specific requirements for such splices in beams of special moment frames. According to that section, lap splices are permitted as long as they are properly confined with hoop or spiral reinforcement over the entire lap length and that they are located away from potential hinge areas. It is possible that the concrete cover surrounding the splice may spall off under conditions of cyclic loading into the inelastic range, so it is very important that proper confinement is provided over the entire lap splice length so that it can perform as intended. Provisions for lap splices are illustrated in Fig. 11.30.

Figure 11.30 Lap splice requirements for beams of special moment frames.



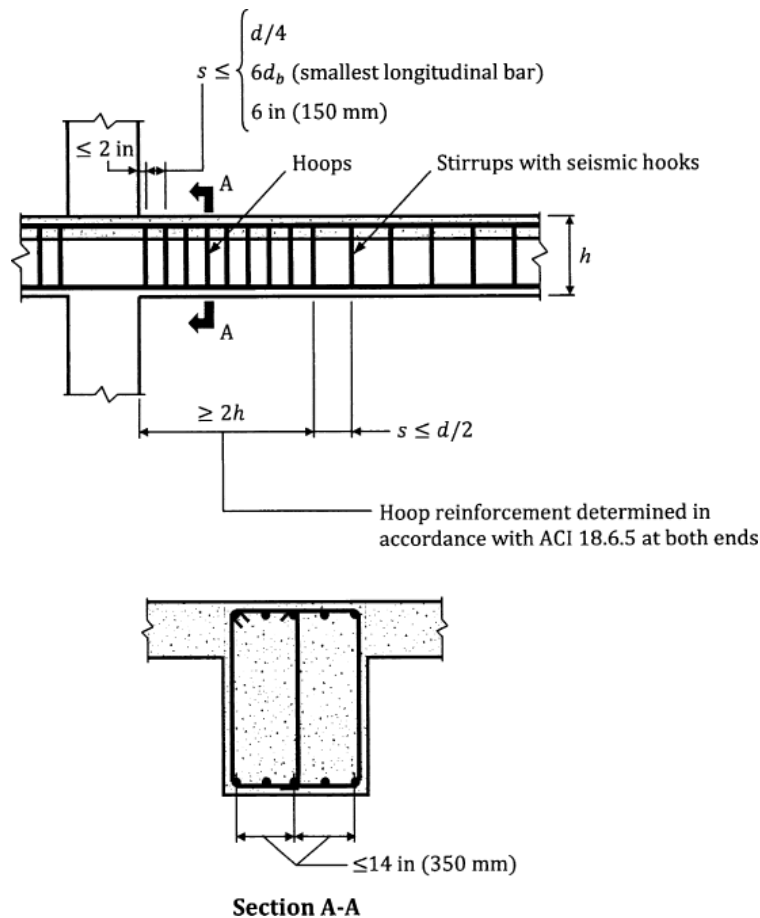
In lieu of lap splices, mechanical and welded splices conforming to ACI 18.2.7 and 18.2.8, respectively, may be used (ACI 18.6.3.4).

### 11.7.2.4. Transverse Reinforcement



A summary of the requirements of ACI 18.6.4.4 for transverse reinforcement is given in Fig. 11.31. One of the main goals in the design of beams in special moment frames is to restrict flexural yielding to specific locations in the span. It is advantageous for the regions of flexural yielding to occur at the ends of the beams adjacent to the beam–column joints: As the building moves back and forth during an earthquake, the plastic hinges undergo reversing cycles of yielding at those locations and no other hinges would likely form anywhere else along the span. It is less desirable for plastic hinges to form at the ends of the beam and at locations in the span; in such cases, which usually occur in beams with long spans and/or relatively large gravity loads compared to the effects from the earthquake, multiple hinges can form at different locations where the positive moment is greatest (due to reversal of frame movement during the earthquake) leading to large rotations and large vertical deflections.

Figure 11.31 Transverse reinforcement requirements for beams of special moment frames.



Adequate confinement of the concrete is required at ends of beams where plastic hinges are likely to form to ensure sufficient ductility of the beams thereby avoiding shear failure. In addition to confining the concrete, transverse reinforcement also assists the concrete in resisting shear forces and maintains lateral support for the longitudinal reinforcing bars.

In regions where flexural yielding is expected, hoops must be used. Hoops are defined in ACI 2.3 as closed ties or continuously wound ties, made up of one or several reinforcement elements having seismic hooks at both ends. ACI 18.6.4.3 permits hoops to be made up of two pieces of reinforcement: (1) a stirrup having seismic hooks at both ends and (2) a crosstie as defined in ACI 2.3 (see Fig. 11.11). The 135-degree hooks at the ends of the bars, which are embedded in the core of the beam, prevent the hoops from opening if the concrete cover spalls off.

The required size and spacing of the hoops at the ends of the beam are determined based on the shear requirements in ACI 18.6.5, which are covered next. For a given hoop size, the required spacing based on shear requirements is compared to the

minimum spacing depicted in Fig. 11.31 and the smaller of the two is provided at both ends of the beam for a distance of at least two times the overall depth of the beam.

The spacing between longitudinal bars that are restrained by legs of crossties or hoops is limited to 14 in (350 mm) [ACI 18.6.4.2]. This provision helps to ensure that proper lateral support is provided for such bars in case they are subjected to compressive forces under moment reversals.

For beams that are subjected to a factored axial compressive force exceeding  $A_g f'_c / 10$ , the anticipated plastic hinge regions must contain hoops that satisfy ACI 18.7.5.2 through 18.7.5.4, which are the transverse reinforcement requirements for columns in special moment frames (ACI 18.6.4.7). These requirements are more stringent than those for beams in special moment frames where  $P_u \leq A_g f'_c / 10$  in order to account for the more brittle behavior that is expected when  $P_u$  is relatively large.

### 11.7.2.5. Shear Requirements

In the event of a design-level earthquake, it is assumed that beams in a special moment frame will yield in flexure. As noted previously, regions of flexural yielding are assumed to form at both ends of the beam over an approximate length of  $2h$  from the face of the columns. In order to assure that a beam will yield in flexure before it can fail in shear, the required shear force should be based on the maximum shear that may develop in the beam. Therefore, instead of designing the required shear reinforcement based solely on the results from a structural analysis using factored load combinations, shear design for beams in a special moment frame is related to the maximum flexural strength that can be developed in the beam, which is defined in ACI 18.6.5.1 as the probable flexural strength  $M_{pr}$ .

The probable flexural strength  $M_{pr}$  is associated with plastic hinging in a beam, and is defined as the strength of a flexural member based on the properties of the member at the joint faces assuming the tensile stress in the longitudinal reinforcing steel is equal to  $1.25f_y$  and the strength reduction factor  $\phi$  is equal 1.0 (ACI 2.2):

$$M_{pr} = A_s(1.25f_y) \left( d - \frac{a}{2} \right)$$

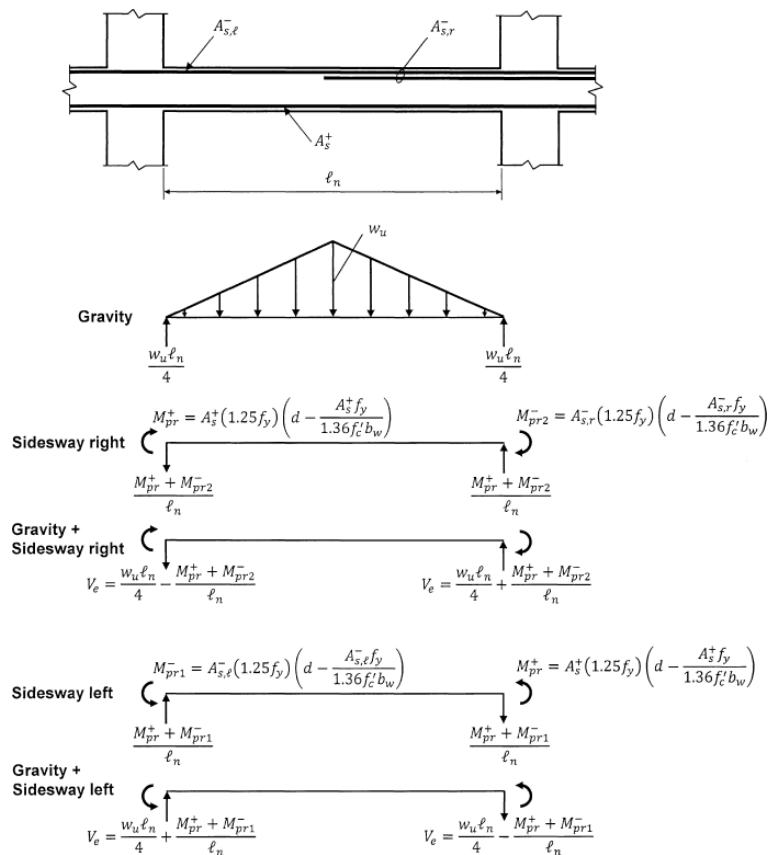
(11.5)

where  $a = A_s(1.25f_y) / 0.85f'_c b_w$ .

The reasons for using 125% of the yield strength  $f_y$  in determining  $M_{pr}$  are twofold: (1) the actual yield strength of the longitudinal reinforcement may exceed the specified yield strength and (2) strain hardening of the longitudinal reinforcement is likely to occur at a joint that undergoes large rotations, which would be expected when the structure is experiencing the design-level earthquake. It has been shown that reinforced concrete beams that are subjected to alternating nonlinear displacements require more shear reinforcement to ensure flexural failure than reinforced concrete beams subjected to displacements in one direction.<sup>9</sup>

Figure 11.32 illustrates the computation of design shear forces  $V_e$  for a beam with a factored triangular gravity load, which is common for framing systems with beam-supported slabs. As shown in the figure, sidesway to the right and sidesway to the left must both be considered to obtain the maximum shear force. Similar equations can be derived for other distributions of gravity load.

Figure 11.32 Design shear forces for beams in special moment frames.



Shear strength in the regions of flexural yielding is provided by both concrete  $V_c$  and transverse reinforcement  $V_s$  in the form of hoops. However, according to ACI 18.6.5.2,  $V_c$  must be taken equal to zero over the lengths identified in ACI 18.6.4.1 when both of the following two conditions occur:

- The earthquake-induced shear force  $(M_{pr}^+ + M_{pr}^-) / \ell_n$  is greater than or equal to one-half of the maximum required shear strength  $V_e$ .
- The factored axial force  $P_u$  on the beam, which includes seismic effects, is less than  $A_g f_c' / 20$ .

This requirement does not imply that concrete does not contribute to shear capacity. In fact, the concrete core that is confined by the surrounding transverse reinforcement plays a vital role in the overall shear strength of a beam. Reducing the dimensions of the concrete core just because the provisions of ACI 18.6.5.2 discount the shear strength of the concrete  $V_c$  is not warranted. Tests have shown that shear strength decreases in beams when subjected to multiple reversals in inelastic deformation, especially when axial forces are low.

The provisions of ACI 18.6.5.2 must be checked over the entire lengths identified in ACI 18.6.4.1. Because the shear force due to earthquake effects is constant along the span, it is possible for concrete to contribute to the total shear capacity at sections near the face of the support where shear forces due to gravity are relatively large and not to contribute at sections away from the support where shear forces due to gravity are smaller. In such cases,  $V_c$  should be set equal to zero over the entire lengths identified in ACI 18.6.4.1. Note that  $V_s$  is limited to  $8 \sqrt{f_c' b_w d}$  (In SI:  $0.66 \sqrt{f_c' b_w d}$ ) regardless if  $V_c$  is included or not (ACI 22.5.1.2).

It is evident from the above discussion that the amount of transverse reinforcement that is required in a beam of a special moment frame is directly related to the amount of longitudinal reinforcement that is provided in the beam. Therefore, it is

important not to needlessly specify more longitudinal reinforcement for flexure than required by the applicable load combinations.

### 11.7.2.6. Development and Cutoff Points of Flexural Reinforcement

Flexural reinforcement must be developed according to ACI 18.8.5 in addition to the applicable provisions of ACI Chap. 25. Reinforcing bars that terminate at columns are developed by providing standard hooks at the ends of the bars, which are to be embedded in the confined core of the column. According to ACI 18.8.5.1, the development length in tension  $\ell_{dh}$  for Nos. 3 through 11 (Nos. 10 through 36) reinforcing bars with a standard hook must satisfy the following (see Fig 11.33, which illustrates the case of a standard 90-degree hook):

- For normal-weight concrete:

$$\ell_{dh} = \text{Larger of } \begin{cases} f_y d_b / 65 \sqrt{f'_c} \text{ [In SI: } f_y d_b / 5.4 \sqrt{f'_c}] \\ 8d_b \\ 6 \text{ in (150 mm)} \end{cases}$$

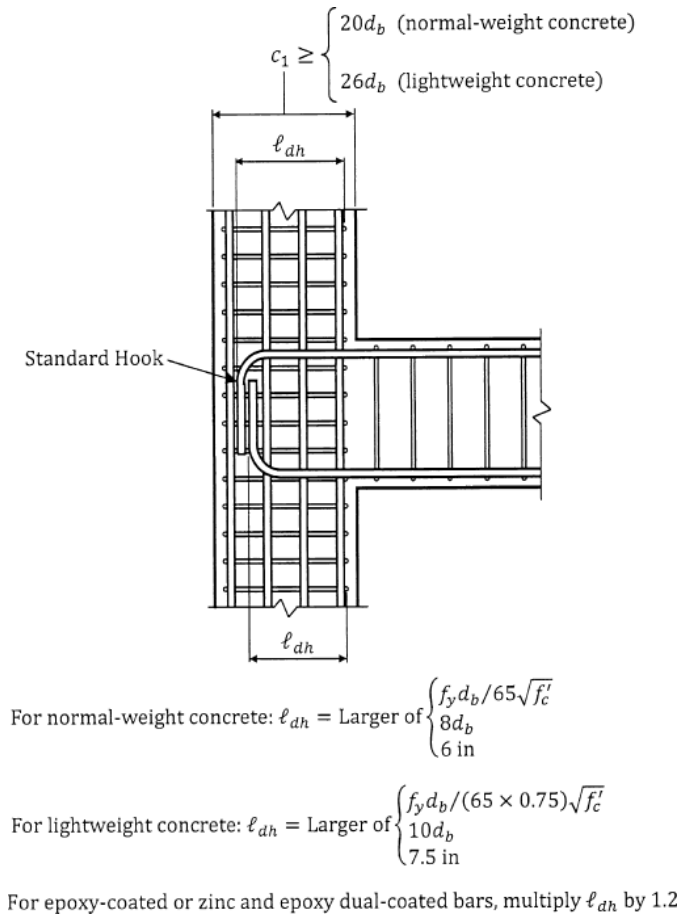
(11.6)

- For lightweight concrete:

$$\ell_{dh} = \text{Larger of } \begin{cases} f_y d_b / (65 \times 0.75) \sqrt{f'_c} \text{ [In SI: } f_y d_b / (5.4 \times 0.75) \sqrt{f'_c}] \\ 10d_b \\ 7.5 \text{ in (190 mm)} \end{cases}$$

(11.7)

**Figure 11.33** Development of flexural reinforcement with standard hooks in beams of special moment frames.



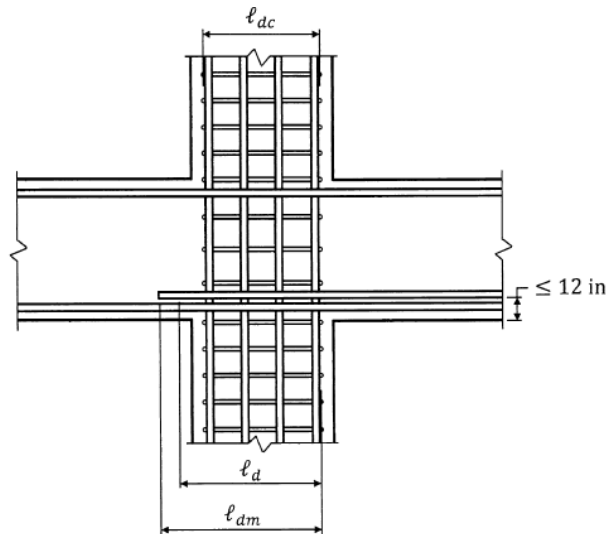
Values of  $\ell_{dh}$  in ACI 18.8.5.1 are derived from the provisions of ACI 25.4.3.1. According to that section, the development length of a standard hook in tension is the following:

$$\ell_{dh} = \text{Larger of } \begin{cases} \psi_e \psi_c \psi_r f_y d_b / 50 \lambda \sqrt{f'_c} \quad [\text{In SI: } 0.24 \psi_e \psi_c \psi_r f_y d_b / \lambda \sqrt{f'_c}] \\ 8d_b \\ 6 \text{ in (150 mm)} \end{cases}$$

where the modification factors  $\psi_e, \psi_c, \psi_r$ , and  $\lambda$  are given in ACI Table 25.4.3.2. Because the hook is to be embedded in the confined core of the column, the modification factors for concrete cover  $\psi_c$  and confining reinforcement  $\psi_r$  are taken equal to 0.7 and 0.8, respectively, from ACI Table 25.4.3.2. To reflect the effect of load reversals at the joint,  $\ell_{dh}$  was also increased to account for factors such as the actual yield stress in the reinforcement being more than the specified yield strength and the effective development length not necessarily starting at the face of the joint.

In lieu of hooked bars, straight bars may be used, as long as they are properly developed in accordance with ACI 18.8.5.3 and 18.8.5.4. The requirements are illustrated in Fig. 11.34. It is evident that the minimum development lengths in tension of straight Nos. 3 through 11 (Nos. 10 through 36) reinforcing bars are a multiple of the lengths in ACI 18.8.5.1 for the same reinforcing bars with standard hooks. In particular, the factor is 2.5 for bottom bars and 3.5 for top bars. Note that Nos. 14 and 18 bars (Nos. 43 and 47 bars) are not included because of the lack of information pertaining to the anchorage of such bars subjected to load reversals.

Figure 11.34 Development of straight flexural reinforcement in beams of special moment frames.



Elevation

$$\text{For normal-weight concrete: } \ell_d = \text{Larger of } \begin{cases} 2.5f_y d_b / 65\sqrt{f'_c} \\ 20d_b \\ 15 \text{ in} \end{cases}$$

$$\text{For lightweight concrete: } \ell_d = \text{Larger of } \begin{cases} 2.5f_y d_b / (65 \times 0.75)\sqrt{f'_c} \\ 25d_b \\ 18.75 \text{ in} \end{cases}$$

For epoxy-coated or zinc and epoxy dual-coated bars, multiply  $\ell_d$  by 1.2

For depth of concrete cast in one lift beneath the bar > 12 in, multiply  $\ell_d$  by 1.3

$$\text{For } \ell_d > \ell_{dc}, \ell_{dm} = 1.6(\ell_d - \ell_{dc}) + \ell_{dc} = 1.6\ell_d - 0.6\ell_{dc}$$

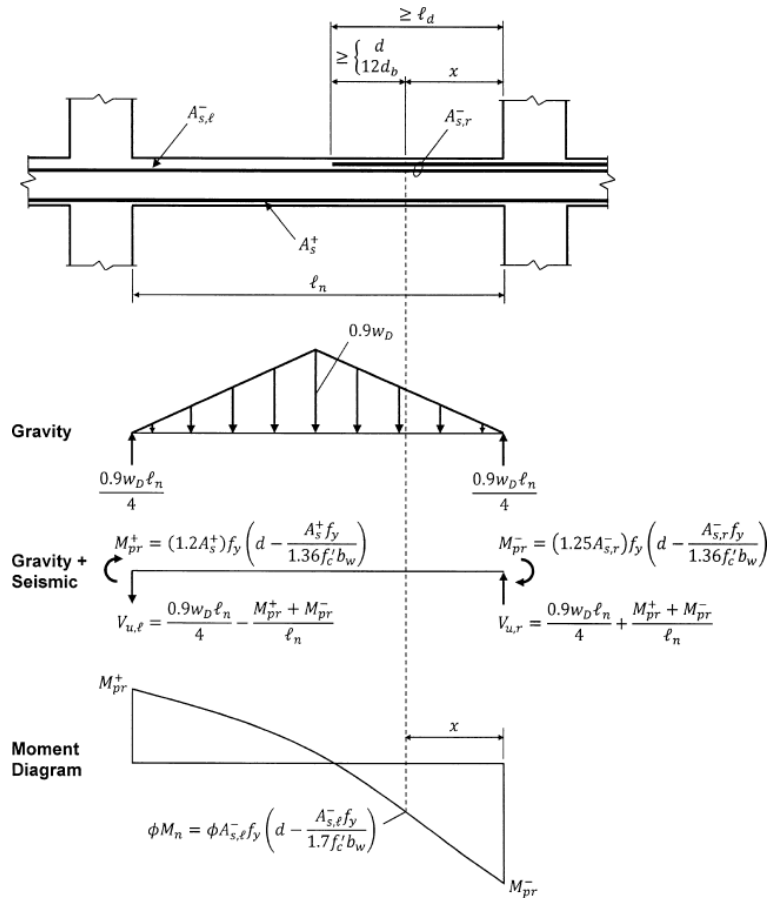
In situations where the straight embedment length of a reinforcing bar extends beyond the confined core of the column, as shown in Fig. 11.34, it is assumed that the limiting bond stress between the reinforcing bars and the concrete outside the confined region is less than that inside the confined region because of the smaller level of confinement. Thus, the required development length outside of the confined core of the column is increased by a factor of 1.6. In Fig. 11.34,  $\ell_{dm}$  is the required development length of the straight bars if the bars are not entirely embedded in the confined column core,  $\ell_d$  is the required development length in tension for straight bars determined in accordance with ACI 18.8.5.3, and  $\ell_{dc}$  is the length of the bars that are embedded in the confined concrete, which in this case, is the length of the confined core of the column.

Flexural bars may be terminated along the span as long as the applicable requirements in ACI 9.7.3 are satisfied. The load combination used to find cutoff points is 0.9 times the dead load with the probable flexural strengths  $M_{pr}$  at the ends of the member, because this combination produces the longest bar lengths.

The following equation can be utilized to determine the theoretical cutoff point  $x$  from the face of the support for negative reinforcing bars (see Fig. 11.35):

$$\frac{x}{2} \left( \frac{0.9w_D x}{\ell_n/2} \right) \frac{x}{3} - V_{u,r} x + M_{pr}^- = \phi M_n$$

Figure 11.35 Cutoff point of negative flexural reinforcement for beams in special moment frames.



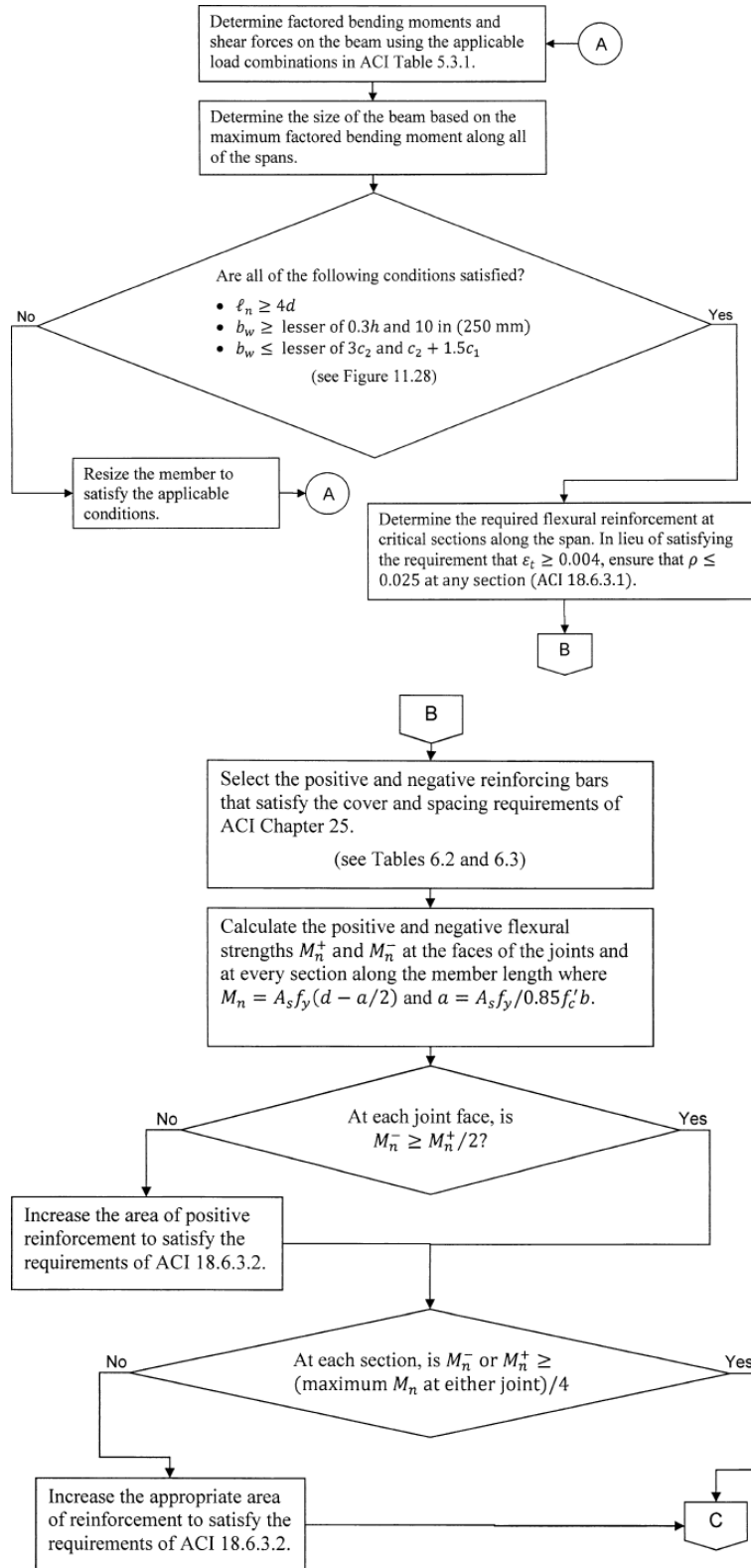
As shown in the figure,  $\phi M_n$  is the design strength of the beam based on the area of negative reinforcement  $A_{s,\ell}^-$  at the left support. The distance  $x$  is the theoretical location where the negative reinforcement  $A_{s,r}^-$  at the right support is no longer required for moment strength, that is,  $A_{s,\ell}^-$  alone is sufficient at this point. Similar equations can be derived for cases other than triangular gravity loads.

According to ACI 9.7.3.3, the negative bars at the right support must extend a distance equal to the larger of  $d$  or  $12d_b$  beyond  $x$ . Additionally, the total length of the bars from the face of the right support must be at least equal to the development length  $\ell_d$  determined by ACI Eq. (25.4.2.3a).

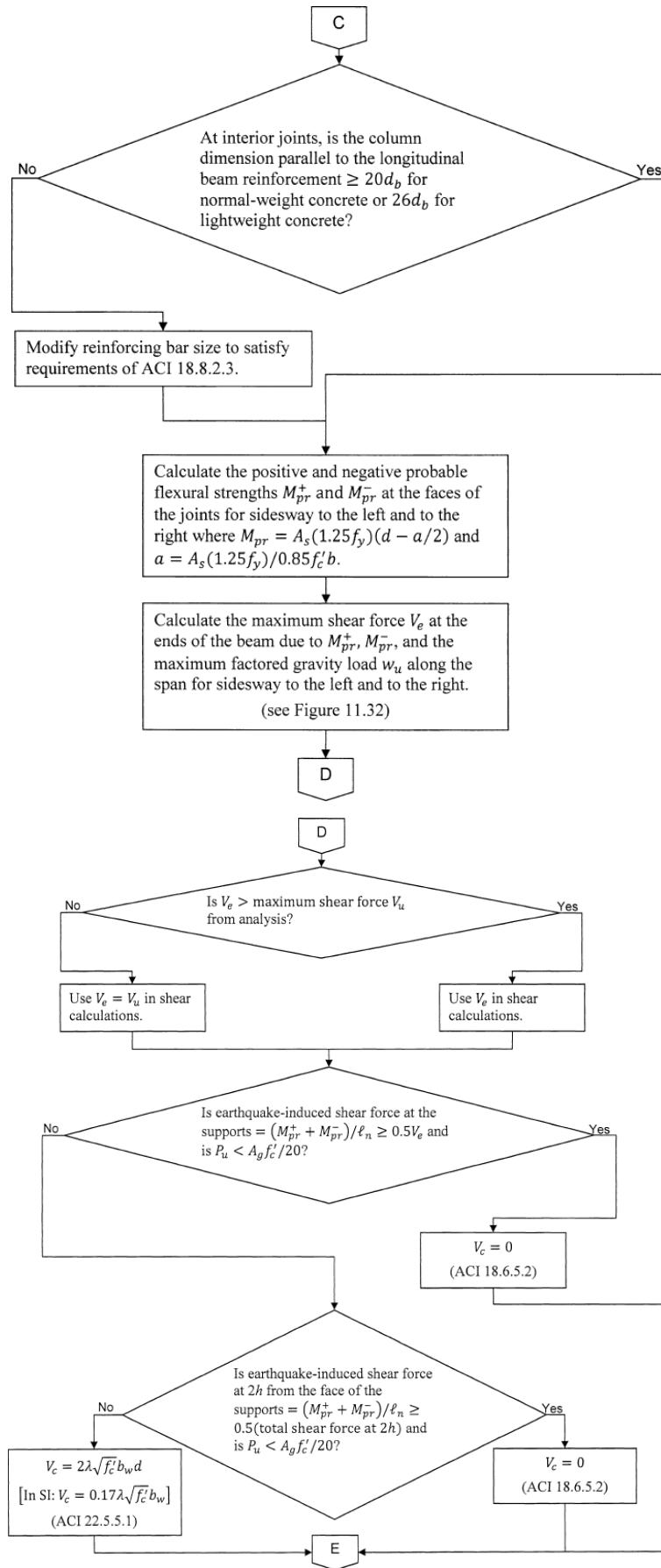
Flexural reinforcement shall not be terminated in a tension zone unless one or more of the conditions of ACI 9.7.3.5 are satisfied. Typically, ACI 9.7.3.5(a) is satisfied at the cutoff point.

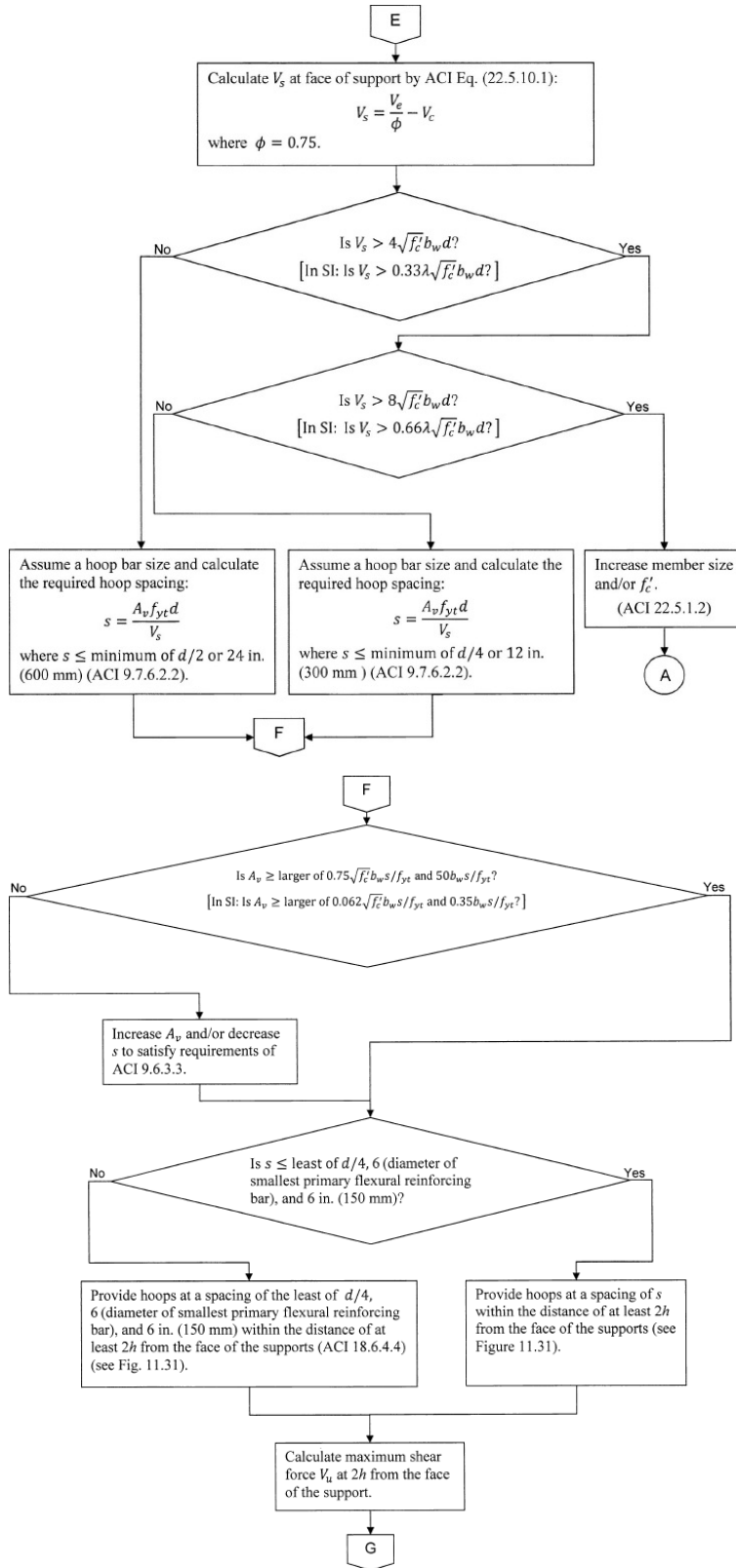
A summary of the overall design procedure for a beam in special moment frame is given in Fig. 11.36 for cases where  $P_u \leq A_g f'_c / 10$ .

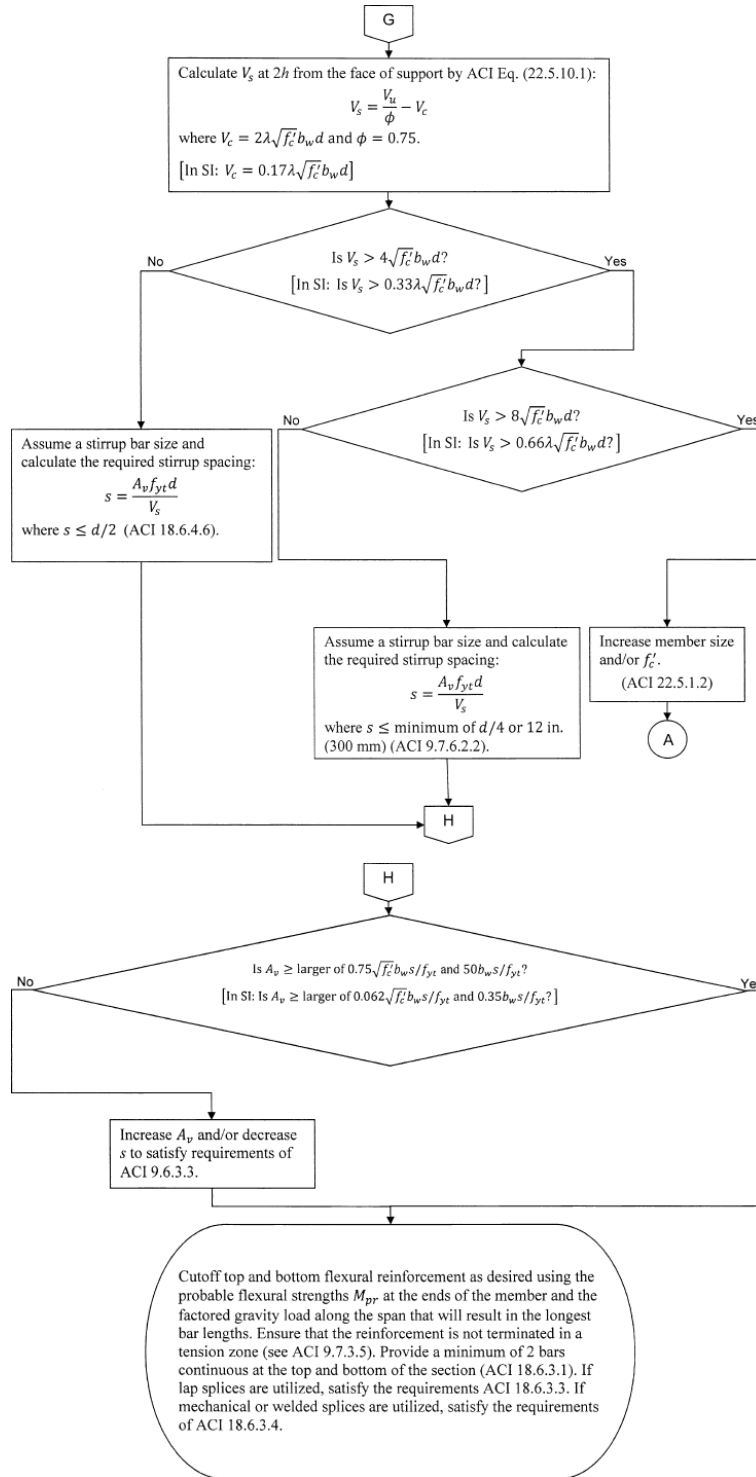
Figure 11.36 Design procedure for beams in special moment frames.





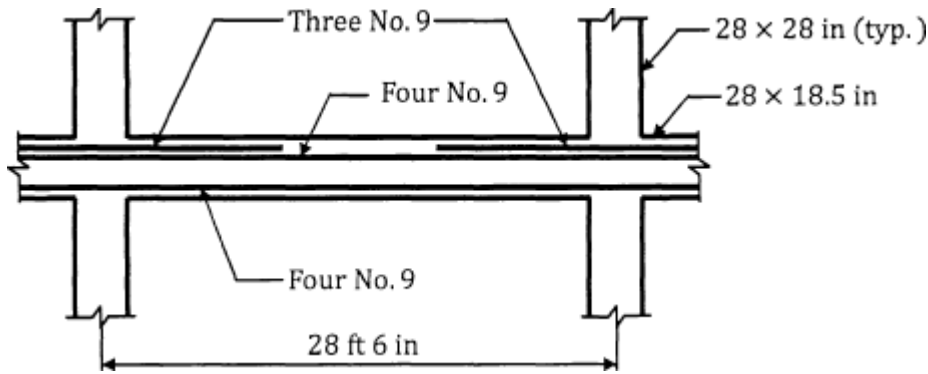






**Example 11.7** Determine the required transverse reinforcement for the beam depicted in Fig. 11.37. Assume normal-weight concrete with  $f'_c = 4,000$  psi and Grade 60 reinforcement. The beam is part of a special moment frame in a building assigned to SDC E. It has been determined that the maximum factored uniformly distributed gravity load along the entire length of the beam that governs for shear is  $w_u = 5.0$  kips/ft.

Figure 11.37 The beam in Example 11.7.



**Solution** According to ACI 18.6.5.1, the design shear forces  $V_e$  are computed from statics assuming that moments of opposite sign corresponding to the probable flexural strength  $M_{pr}$  act at the joint faces and that the member is loaded with tributary factored gravity load along its span. Sidesway to the right and to the left must be considered when calculating the maximum design shear forces.

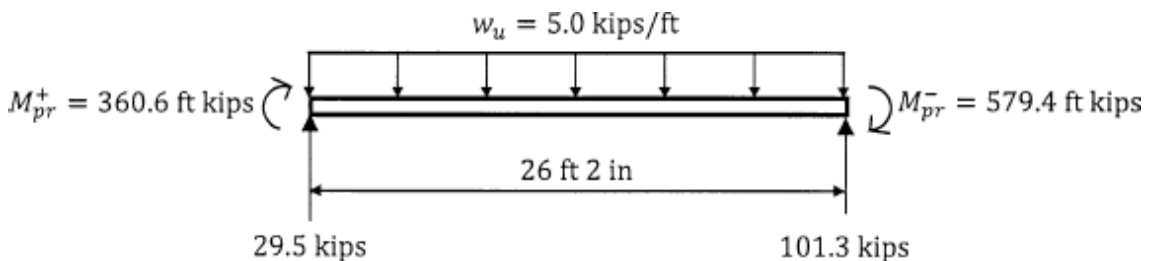
The probable flexural strengths  $M_{pr}$  for the top and bottom bars in this beam are the following:

$$\begin{aligned} \text{For seven No. 9 top bars: } M_{pr}^- &= A_s^- (1.25 f_y) \left( d - \frac{A_s^- f_y}{1.36 f_c' b_w} \right) \\ &= 7.0 \times 1.25 \times 60 \left( 16 - \frac{7.0 \times 60}{1.36 \times 4 \times 28} \right) / 12 = 579.4 \text{ ft kips} \end{aligned}$$

$$\begin{aligned} \text{For four No. 9 bottom bars: } M_{pr}^+ &= A_s^+ (1.25 f_y) \left( d - \frac{A_s^+ f_y}{1.36 f_c' b_w} \right) \\ &= 4.0 \times 1.25 \times 60 \left( 16 - \frac{4.0 \times 60}{1.36 \times 4 \times 28} \right) / 12 \\ &= 360.6 \text{ ft kips} \end{aligned}$$

Figure 11.38 shows the beam and shear forces due to gravity loads plus probable flexural strengths for sidesway to the right. Due to the symmetric distribution of the negative longitudinal reinforcement in the beam, sidesway to the left gives the same maximum shear force.

Figure 11.38 Design shear forces for the beam in Example 11.7.



In general, shear strength is provided by both concrete  $V_c$  and reinforcing steel  $V_s$ . However, according to ACI 18.6.5.2,  $V_c$  is to be taken as zero when the earthquake-induced shear force calculated in accordance with ACI 18.6.5.1 is greater than or equal to 50% of the total shear force and the factored axial compressive force including earthquake effects is less than  $A_g f_c' / 20$ . In this example, the beam carries negligible axial forces and the maximum earthquake-induced shear force at the face of the support is equal to  $(579.4 + 360.6) / 26.167 = 35.9$  kips, which is less than 50% of the total shear force =  $101.3 / 2 = 50.7$  kips. At a distance  $2h = 2 \times 18.5 = 37.0$  in from the face of the support, the earthquake-induced shear force is equal to 35.9 kips (constant along span), and the total shear force =  $101.3 - [5.0 \times (37/12)] = 85.9$  kips. Therefore, at this location,  $35.9 \text{ kips} < 85.9 / 2 = 43.0$  kips. Thus, the nominal shear strength provided by the concrete  $V_c$  can be included:

$$V_c = 2\lambda \sqrt{f_c'} b_w d = 2 \times 1.0 \sqrt{4,000} \times 28 \times 16 / 1,000 = 56.7 \text{ kips}$$

Thus,

$$V_s = \frac{V_u}{\phi} - V_c = \frac{101.3}{0.75} - 56.7 = 78.4 \text{ kips} < 8\sqrt{f'_c b_w d} = 226.8 \text{ kips}$$

Also,  $V_s$  is less than  $4\sqrt{f'_c b_w d} = 113.4$  kips.

Because the clear space between the four No. 9 bottom bars is approximately 6.6 in (assuming No. 3 hoops), which is greater than 6 in, crossties are required on each of the interior bars in order to satisfy the lateral support requirements of ACI 25.7.2.3 (ACI 18.6.4.2). The required spacing of No. 3 hoops with two crossties is the following:

$$s = \frac{A_v f_y d}{V_s} = \frac{(4 \times 0.11) \times 60 \times 16}{78.4} = 5.4 \text{ in}$$

Maximum allowable hoop spacing within a distance of  $2h = 37.0$  in from the face of the support at each end of the member is the least of the following (ACI 18.6.4.4):

- $d/4 = 16/4 = 4$  in (governs)
- 6 (diameter of smallest longitudinal bar) =  $6 \times 1.128 = 6.8$  in
- 6 in

Use 10 No. 3 hoops and crossties at each end of the beam spaced at 4 in on center with the first set located 2 in from the face of the support (ACI 18.6.4.4).

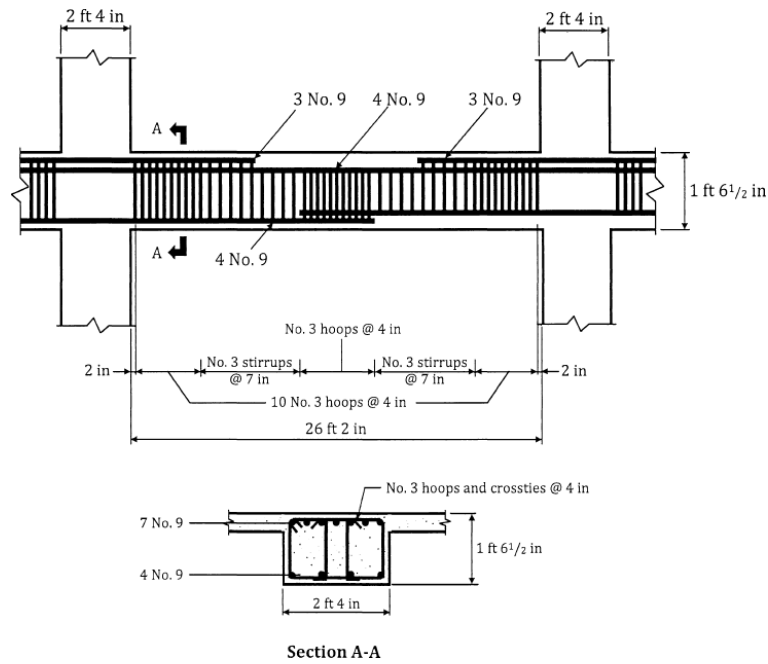
Where hoops are no longer required, stirrups with seismic hooks at both ends may be used (ACI 18.6.4.6). At a distance of 38 in from the face of the support,  $V_u = 85.5$  kips. Therefore, the required stirrup spacing for No. 3 stirrups with two crossties is the following:

$$\begin{aligned} s &= \frac{A_v f_y d}{V_s} = \frac{(4 \times 0.11) \times 60 \times 16}{(85.5/0.75) - 56.7} = 7.4 \text{ in} \\ &\leq \frac{A_v f_y}{0.75\sqrt{f'_c b_w}} = \frac{(4 \times 0.11) \times 60,000}{0.75\sqrt{4,000} \times 28} = 19.9 \text{ in} \\ &\leq \frac{A_v f_y}{50b_w} = \frac{(4 \times 0.11) \times 60,000}{50 \times 28} = 18.9 \text{ in} \end{aligned}$$

The maximum allowable spacing of the stirrups is  $d/2 = 8$  in (ACI 18.6.4.6), which is greater than 7.4 in. A 7-in spacing, starting at 38 in from the face of the supports is sufficient for the remaining portion of the beam.

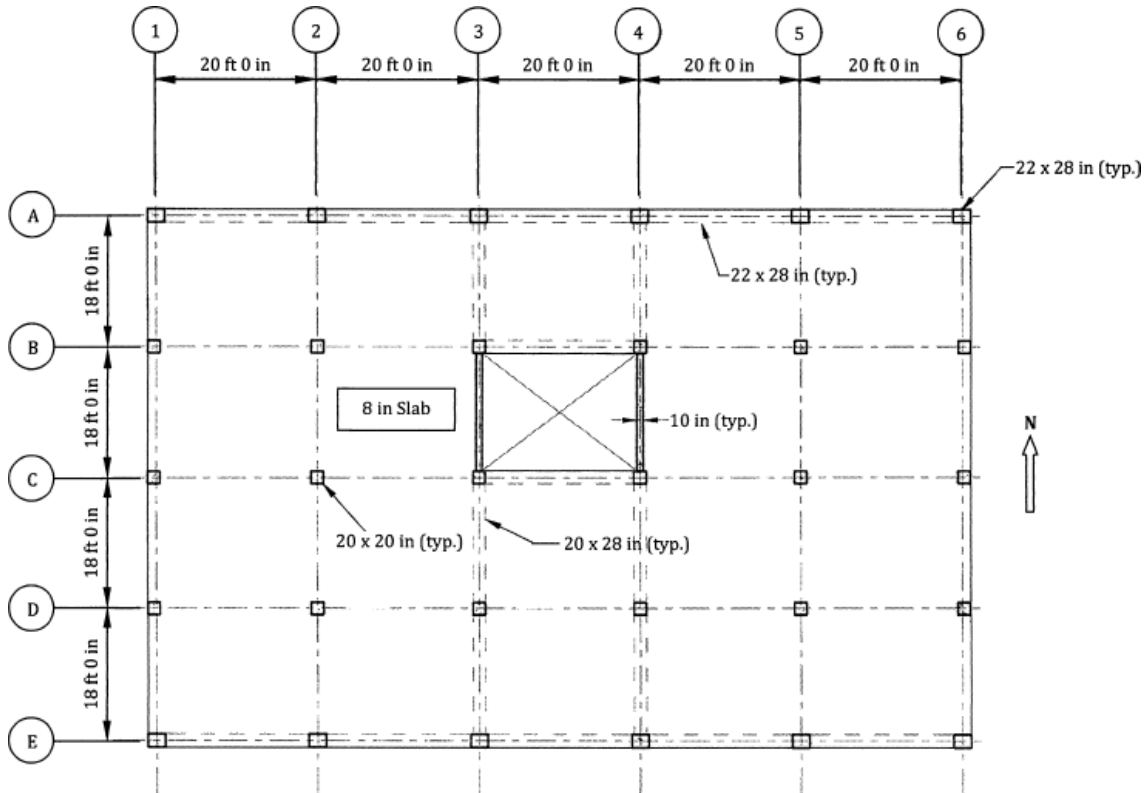
Reinforcement details for the beam are shown in [Fig. 11.39](#).

Figure 11.39 Reinforcement details for the beam in Example 11.7.



**Example 11.8** Illustrated in Fig. 11.40 is a typical floor in a building assigned to SDC D. Special moment frames are used as the SFRS in the E-W direction. Design the beam on column line A between lines 3 and 4 given the bending moments and shear forces in Table 11.13, which were obtained from an analysis of the building. Assume normal-weight concrete with  $f'_c = 4,000$  psi and Grade 60 reinforcement. It has been determined that the design spectral acceleration  $S_{DS} = 1.0$ .

Figure 11.40 Typical floor plan in Example 11.8.



**Table 11.13** Bending Moments and Shear Forces for the Beam in [Example 11.8](#)

Load Case	Location	Bending Moment (ft kips)	Shear Force (kips)
Dead ( $D$ )	Support	-37.7	19.5
	Midspan	27.4	–
Live ( $L$ )	Support	-15.1	7.8
	Midspan	10.7	–
Seismic ( $Q_E$ )	Support	$\pm 83.7$	$\pm 9.4$

**Solution** The flowchart in [Fig. 11.36](#) is used to design this beam. Because this is an edge beam, it is subjected to torsional effects, and requirements in addition to those provided in [Fig. 11.36](#) must be satisfied to complete the design (see [Chap. 6](#) for information on designing for torsional effects).

**Step 1. Determine the factored bending moments, shear forces, and torsional moments.** A summary of the applicable load combinations for the bending moments and shear forces is given in [Table 11.14](#). Due to symmetry, the factored reactions are the same at both supports.

**Table 11.14** Design Bending Moments and Shear Forces for the Beam in [Example 11.8](#)

Load Combination	Location	Bending Moment (ft kips)	Shear Force (kips)	
1.4 $D$	Support	-52.8	27.3	
	Midspan	38.4	–	
1.2 $D$ + 1.6 $L$	Support	-69.4	35.9	
	Midspan	50.0	–	
1.4 $D$ + 0.5 $L$ + $Q_E$	Support	SSR	-144.0	40.6
		SSL	23.4	21.8
	Midspan	43.7	–	
0.7 $D$ + $Q_E$	Support	SSR	-110.1	23.1
		SSL	57.3	4.3
	Midspan	19.2	–	

SSR = sidesway right, SSL = sidesway left

Further explanation is needed on how the load factors in ACI Eqs. (5.3.1e) and (5.3.1g) were obtained. According to ASCE/SEI 12.4.2, the seismic load effect  $E$  is the combination of horizontal and vertical seismic load effects. In ACI Eq. (5.3.1e) where the effects of gravity and seismic ground motion are additive,  $E = \rho Q_E + 0.2S_{DS}D$ , and in ACI Eq. (5.3.1g) where the effects of gravity and seismic ground motion counteract,  $E = \rho Q_E - 0.2S_{DS}D$ .

The redundancy factor  $\rho$  is equal to 1.3 for structures assigned to SDC D and above unless one of the two conditions in ASCE/SEI 12.3.4.2 is met. According to the first condition, each story resisting more than 35% of the base shear in the direction of analysis must comply with the applicable provisions in ASCE/SEI Table 12.3-3. In the E-W direction, the loss of moment resistance at both ends of a single beam in the special moment frames would not result in more than a 33% reduction in story strength, nor would the resulting system have a Type 1b extreme torsional irregularity (see ASCE/SEI Table 12.3-1). Thus, the first of the two conditions under ASCE/SEI 12.3.4.2 is met and  $\rho = 1.0$ .

Therefore, substituting  $\rho$  and  $S_{DS}$  into the equations for  $E$  results in the following:

- ACI Eq. (5.3.1e):

$$E = \rho Q_E + 0.2S_{DS}D = Q_E + (0.2 \times 1.0)D = Q_E + 0.2D$$

$$1.2D + 0.5L + 1.0E = 1.2D + 0.5L + Q_E + 0.2D = 1.4D + 0.5L + Q_E$$

- ACI Eq. (5.3.1g):

$$E = \rho Q_E - 0.2S_{DS}D = Q_E - 0.2D$$

$$0.9D + 1.0E = 0.9D + Q_E - 0.2D = 0.7D + Q_E$$

Both sidesway to the right and sidesway to the left must be considered for the effects due to  $Q_E$  as shown in the table.

The torsional moments are due to gravity loads only. Because the beams are part of an indeterminate framing system where redistribution of internal forces can occur following torsional cracking, the maximum factored torsional moment  $T_u$  at the critical section located at a distance  $d$  from the face of the support need not exceed the following (see ACI 22.7.5):

$$T_u = \phi T_{cr} = \phi 4\lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right)$$

where  $b_e = h - t_f = 28 - 8 = 20 \text{ in} < 4 t_f = 32 \text{ in}$

$A_{cp} = b_w h + b_e t_f = 776 \text{ in}^2$

$p_{cp} = 2(h + b_w + b_e) = 140 \text{ in}$

Therefore,

$$T_u = \phi T_{cr} = 0.75 \times 4 \times 1.0 \sqrt{4,000} \left( \frac{776^2}{140} \right) / 12,000 = 68.0 \text{ ft kips}$$

**Step 2. Determine the size of the beam.** The size of the beam in this example has been given as 22 in wide and 28 in deep (see Fig. 11.40).

**Step 3. Check the dimensional limits of the beam.**

- $\ell_n = (20 \times 12) - 28 = 212 \text{ in} > 4 d = 4 \times (28 - 2.5) = 102 \text{ in}$
- $b_w = 22 \text{ in} > \text{lesser of } 0.3 h = 8.4 \text{ and } 10 \text{ in}$
- $b_w = 22 \text{ in} < \text{lesser of } 3 c_2 = 3 \times 22 = 66 \text{ in and } c_2 + 1.5c_1 = 64 \text{ in}$

This beam meets the dimensional limits of ACI 18.6.2.

The cross-sectional limits of ACI 22.7.7 are checked in Step 11 below after the maximum shear force has been determined and after it has been established whether or not  $V_c$  can contribute to  $V_n$ .

**Step 4. Determine the required flexural reinforcement.** The required flexural reinforcement is determined in accordance with the provisions of ACI Chap. 9 (see Chap 6) and ACI 18.6.3.1. A summary is given in Table 11.15.

**Table 11.15** Summary of Required Flexural Reinforcement for the Beam in Example 11.8

Location	$M_u$ (ft kips)	$A_s$ (in <sup>2</sup> )	Reinforcement	$M_n$ (ft kips)
Support	-144.0	1.87	3 No. 8	290.9
	57.3	1.87	3 No. 8	290.9
Midspan	50.0	1.87	3 No. 8	290.9

Note that the minimum reinforcement requirements of ACI 9.6.1 govern at all sections:

$$A_{s,min} = \frac{200b_w d}{f_y} = \frac{200 \times 22 \times 25.5}{60,000} = 1.87 \text{ in}^2$$



The provided reinforcement is also less than the maximum permitted:

$$A_{s,max} = \rho_{max} b_w d = 0.025 \times 22 \times 25.5 = 14.0 \text{ in}^2$$

The selected reinforcement satisfies all of the requirements of ACI Chap. 25 pertaining to cover and spacing (see Tables 6.2 and 6.3).

Table 11.15 also contains the nominal moment strengths at the faces of the joint and at any section along the length of the beam.

**Step 5. Check the flexural strength requirements of ACI 18.6.3.2.**

- At the face of each joint:  $M_n^+ = 290.9 \text{ ft kips} > M_n^- / 2 = 290.9 / 2 = 145.5 \text{ ft kips}$
- At any section along the span:  $(M_n^+ \text{ or } M_n^-) / 4 = 290.9 / 4 = 72.7 \text{ ft kips}$

Providing at least two No. 8 bars ( $M_n = 196.4 \text{ ft kips}$ ) along the length of the beam satisfies this provision. This also satisfies the minimum number of continuous bars that must be provided in accordance with ACI 18.6.3.1.

**Step 6. Check the dimension of the column parallel to the longitudinal reinforcement.** Where reinforcing bars extend through an interior joint, the column dimension parallel to the beam reinforcement must be at least 20 times the diameter of the largest longitudinal bar for normal-weight concrete (ACI 18.8.2.3). The minimum required column dimension =  $20 \times 1.0 = 20 \text{ in}$ , which is less than the 28-in column width that is provided.

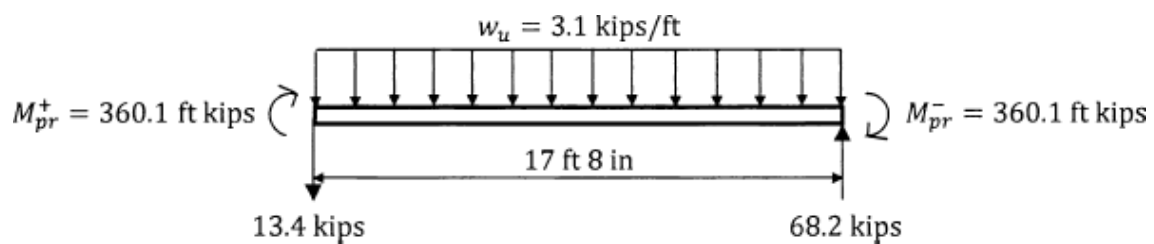
**Step 7. Calculate the probable flexural strengths  $M_{pr}$  at the faces of the joints.** Because the top and bottom reinforcement in this beam are the same,  $M_{pr}^-$  and  $M_{pr}^+$  are the same:

$$M_{pr} = A_s (1.25 f_y) \left( d - \frac{a}{2} \right) \text{ where } a = A_s (1.25 f_y) / 0.85 f'_c b_w$$

$$M_{pr} = (3 \times 0.79) \times (1.25 \times 60) \left[ 25.5 - \frac{(3 \times 0.79)(1.25 \times 60)}{2 \times 0.85 \times 4 \times 22} \right] / 12 = 360.1 \text{ ft kips}$$

**Step 8. Calculate the maximum shear force  $V_e$  at the ends of the beam.** The largest shear force including factored gravity effects and the effects from  $M_{pr}$  is depicted in Fig. 11.41 for sidesway to the right. Shown in the figure is the equivalent maximum factored uniformly distributed gravity load on the beam, which was obtained from a finite element analysis of the floor system. Sidesway to the left yields the same maximum shear force because of the symmetric distribution of the longitudinal reinforcement in the beam. The maximum shear force  $V_e = 68.2 \text{ kips}$  is greater than the maximum shear force obtained from the structural analysis, which is equal to 40.6 kips (see Table 11.14).

Figure 11.41 Design shear forces for the beam in Example 11.8.



**Step 9. Determine if the nominal shear strength of the concrete  $V_c$  can be utilized.** The earthquake-induced shear force at the supports is equal to  $(M_{pr}^+ + M_{pr}^-) / \ell_n = (360.1 + 360.1) / 17.67 = 40.8 \text{ kips}$ , which is greater than  $0.5 V_e = 34.1 \text{ kips}$ . Additionally, the beam is subjected to a negligible factored axial compressive force. Therefore,  $V_c = 0$  in accordance with ACI 18.6.5.2.

**Step 10. Calculate  $V_s$  at the face of the support.**

$$V_s = \frac{V_u}{\phi} - V_c = \frac{68.2}{0.75} - 0 = 90.9 \text{ kips}$$

According to ACI 22.5.1.2, shear strength contributed by shear reinforcement  $V_s$  shall not exceed  $8 \sqrt{f'_c} b_w d = 283.9 \text{ kips}$ , which is satisfied in this case. Also,  $V_s$  is less than  $4 \sqrt{f'_c} b_w d = 142.0 \text{ kips}$ .

**Step 11. Check the cross-sectional limits of ACI 22.7.7.**

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7 A_{oh}^2}\right)^2} = \sqrt{\left(\frac{68,200}{22 \times 25.5}\right)^2 + \left(\frac{68.0 \times 12,000 \times 86.0}{1.7 \times 453.25^2}\right)^2} = 234.9 \text{ psi}$$

$$\phi \left(\frac{V_c}{b_w d} + 8\sqrt{f'_c}\right) = 0.75(0 + 8)\sqrt{4,000} = 379.5 \text{ psi} > 234.9 \text{ psi}$$

where  $x_1 = b_w - 2c - d_s = 22 - (2 \times 1.5) - 0.5 \text{ No. 4 hoop} = 18.5 \text{ in}$

$y_1 = h - 2c - d_s = 28 - (2 \times 1.5) - 0.5 = 24.5 \text{ in}$

$p_h = 2(x_1 + y_1) = 86.0 \text{ in}$

$A_{oh} = x_1 y_1 = 453.25 \text{ in}^2$

Therefore, the cross-sectional limits of ACI 22.7.7 are satisfied.

**Step 12. Determine the required transverse reinforcement for shear within 2 h from the face of the support.**

$$\frac{A_v}{s} = \frac{V_u - \phi V_c}{\phi f_{yt} d} = \frac{68,200 - 0}{0.75 \times 60,000 \times 25.5} = 0.059 \text{ in}^2/\text{in}$$

**Step 13. Determine the required transverse reinforcement for torsion.**

$$\frac{2A_t}{s} = \frac{2T_u}{2\phi \cot \theta A_o f_{yt}} = \frac{2 \times 68.0 \times 12,000}{2 \times 0.75 \times \cot 45^\circ \times (0.85 \times 453.25) \times 60,000} = 0.047 \text{ in}^2/\text{in}$$

**Step 14. Determine the total transverse reinforcement.**

Total required transverse reinforcement =  $0.059 + 0.047 = 0.106 \text{ in}^2/\text{in}$

Minimum transverse reinforcement is the greater of the following (ACI 9.6.4.2):

$$0.75\sqrt{f'_c} \frac{b_w}{f_{yt}} = 0.75\sqrt{4,000} \frac{22}{60,000} = 0.017 \text{ in}^2/\text{in}$$

$$\frac{50b_w}{f_{yt}} = \frac{50 \times 22}{60,000} = 0.018 \text{ in}^2/\text{in (governs)}$$

As expected, minimum transverse reinforcement does not govern in this example.

Maximum allowable hoop spacing within a distance of  $2h = 56.0 \text{ in}$  from the face of the support at each end of the member is the lesser of the following (ACI 18.6.4.4):

- $d/4 = 25.5/4 = 6.4 \text{ in}$
- 6 (diameter of smallest longitudinal bar) =  $6 \times 1.0 = 6.0 \text{ in}$
- 6.0 in

Assuming No. 4 hoops and one No. 4 crosstie, the required spacing  $s$  at the critical section =  $3 \times 0.20/0.106 = 5.7 \text{ in} < 6.0 \text{ in}$ . Note that a No. 4 crosstie is required around the center No. 8 longitudinal bars in the cross-section to satisfy the lateral support requirements of ACI 18.6.4.2.

Use 12 No. 4 hoops with 1 No. 4 crosstie spaced 5 in on center at each end of the beam with the first set of hoops and crossties located 2 in from the face of the support (ACI 18.6.4.4).

**Step 15. Determine required size and spacing of stirrups with seismic hooks where hoops are no longer required.** Outside the region of flexural yielding, stirrups with seismic hooks can be utilized, and the contribution of the nominal shear strength of the concrete can be used. At a distance of 57 in from the face of the support,  $V_u = 68.2 - 3.1(57/12) = 53.5 \text{ kips}$ .

$$\frac{A_v}{s} = \frac{V_u - \phi V_c}{\phi f_{yt} d} = \frac{53,500 - (0.75 \times 2 \times \sqrt{4,000} \times 22 \times 25.5)}{0.75 \times 60,000 \times 25.5} = 0.0002 \text{ in}^2/\text{in}$$

Because the torsional moment is constant over the entire span,  $2A_t/s = 0.047 \text{ in}^2/\text{in}$ .

Therefore, the required spacing for combined shear and torsion is equal to  $2 \times 0.2/0.047 = 8.5 \text{ in}$ .

Maximum spacing of transverse reinforcement where hoops are not required is the lesser of the following (see ACI 9.7.6.3.3 and 18.6.4.6):

- $p_h/8 = 86.0/8 = 10.8$  in (governs)
- 12 in
- $d/2 = 25.5/2 = 12.8$  in

Provide No. 4 stirrups with seismic hooks spaced at 8 in along the length of the beam where hoops are not required. These stirrups also satisfy the minimum shear reinforcement requirements in ACI 9.6.3.3. Note that the crosstie around the middle No. 8 bars is not required for lateral support of the longitudinal bars outside the plastic hinge zone.

**Step 16. Determine required longitudinal reinforcement for torsion.**

$$A_\ell = \frac{A_t}{s} p_h \left( \frac{f_{yt}}{f_y} \right) \cot^2 \theta = \frac{0.047}{2} \times 86 = 2.02 \text{ in}^2$$

$$A_{\ell, \min} = \begin{cases} \frac{5\sqrt{f'_c} A_{cp}}{f_y} - \left( \frac{A_t}{s} \right) p_h \left( \frac{f_{yt}}{f_y} \right) = \frac{5\sqrt{4,000} \times 776}{60,000} - \left( \frac{0.047}{2} \times 86 \right) = 2.07 \text{ in}^2 \\ \frac{5\sqrt{f'_c} A_{cp}}{f_y} - \left( \frac{25b_w}{f_{yt}} \right) p_h \left( \frac{f_{yt}}{f_y} \right) = \frac{5\sqrt{4,000} \times 776}{60,000} - \left( \frac{25 \times 22}{60,000} \times 86 \right) = 3.30 \text{ in}^2 \end{cases}$$

Therefore,  $A_\ell = 2.07 \text{ in}^2$ .

**Step 17. Determine total required longitudinal reinforcement.** The longitudinal reinforcement required for torsion must be combined with the longitudinal reinforcement required for flexure. The longitudinal torsion reinforcement must be distributed around the perimeter of the section with a maximum spacing of 12 in (ACI 9.7.5.1). In order to have a uniform distribution of reinforcement around the perimeter, assign  $2.07/4 = 0.52 \text{ in}^2$  to each face.

Use one No. 7 bar on each side (area =  $0.60 \text{ in}^2$ , bar diameter =  $0.875 \text{ in} > 0.042s = 0.21 \text{ in}$ ). The maximum spacing requirement of ACI 9.7.5.1 is satisfied because the spacing =  $\{28 - 2[1.5 + 0.5 + (1.0/2)]\}/2 = 11.5 < 12.0 \text{ in}$ .

The remaining  $0.52 \text{ in}^2$  of longitudinal steel required for torsion at the top and bottom of the section is added to the area of steel required for flexure (see Table 11.15):

- Face of support

$$1.87 + 0.52 = 2.39 \text{ in}^2 \approx 2.37 \text{ in}^2$$

- Midspan

$$1.87 + 0.52 = 2.39 \text{ in}^2 \approx 2.37 \text{ in}^2$$

Thus, the reinforcing bars in Table 11.15 can be used for combined flexure and torsion.

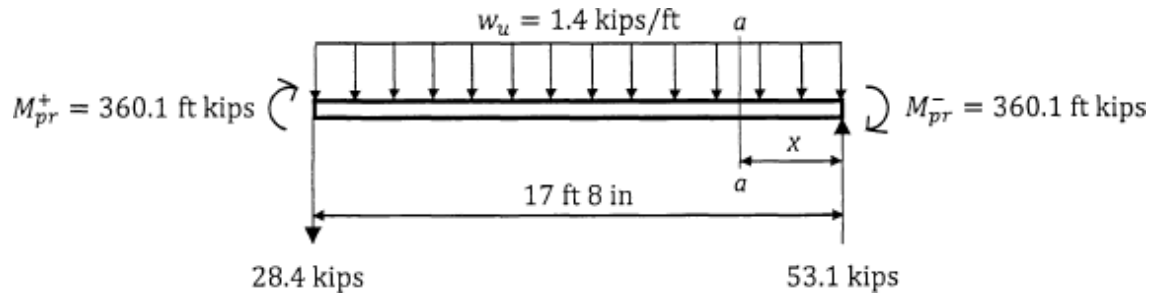
**Step 18. Determine reinforcing bar cutoff points.** The negative reinforcement at the supports is three No. 8 bars. The location where one of the three bars can be terminated will be determined.

The load combination that is used to determine the cutoff point of the one No. 8 bar is 0.7 times the dead load, which is equal to 1.4 kips/ft in this example, in combination with the probable flexural strengths  $M_{pr}$  at the ends of the member because this combination produces the longest bar lengths. The design flexural strength  $\phi M_n$ , provided by two No. 8 bars is 176.8 ft kips. Therefore, the one No. 8 bar can be terminated after the required moment strength  $M_u$  has been reduced to 176.8 ft kips.

The distance  $x$  from the support to the location where the moment is equal to 176.8 ft kips can readily be determined by summing moments about section a-a in Fig. 11.42:

$$\frac{1.4x^2}{2} - 53.1x + 360.1 - 176.8 = 0$$

Figure 11.42 Cutoff location of negative reinforcement in beam in Example 11.8.



Solving for  $x$  gives a distance of 3.6 ft from the face of the support.

The one No. 8 bar must extend a distance  $d = 25.5$  in (governs) or  $12 d_b = 12$  in beyond the distance  $x$  (ACI 9.7.3.3). Thus, from the face of the support, the total bar length must be at least equal to  $3.6 + (25.5/12) = 5.7$  ft.

Also, the bars must extend a full development length  $\ell_d$  beyond the face of the support, which is determined by ACI Eq. (25.4.2.3a):

$$\ell_d = \left( \frac{3}{40} \frac{f_y}{\lambda \sqrt{f'_c}} \frac{\psi_t \psi_e \psi_s}{\left( \frac{c_b + K_{tr}}{d_b} \right)} \right) d_b$$

where  $\psi_t$  = modification factor for casting position = 1.3 for top bars

$\psi_e$  = modification factor for reinforcement coating = 1.0 for uncoated bars

$\psi_s$  = modification factor for reinforcement size = 1.0 for No. 8 bars

$\lambda$  = modification factor for lightweight concrete = 1.0 for normal-weight concrete

$c_b$  = spacing or cover dimension

$$= 1.5 + 0.5 + \frac{1.0}{2} = 2.5 \text{ in (governs)}$$

$$= \frac{22 - 2(1.5 + 0.5) - 1.0}{2 \times 2} = 4.3 \text{ in}$$

$K_{tr}$  = transverse reinforcement index

$$= \frac{40 A_{tr}}{sn} = \frac{40 \times 3 \times 0.2}{5 \times 3} = 1.6$$

$$\frac{c_b + K_{tr}}{d_b} = \frac{2.5 + 1.6}{1.0} = 4.1 > 2.5, \text{ use } 2.5$$

Therefore,

$$\ell_d = \left( \frac{3}{40} \frac{60,000}{1.0 \sqrt{4,000}} \frac{1.3 \times 1.0 \times 1.0}{2.5} \right) \times 1.0 = 37.0 \text{ in} = 3.1 \text{ ft} < 5.7 \text{ ft}$$

Thus, the total required length of the one No. 8 bar must be at least 5.7 ft beyond the face of the support.

Flexural reinforcement is not permitted to be terminated in a tension zone unless one or more of the conditions of ACI 9.7.3.5 are satisfied. In this case, the point of inflection is approximately 7.5 ft from the face of the support, which is greater than 5.7 ft. Thus, the one No. 8 bar cannot be terminated here unless one of the conditions of ACI 9.7.3.5 is satisfied. Check if the factored shear force  $V_u$  at the cutoff point does not exceed  $2\phi V_n/3$  [ACI 9.7.3.5(a)]. With No. 4 stirrups spaced at 8 in on center that are provided in this region of the beam,  $\phi V_n$  is determined by the following:

$$\phi V_n = \phi (V_c + V_s) = 0.75 \left( \frac{2\sqrt{4,000} \times 22 \times 25.5}{1,000} + \frac{0.4 \times 60 \times 25.5}{8} \right) = 110.6 \text{ kips}$$

At 5.7 ft from the face of the support,  $V_u = 53.1 - (1.4 \times 5.7) = 45.1$  kips, which is less than  $2 \times 110.6/3 = 73.7$  kips. Therefore, the one No. 8 bar can be terminated at 5.7 ft from the face of the support.

**Step 19. Determine flexural reinforcement splices.** Lap splices are determined for the No. 8 bottom bars. Because all of the bars are to be spliced within the required length, a Class B splice must be used (ACI 25.5.2.1):

$$\ell_{st} = \text{Class B splice length} = 1.3\ell_d$$

The development length  $\ell_d$  is determined by ACI Eq. (25.4.2.3a):

$$\ell_d = \left( \frac{3}{40} \frac{f_y}{\lambda \sqrt{f'_c}} \frac{\psi_t \psi_e \psi_s}{\left( \frac{c_b + K_{tr}}{d_b} \right)} \right) d_b$$

where  $\psi_t$  = modification factor for casting position = 1.0 for other than top bars

$\psi_e$  = modification factor for reinforcement coating = 1.0 for uncoated bars

$\psi_s$  = modification factor for reinforcement size = 1.0 for No. 8 bars

$\lambda$  = modification factor for lightweight concrete = 1.0 for normal-weight concrete

$c_b$  = spacing or cover dimension

$$= 1.5 + 0.5 + \frac{1.0}{2} = 2.5 \text{ in (governs)}$$

$$= \frac{22 - 2(1.5 + 0.5) - 1.0}{2 \times 2} = 4.3 \text{ in}$$

$K_{tr}$  = transverse reinforcement index

$$= \frac{40A_{tr}}{sn} = \frac{40 \times 2 \times 0.2}{4 \times 3} = 1.3$$

$$\frac{c_b + K_{tr}}{d_b} = \frac{2.5 + 1.3}{1.0} = 3.8 > 2.5, \text{ use } 2.5$$

Therefore,

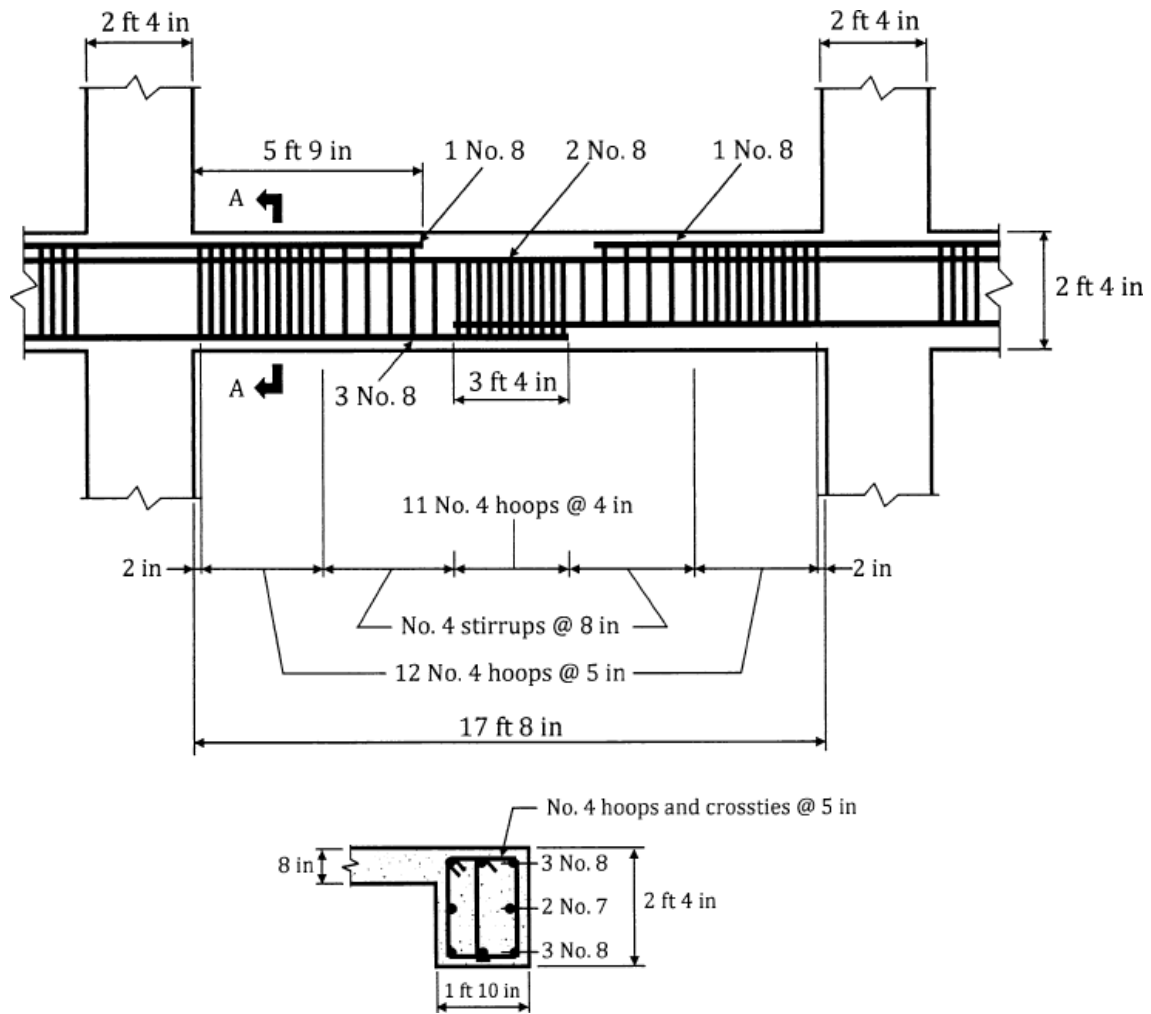
$$\ell_d = \left( \frac{3}{40} \frac{60,000}{1.0 \sqrt{4,000}} \frac{1.0 \times 1.0 \times 1.0}{2.5} \right) \times 1.0 = 28.5 \text{ in} = 2.4 \text{ ft}$$

Class B splice length =  $1.3 \times 2.4 = 3.1$  ft

Use a 3 ft 4 in splice length with No. 4 hoops spaced at the smaller of  $d/4 = 6$  in or 4 in (governs) on center (ACI 18.6.3.3).

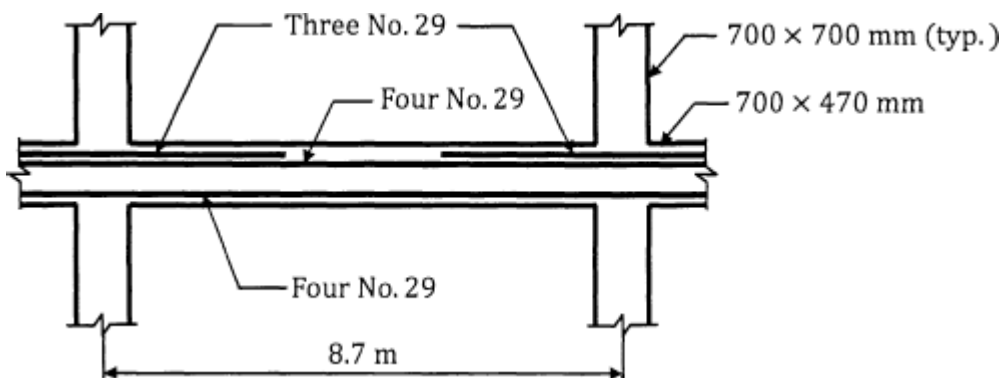
Figure 11.43 shows the reinforcement details for this beam.

Figure 11.43 Reinforcement details for the beam in Example 11.8.



**Example 11.9** Determine the required transverse reinforcement for the beam shown in Fig. 11.44. Assume normal-weight concrete with  $f'_c = 28 \text{ MPa}$  and Grade 420 reinforcement. The beam is part of a special moment frame in a building assigned to SDC E. It has been determined that the maximum factored uniformly distributed gravity load along the entire length of the beam that governs for shear is  $w_u = 73.0 \text{ kN/m}$ .

Figure 11.44 The beam in Example 11.9.



**Solution** According to ACI 18.6.5.1, the design shear forces  $V_e$  are computed from statics assuming that moments of opposite sign corresponding to the probable flexural strength  $M_{pr}$  act at the joint faces and that the member is loaded with tributary factored gravity load along its span. Sidesway to the right and to the left must be considered when calculating the maximum design shear forces.

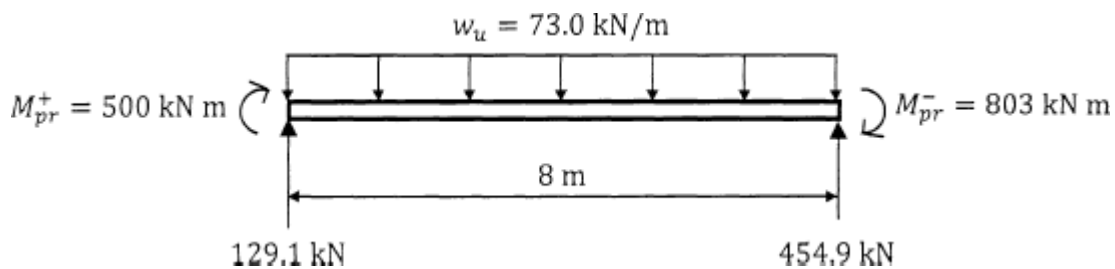
The probable flexural strengths  $M_{pr}$  for the top and bottom bars in this beam are the following:

$$\begin{aligned} \text{For seven No. 29 top bars: } M_{pr}^- &= A_s^- (1.25 f_y) \left( d - \frac{A_s^- f_y}{1.36 f_c' b_w} \right) \\ &= 4,515 \times 1.25 \times 420 \left( 410 - \frac{4,515 \times 420}{1.36 \times 28 \times 700} \right) / 1 \times 10^6 \\ &= 803 \text{ kN m} \end{aligned}$$

$$\begin{aligned} \text{For four No. 29 bottom bars: } M_{pr}^+ &= A_s^+ (1.25 f_y) \left( d - \frac{A_s^+ f_y}{1.36 f_c' b_w} \right) \\ &= 2,580 \times 1.25 \times 420 \left( 410 - \frac{2,580 \times 420}{1.36 \times 28 \times 700} \right) / 1 \times 10^6 \\ &= 500 \text{ kN m} \end{aligned}$$

Figure 11.45 shows the beam and shear forces due to gravity loads plus probable flexural strengths for sidesway to the right. Due to the symmetric distribution of the negative longitudinal reinforcement in the beam, sidesway to the left gives the same maximum shear force.

Figure 11.45 Design shear forces for the beam in Example 11.9.



In general, shear strength is provided by both concrete  $V_c$  and reinforcing steel  $V_s$ . However, according to ACI 18.6.5.2,  $V_c$  is to be taken as zero when the earthquake-induced shear force calculated in accordance with ACI 18.6.5.1 is greater than or equal to 50% of the total shear force and the factored axial compressive force including earthquake effects is less than  $A_{gf} c' / 20$ . In this example, the beam carries negligible axial forces and the maximum earthquake-induced shear force at the face of the support is equal to  $(803 + 500) / 8 = 162.9$  kN, which is less than 50% of the total shear force  $= 454.9 / 2 = 227.5$  kN. At a distance  $2h = 2 \times 0.47 = 0.94$  m from the face of the support, the earthquake-induced shear force is equal to 162.9 kN (constant along span), and the total shear force  $= 454.9 - (73.0 \times 0.94) = 386.3$  kN. Therefore, at this location,  $162.9 \text{ kN} < 386.3 / 2 = 193.1$  kN. Thus, the nominal shear strength provided by the concrete  $V_c$  can be included:

$$V_c = 0.17 \lambda \sqrt{f_c'} b_w d = 0.17 \times 1.0 \sqrt{28} \times 700 \times 410 / 1,000 = 258.2 \text{ kN}$$

Thus,

$$V_s = \frac{V_u}{\phi} - V_c = \frac{454.9}{0.75} - 258.2 = 348.3 \text{ kN} < 0.66 \sqrt{f_c'} b_w d = 1,002.3 \text{ kN}$$

Also,  $V_s$  is less than  $0.33 \sqrt{f_c'} b_w d = 501.2$  kN.

Because the clear space between the four No. 29 bottom bars is approximately 162 mm (assuming No. 10 hoops), which is greater than 150 mm, crossties are required on each of the interior bars in order to satisfy the lateral support requirements of ACI 25.8.2.3 (ACI 18.6.4.2). The required spacing of No. 10 hoops with two crossties is the following:

$$s = \frac{A_v f_y d}{V_s} = \frac{(4 \times 71) \times 420 \times 410}{348.3 \times 1,000} = 140 \text{ mm}$$

Maximum allowable hoop spacing within a distance of  $2h = 940$  mm from the face of the support at each end of the member is the least of the following (ACI 18.6.4.4):

- $d/4 = 410/4 = 103$  mm (governs)
- 6 diameter of smallest longitudinal bar  $= 6 \times 28.7 = 172$  mm



- 150 mm

Use 10 No. 10 hoops at each end of the beam spaced at 100 mm on center with the first hoop located 50 mm from the face of the support (ACI 18.6.4.4).

Where hoops are no longer required, stirrups with seismic hooks at both ends may be used (ACI 18.6.4.6). At a distance of 950 mm from the face of the support,  $V_u = 385.6$  kN. Therefore, the required stirrup spacing for No. 10 stirrups with two crossties is the following:

$$s = \frac{A_v f_y d}{V_s} = \frac{(4 \times 71) \times 420 \times 410}{[(385.6/0.75) - 258.2] \times 1,000} = 191 \text{ mm}$$

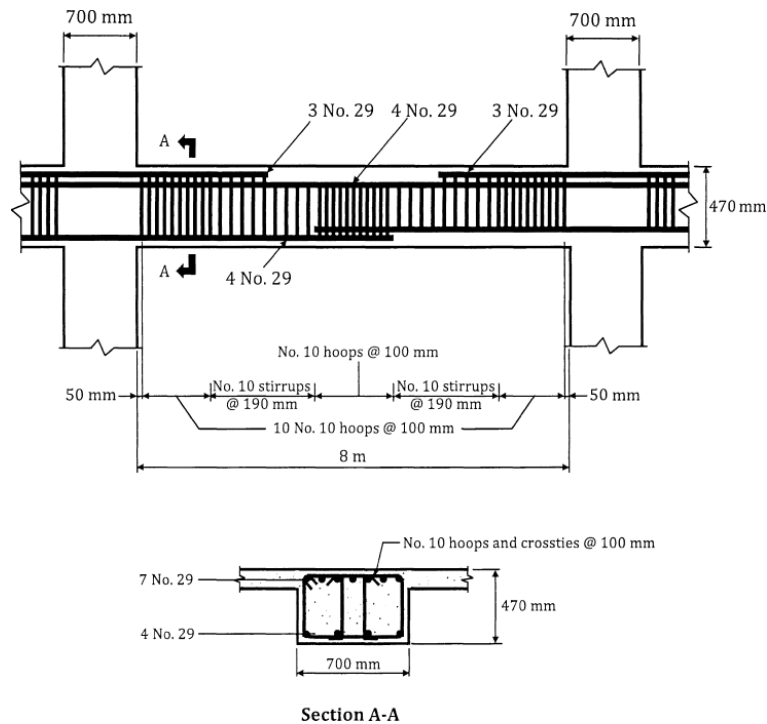
$$\leq \frac{A_v f_y}{0.062 \sqrt{f'_c b_w}} = \frac{(4 \times 71) \times 420}{0.062 \sqrt{28 \times 700}} = 519 \text{ mm}$$

$$\leq \frac{A_v f_y}{0.35 b_w} = \frac{(4 \times 71) \times 420}{0.35 \times 700} = 487 \text{ mm}$$

The maximum allowable spacing of the stirrups is  $d/2 = 205$  mm (ACI 18.6.4.6), which is greater than 191 mm. A 190-mm spacing, starting at 950 mm from the face of the supports is sufficient for the remaining portion of the beam.

Reinforcement details for the beam are shown in Fig. 11.46.

Figure 11.46 Reinforcement details for the beam in Example 11.9.



### 11.7.3. Columns

The provisions for columns that are part of the SFRS in a special moment frame are given in ACI 18.7. These provisions are applicable to any column regardless of the magnitude of the axial force on the column. Table 11.16 contains a summary of the requirements in ACI 18.7.

Table 11.16 Design and Detailing Requirements for Columns in Special Moment Frames

	Requirement	ACI Section
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		Number(s)
<b>Dimensional Limits</b>	Shortest cross-sectional dimension measured on a straight line passing through the geometric centroid of the section $\geq 12$ in (300 mm)	18.7.2.1(a)
	Ratio of the shortest cross-sectional dimension to the perpendicular dimension $\geq 0.4$	18.7.2.1(b)
<b>Minimum Flexural Strength</b>	Flexural strength of columns shall satisfy ACI Eq. (18.7.3.2): $\Sigma M_{nc} \geq (6/5) \Sigma M_{nb}$ $\Sigma M_{nc}$ = sum of nominal flexural strengths of columns framing into the joint, evaluated at the faces of the joint $\Sigma M_{nb}$ = sum of nominal flexural strengths of the beams framing into the joint, evaluated at the faces of the joint. In T-beam construction, where the slab is in tension under moments at the face of the joint, slab reinforcement within an effective slab width defined in ACI 6.3.2 shall contribute to $M_{nb}$ if the slab reinforcement is developed at the critical section for flexure.	18.7.3.2
	If the requirements of ACI 18.7.3.2 are not satisfied at a joint, the lateral strength and stiffness of the columns framing into that joint are to be ignored when determining the strength and stiffness of the structure. These columns must satisfy the provisions of ACI 18.14.	18.7.3.3
<b>Longitudinal Reinforcement</b>	Area of longitudinal reinforcement $A_{st}$ shall satisfy the following: $0.01 A_g \leq A_{st} \leq 0.06 A_g$ .	18.7.4.1
	In columns with circular hoops, a minimum of six longitudinal bars are required in the section.	18.7.4.2
	Mechanical splices shall conform to ACI 18.2.7 and welded splices shall conform to ACI 18.2.8.	18.7.4.3
	Lap splices are permitted only within the center half of the member length and shall be designed as tension lap splices. Lap splices must be enclosed by transverse reinforcement conforming to ACI 18.7.5.2 and 18.7.5.3.	18.7.4.3
<b>Transverse Reinforcement</b>	Transverse reinforcement required in ACI 18.7.5.2 through 18.7.5.4 must be provided over the length $\ell_o$ from each joint face and on both sides of any section where flexural yielding is likely to occur as a result of displacements beyond the elastic range of behavior. Length $\ell_o$ must be greater than or equal to the greatest of the following: (a) Depth of column at the joint face or at the section where flexural yielding is likely to occur (b) Clear span of the column/6 (c) 18 in (450 mm)	18.7.5.1
	Transverse reinforcement is to be provided by: (a) Single or overlapping spirals (b) Circular hoops	18.7.5.2(a)

	(c) Rectilinear hoops with or without cross ties	
Transverse Reinforcement	Bends of rectilinear hoops and cross ties are to engage peripheral longitudinal reinforcing bars.	18.7.5.2(b)
	Cross ties of the same or smaller bar size as the hoops shall be permitted, subject to the limitation of ACI 25.7.2.2. Consecutive cross ties shall be alternated end for end along the longitudinal reinforcement and around the perimeter of the cross-section.	18.7.5.2(c)
	Where rectilinear hoops or cross ties are used, they are to provide lateral support to longitudinal reinforcement in accordance with ACI 25.7.2.2 and 25.7.2.3.	18.7.5.2(d)
	Spacing of cross ties or legs of rectilinear hoops $h_x$ within a cross-section of the column must be less than or equal to 14 in (350 mm).	18.7.5.2(e)
	In columns with rectilinear hoops where the factored axial compressive force $P_u > 0.3A_g f'_c$ or where $f'_c > 10,000$ psi (70 MPa), every longitudinal bar or bundle of bars around the perimeter of the column core must have lateral support provided by the corner of a hoop or by a seismic hook. The value of $h_x$ in such cases must be less than or equal to 8 in (200 mm).	18.7.5.2(f)
	The spacing of transverse reinforcement along the length $\ell_o$ shall not exceed the smallest of the following: (a) Minimum column dimension/4 (b) $6 \times$ diameter of smallest longitudinal bar (c) $s_o$ defined in ACI Eq. (18.7.5.3): $4 \text{ in} \leq s_o = 4 + \left( \frac{14 - h_x}{3} \right) \leq 6 \text{ in}$ $\left[ \text{In SI: } 100 \text{ mm} \leq s_o = 100 + \left( \frac{350 - h_x}{3} \right) \leq 150 \text{ mm} \right]$	18.7.5.3
	Minimum transverse reinforcement shall be in accordance with ACI Table 18.7.5.4: 1. Rectilinear hoops ( $A_{sh}$ ) (a) $P_u \leq 0.3A_g f'_c$ and $f'_c \leq 10,000$ psi (70 MPa) $A_{sh} \geq \text{Greater of } \begin{cases} 0.3s_b c \left( \frac{A_g}{A_{ch}} - 1 \right) \frac{f'_c}{f_{yt}} \\ 0.09s_b c \frac{f'_c}{f_{yt}} \end{cases}$ (b) $P_u > 0.3A_g f'_c$ or $f'_c > 10,000$ psi (70 MPa) $A_{sh} \geq \text{Greater of } \begin{cases} 0.3s_b c \left( \frac{A_g}{A_{ch}} - 1 \right) \frac{f'_c}{f_{yt}} \\ 0.09s_b c \frac{f'_c}{f_{yt}} \end{cases}$	18.7.5.4

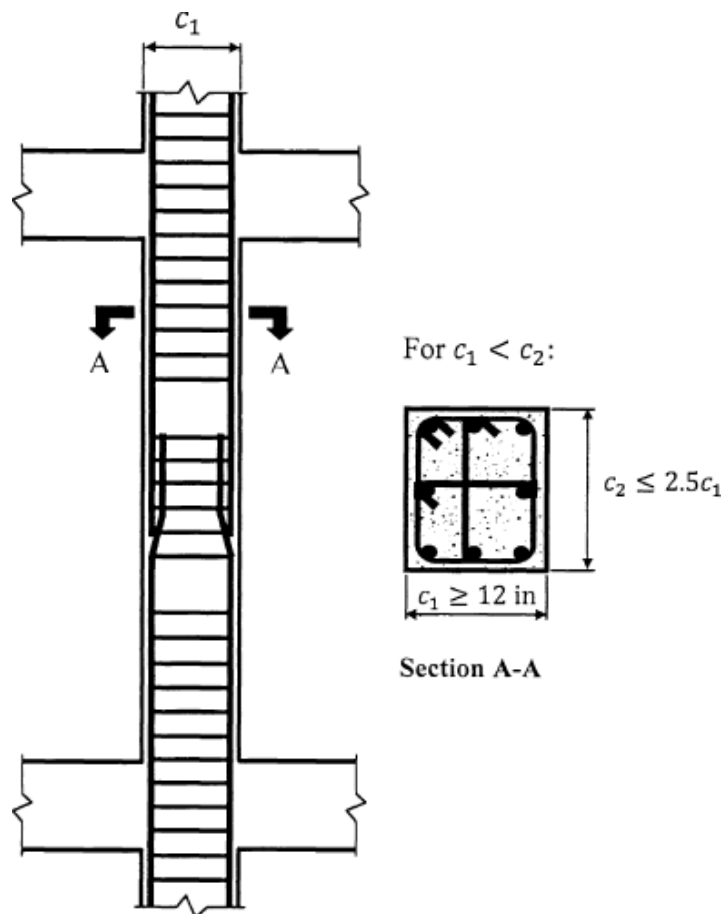
	$\left[ 0.2k_f k_n \frac{P_u}{f_{yt} A_{ch}} s b_c \right]$	
<p><b>Transverse Reinforcement</b></p>	<p>2. Spiral or circular hoops (<math>\rho_s</math>)</p> <p>(a) <math>P_u \leq 0.3A_g f'_c</math> and <math>f'_c \leq 10,000</math> psi (70 MPa)</p> $\rho_s \geq \text{Greater of } \begin{cases} 0.45 \left( \frac{A_g}{A_{ch}} - 1 \right) \frac{f'_c}{f_{yt}} \\ 0.12 \frac{f'_c}{f_{yt}} \end{cases}$ <p>(b) <math>P_u &gt; 0.3A_g f'_c</math> or <math>f'_c &gt; 10,000</math> psi (70 MPa)</p> $\rho_s \geq \text{Greater of } \begin{cases} 0.45 \left( \frac{A_g}{A_{ch}} - 1 \right) \frac{f'_c}{f_{yt}} \\ 0.12 \frac{f'_c}{f_{yt}} \\ 0.35k_f \frac{P_u}{f_{yt} A_{ch}} \end{cases}$	
	<p>Beyond the length <math>\ell_o</math>, spiral or hoop reinforcement satisfying the provisions of ACI 25.7.2 through 25.7.4 must be provided. The spacing of the transverse reinforcement <math>s</math> shall not exceed the smallest of the following:</p> <p>(a) <math>6 \times</math> diameter of smallest longitudinal bar</p> <p>(b) 6 in (150 mm)</p> <p>A larger amount of transverse reinforcement may be required based on the provisions of ACI 18.7.4.3 or 18.7.6.</p>	<p>18.7.5.5</p>
	<p>Columns supporting reactions from discontinued stiff members, such as walls, shall have transverse reinforcement as required in ACI 18.7.5.2 through 18.7.5.4 over their full height at all levels beneath the discontinuity if the factored axial compressive force in these columns related to earthquake effects exceeds <math>A_g f'_c / 10</math>. The limit of <math>A_g f'_c / 10</math> shall be increased to <math>A_g f'_c / 4</math> where design forces have been magnified to account for the overstrength of the vertical elements of the seismic force-resisting system. Transverse reinforcement shall extend above and below the column as required in ACI 18.7.5.6(b).</p>	<p>18.7.5.6</p>
	<p>Additional transverse reinforcement is required in columns where the concrete cover outside of the transverse reinforcement specified in ACI 18.7.5.1, 18.7.5.5, and 18.7.5.6 exceeds 4 in (100 mm). Concrete cover for additional transverse reinforcement shall not exceed 4 in (100 mm) and the spacing of the additional transverse reinforcement shall not exceed 12 in (300 mm).</p>	<p>18.7.5.7</p>

<b>Shear Strength</b>	Design shear force $V_e$ is to be determined considering the maximum forces that can be generated at the faces of the joints at each end of the column. The joint forces are to be determined using $M_{pr}$ at each end of the column associated with the range of factored axial forces $P_u$ . Member shear forces need not exceed those determined from joint strengths based on $M_{pr}$ of the transverse beams framing into the joint.	18.7.6.1
	Transverse reinforcement over the lengths $\ell_o$ shall be proportioned to resist shear forces assuming $V_c = 0$ when both (a) and (b) occur: (a) The earthquake-induced shear force calculated by ACI 18.7.6.1 is greater than or equal to one-half of the maximum required shear strength within $\ell_o$ . (b) The factored axial compressive force $P_u$ , including earthquake effects, is less than $A_g f'_c/20$ .	18.7.6.2

### 11.7.3.1. Dimensional Limits

The geometric constraints prescribed in ACI 18.7.2, which are illustrated in Fig. 11.47, follow from previous practice. The 0.4 limit on the cross-sectional dimensions of a column results in a section that is more compact as opposed to one that is long and rectangular. The 12-inch (300-mm) minimum column dimension is not practical in most special moment frames except possibly for columns in low-rise buildings.

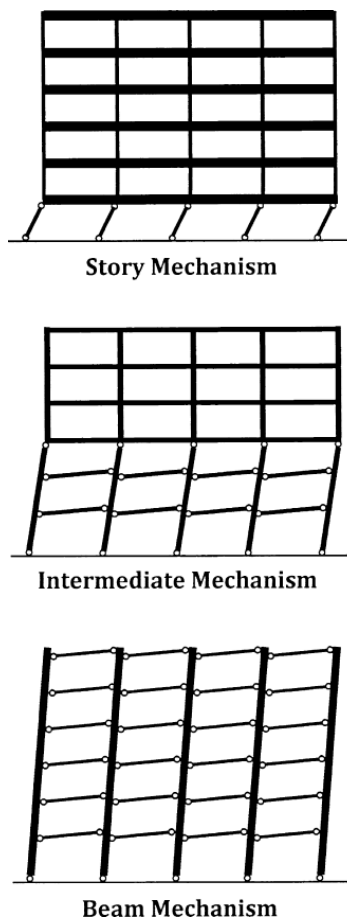
Figure 11.47 Dimensional limits of columns in special moment frames.



### 11.7.3.2. Minimum Flexural Strength

The intent of ACI 18.7.3 is to reduce the possibility of yielding in columns that are part of the SFRS. There is an increased likelihood that inelastic action will occur if the columns in a frame are not stronger than the beams that are framing into the joint. If yielding takes place at both ends of all the columns in a story of a building, a story mechanism could occur that can lead to collapse (see Fig. 11.48). A more desirable situation is where the columns are much stronger than the beams, resulting in yielding at the ends of the beams and not in the columns. This is referred to as a beam mechanism (see Fig. 11.48), and it has been shown that for this type of mechanism to occur, the strengths of the columns must be at least three to four times that of the beams.<sup>10</sup> Because this strength ratio is usually impractical to achieve in practice, ACI 18.7.3 specifies a strength ratio of (6/5), which means that some of the columns are expected to yield (i.e., an intermediate mechanism). As such, the ends of the columns must be properly detailed for possible hinge formation.

Figure 11.48 Mechanisms in a special moment frame.



The flexural strengths of the columns at a joint shall satisfy ACI Eq. (18.7.3.2):

$$\Sigma M_{nc} \geq (6/5) \Sigma M_{nb}$$

(11.8)

where  $\Sigma M_{nc}$  = sum of nominal flexural strengths of columns framing into the joint, evaluated at the faces of the joint

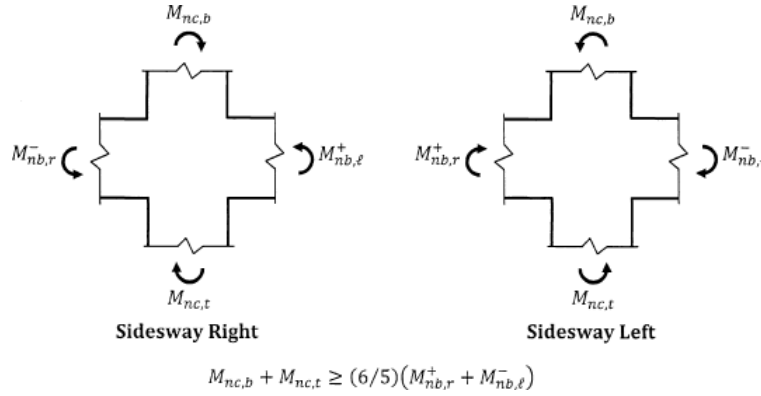
$\Sigma M_{nb}$  = sum of nominal flexural strengths of beams framing into the joint, evaluated at the faces of the joint

Figure 11.49 illustrates this requirement for sidesway to the right and sidesway to the left; this requirement must also be checked in the direction perpendicular to the one shown in the figure. When determining  $M_{nb}$ , ACI 18.7.3.2 requires that the



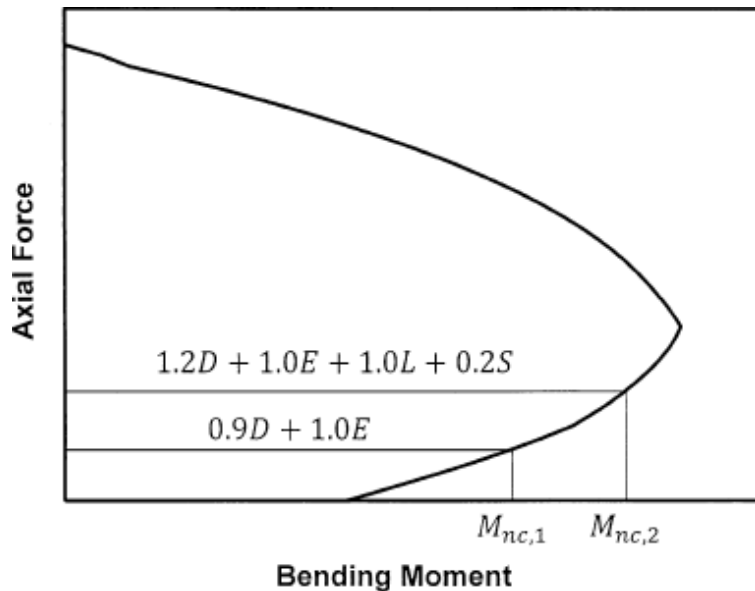
slab reinforcement within the effective slab width defined in ACI 6.3.2 contribute to the flexural strength. It is important to note that even with minimum reinforcement in the slab, the contribution of this reinforcement to  $M_{nb}$  is significant and needs to be included in all cases.

Figure 11.49 Minimum flexural strength requirements for columns in special moment frames.



The nominal moment strength of the column  $M_{nc}$  is dependent on the magnitude of the axial load in the column. Illustrated in Fig. 11.50 is a nominal strength interaction diagram with axial forces corresponding to ACI Eqs. (5.3.1e) and (5.3.1g), which include seismic effects  $E$ . According to ACI 18.7.3.2,  $M_{nc}$  is to be calculated for the factored axial force that is consistent with the direction of analysis that results in the lowest flexural strength. In the example depicted in Fig. 11.50, the lowest flexural strength is  $M_{nc,1}$ , which corresponds to the axial force associated with ACI Eq. (5.3.1g).

Figure 11.50 Nominal strength interaction diagram for a column in a special moment frame.



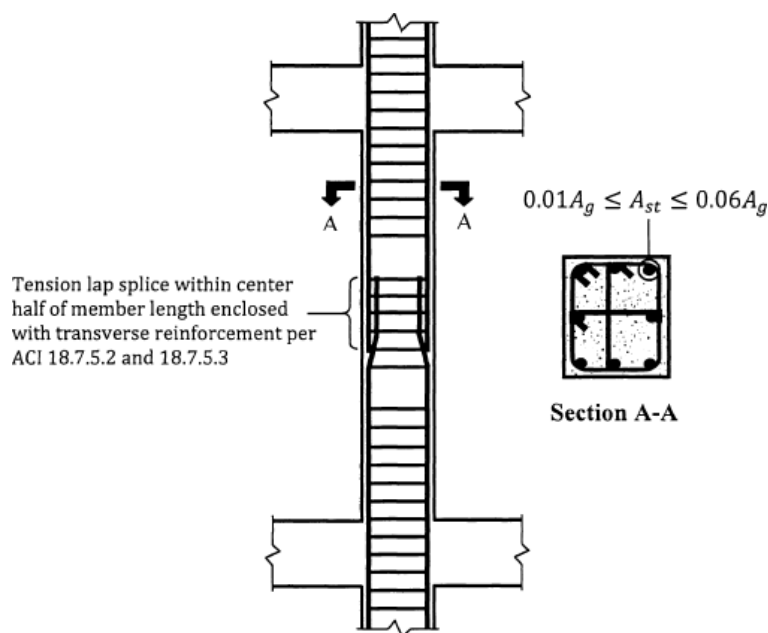
If the provisions of ACI 18.7.3.2 are not satisfied at a joint, any contribution the column at that joint has to the lateral strength and stiffness of the structure is to be ignored, and the column must be designed and detailed in accordance with the requirements of ACI 18.14 for members that are not part of the SFRS; this includes providing transverse reinforcement over the entire length similar to that for columns that are part of the SFRS. Note that there are certain circumstances where these types of columns should not be ignored. For example, the seismic base shear and overall torsional effects should not be reduced because the stiffness of these columns has been disregarded in the overall model of the structure.

### 11.7.3.3. Longitudinal Reinforcement Requirements

Columns must be designed for the combined effects of axial loads and bending moments according to ACI Chap. 10. All factored load combinations must fall within the design axial load–bending moment interaction diagram that is associated with the section (see Chap. 8).

The longitudinal reinforcement requirements of ACI 18.7.4 are illustrated in Fig. 11.51. According to ACI 18.7.4.1, the area of longitudinal reinforcement  $A_{st}$  must not be less than  $0.01A_g$  and must not be greater than  $0.06A_g$  where  $A_g$  is the gross area of the cross-section of the column. In addition to controlling time-dependent deformations, the lower limit ensures that the yield moment exceeds the cracking moment. The upper limit helps control steel congestion and the development of high shear stresses. Because of the potential for reinforcement congestion within the beam–column joints, it is good practice to use a longitudinal reinforcement ratio in the columns of no more than about 2%. Providing a ratio larger than 2% is usually not practical or economical.

Figure 11.51 Longitudinal reinforcement requirements for columns in special moment frames.



Requirements for lap splices of the longitudinal reinforcement are also shown in Fig. 11.51. Because spalling of the concrete shell surrounding the transverse reinforcement is likely to occur at the column ends where the bending moments are expected to be high, lap splices must be located within the center half of the column length. Special transverse reinforcement is required over the lap splice length mainly to help the splice perform as intended when subjected to stress reversals.

Mechanical and welded splices conforming to ACI 18.2.7 and 18.2.8, respectively, may be used instead of lap splices.

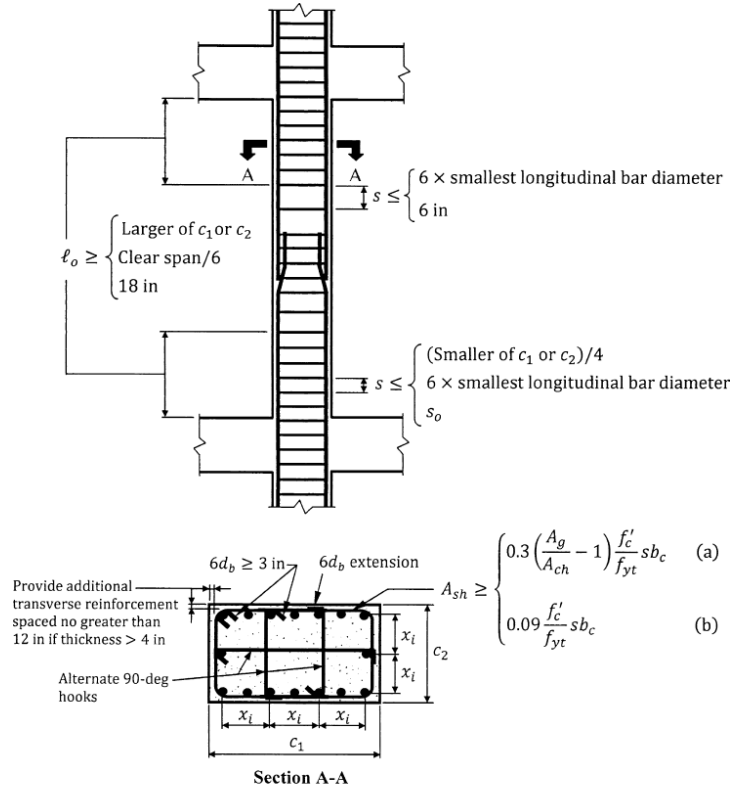
### 11.7.3.4. Transverse Reinforcement Requirements

Closely spaced transverse reinforcement is required over the length  $\ell_o$  at each end of a column to confine the concrete because it is at these locations where the largest bending moments are expected to occur, which could lead to flexural yielding. Rectilinear hoops with or without crossties, circular hoops, and single or overlapping spirals are common forms of transverse reinforcement utilized in columns. At locations where axial loads and bending moments are higher than at other locations in a moment frame, like at the base of a column in the first story of a building, research has shown that  $\ell_o$  should be increased by 50% or more.<sup>11</sup>

Transverse reinforcement requirements are illustrated in Fig. 11.52 for rectilinear hoops where  $P_u \leq 0.3A_g f'_c$  and  $f'_c \leq 10,000$  psi (70 MPa) and in Fig. 11.53 where  $P_u > 0.3A_g f'_c$  and/or  $f'_c > 10,000$  psi (70 MPa). Similar

requirements for spiral and circular hoop reinforcement are given in Fig. 11.54. Spiral reinforcement is generally the most efficient form of confinement reinforcement; however, the extension of the spirals into the beam–column joint usually causes construction difficulties, especially when placing longitudinal reinforcement from the beam through the joint.

**Figure 11.52** Transverse reinforcement requirements for rectilinear hoops in columns of special moment frames [ $P_u > 0.3 A_g f'_c$  and  $f'_c > 10,000$  psi (70 MPa)].

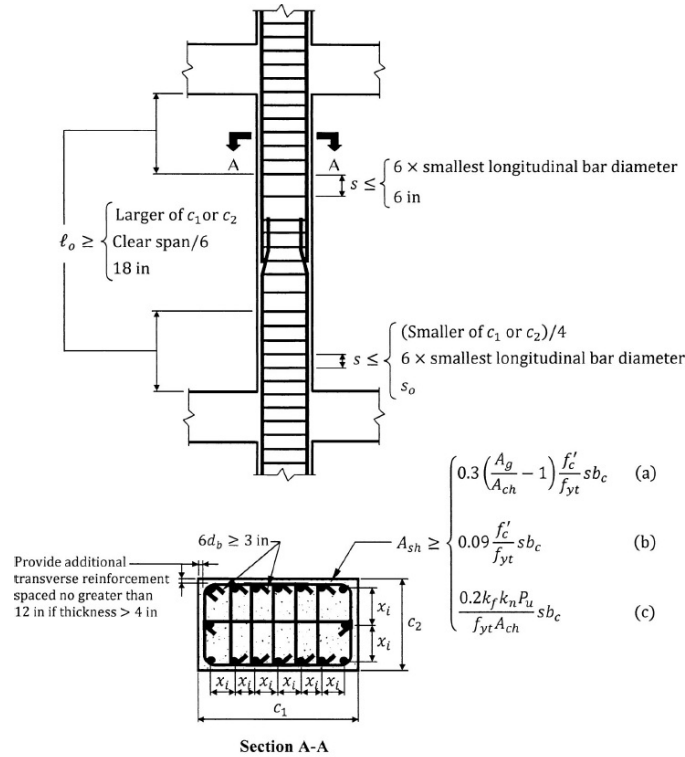


**Notes**

1. In determining  $A_{sh}$ , provisions of ACI 18.7.6 must also be satisfied.
2.  $x_l \leq 14$  in
3.  $h_x =$  largest value of  $x_l$
4.  $4 \text{ in} \leq s_o = 4 + [(14 - h_x)/3] \leq 6 \text{ in}$



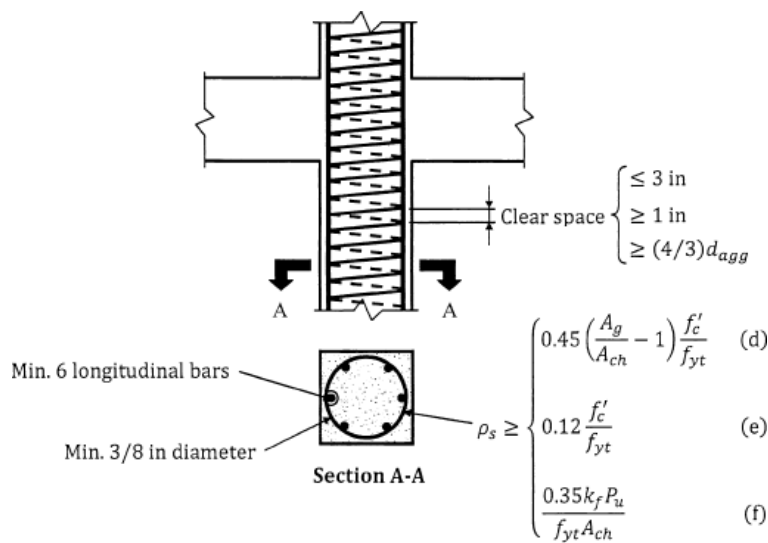
**Figure 11.53** Transverse reinforcement requirements for rectilinear hoops in columns of special moment frames [ $P_u > 0.3 A_g f'_c$  and/or  $f'_c > 10,000$  psi (70 MPa)].



**Notes**

1. In determining  $A_{sh}$ , provisions of ACI 18.7.6 must also be satisfied.
2.  $k_f = (f'_c / 25,000) + 0.6 \geq 1.0$
3.  $k_n = n_t / (n_t - 2)$
4.  $x_1 \leq 8$  in
5.  $h_x =$  largest value of  $x_i$
6.  $4 \text{ in} \leq s_o = 4 + [(14 - h_x) / 3] \leq 6$  in

**Figure 11.54** Transverse reinforcement requirements for spiral or circular hoops in columns of special moment frames.



**Notes**

1. Expression (f) for  $\rho_s$  is applicable only where  $P_u > 0.3 A_g f'_c$  or  $f'_c > 10,000$  psi.
2. In determining  $A_{sh}$ , provisions of ACI 18.7.6 must also be satisfied.
3.  $k_f = (f'_c / 25,000) + 0.6 \geq 1.0$

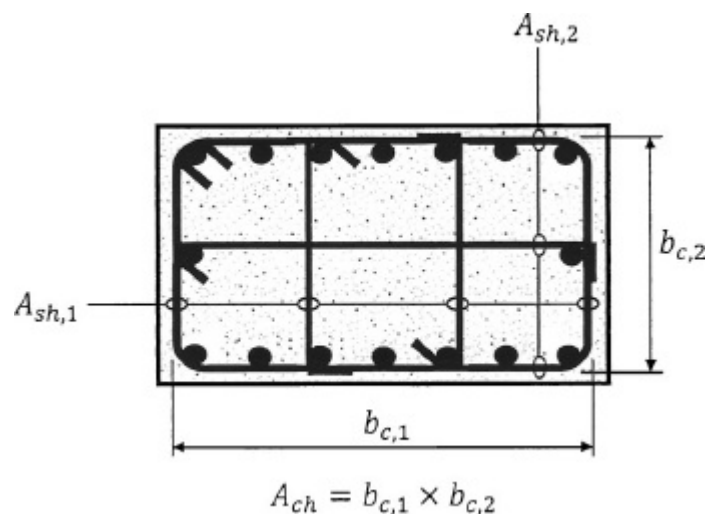
To obtain adequate concrete confinement, the spacing of the transverse reinforcement along the lengths  $\ell_o$  at the ends of the column must not exceed one-quarter of the minimum column dimension. In the event that the concrete shell around the confined core spalls off, the spacing must be less than or equal to six times the diameter of the smallest longitudinal bar in the column; this helps to prevent the longitudinal reinforcement from buckling. The spacing  $s_o$  is also related to concrete confinement. It is permitted to use a 6-in (150-mm) spacing instead of a 4-in (100-mm) spacing along the length  $\ell_o$  where crossties or legs of overlapping hoops are provided at a horizontal spacing no greater than 8 in (200 mm). This provision recognizes the benefits of having additional lateral confinement around the longitudinal bars in the column.

Minimum amounts of transverse reinforcement  $A_{sh}$  are provided in ACI Table 18.7.5.4 and are summarized in Table 11.16. Expressions (a) and (b) in ACI Table 18.7.5.4 for rectilinear hoops and Expressions (d) and (e) for spiral or circular hoops are meant to ensure that spalling of the concrete shell outside of the confined core does not result in a loss of column axial load strength. These expressions are valid where  $P_u \leq 0.3A_g f'_c$  and  $f'_c \leq 10,000$  psi (70 MPa).

Expressions (c) and (f), which are intended to help columns perform as intended when subjected to a lateral drift ratio of 3%, are valid where  $P_u > 0.3A_g f'_c$ ; this corresponds roughly to the onset of compression-controlled behavior for columns with a symmetrical distribution of longitudinal reinforcement. These expressions are also valid where the compressive strength of the concrete in the column exceeds 10,000 psi (70 MPa). The term  $k_n$ , which appears in Expression (c) for rectilinear hoops, is equal to  $n_l / (n_l - 2)$  where  $n_l$  is the number of longitudinal bars around the perimeter of the column core that are laterally supported by the corner of hoops or by seismic hooks. It is evident from Expression (c) that as  $n_l$  increases,  $k_n$  decreases, which results in a smaller amount of minimum required  $A_{sh}$ . The term  $k_f$ , which appears in Expressions (c) and (f), is equal to  $(f'_c / 25,000) + 0.6 \geq 1.0$  [In SI:  $(f'_c / 175) + 0.6 \geq 1.0$ ]. This term increases the minimum required  $A_{sh}$  where  $f'_c$  exceeds 10,000 psi (70 MPa) to help in preventing brittle failure of the column. ACI R18.7.5.4 recommends that concrete strengths greater than 15,000 psi (100 MPa) should be used with caution in columns of special moment frames because of the limited test data that are available on their overall performance.

Minimum required  $A_{sh}$  is determined in both directions of the rectangular core of the column. The term  $b_c$  in these expressions is the dimension of the confined core of the column (measured to the outside edge of the transverse reinforcement) that is perpendicular to the tie legs that make up  $A_{sh}$ . The appropriate  $b_c$  must be used when calculating  $A_{sh}$ . Illustrated in Fig. 11.55 is  $b_c$  and the corresponding  $A_{sh}$  that is to be provided perpendicular to each face of the column. Also shown in the figure is  $A_{ch}$ , which is defined as the cross-sectional area of the column measured to the outside edges of the transverse reinforcement; it is equal to the product of  $b_c$  in both directions and is the area of the confined core of the column.

Figure 11.55 Definition of  $b_c$  for rectilinear hoops in columns in special moment frames.



To ensure a relatively uniform toughness of the column along its full height, spiral or hoop reinforcement satisfying the transverse reinforcement requirements of ACI 25.7.2 and 10.7.4 must be spaced no more than six longitudinal bar diameters

or 6 in (150 mm) in the region beyond the length  $\ell_o$ , unless a larger amount is required to confine lap splices or to increase shear strength.

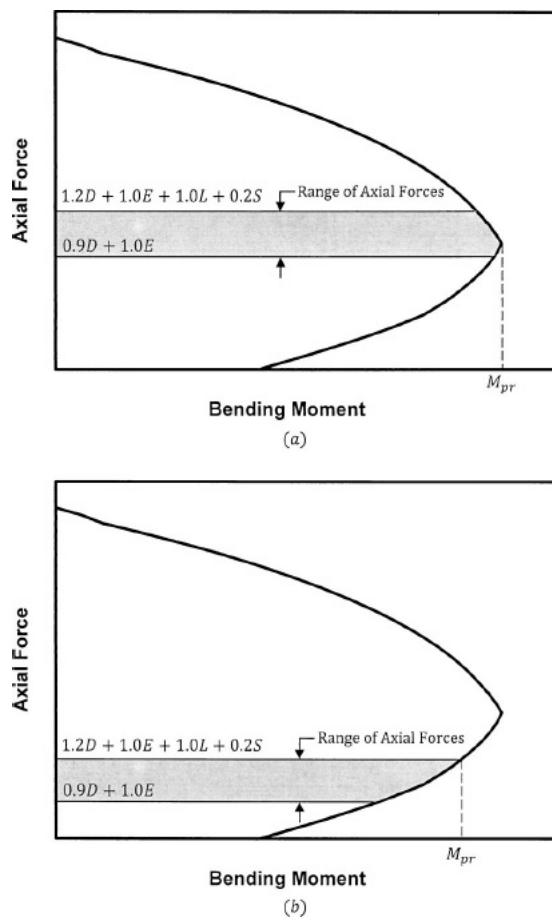
Similar to columns in intermediate moment frames, the transverse reinforcement requirements of ACI 18.7.5.6 must be satisfied for columns in special moment frames that support reactions from discontinuous stiff members, such as walls. See Section 11.6.3 and Fig. 11.20 for information on the appropriate design and detailing requirements for these columns.

### 11.7.3.5. Shear Requirements

Like beams in special moment frames, columns in special moment frames must be designed for the shear forces associated with probable flexural strengths  $M_{pr}$  acting at the ends of the column associated with the range of factored axial loads  $P_u$  acting on the member. Sidesway to the left and sidesway to the right must both be considered to obtain the maximum shear force. One such case is illustrated in ACI Fig. R18.6.5.

Shown in Fig. 11.56a is an interaction diagram of a column that has been constructed using the tensile stress in the longitudinal reinforcing steel equal to  $1.25f_y$  and the strength reduction factor  $\phi$  equal to 1.0. This diagram is a representation of the probable flexural strengths  $M_{pr}$  of a column as a function of factored axial loads. According to ACI 18.7.6.1.1,  $M_{pr}$  at each end of a column must be determined for the range of factored axial forces  $P_u$  acting on the column in the direction of analysis. The range corresponds to the load combinations that include the earthquake effect  $E$ . For the range of  $P_u$  shown in Fig. 11.56a, the largest probable flexural strength occurs at the balanced point even though the factored axial force that corresponds to this moment is not obtained from any of the applicable load combinations. For the range of factored axial forces shown in Fig. 11.56b,  $M_{pr}$  is equal to the moment that is associated with the factored axial force from ACI Eq. (5.3.1e) because that moment is greater than the moment associated with any other factored axial force within that range.

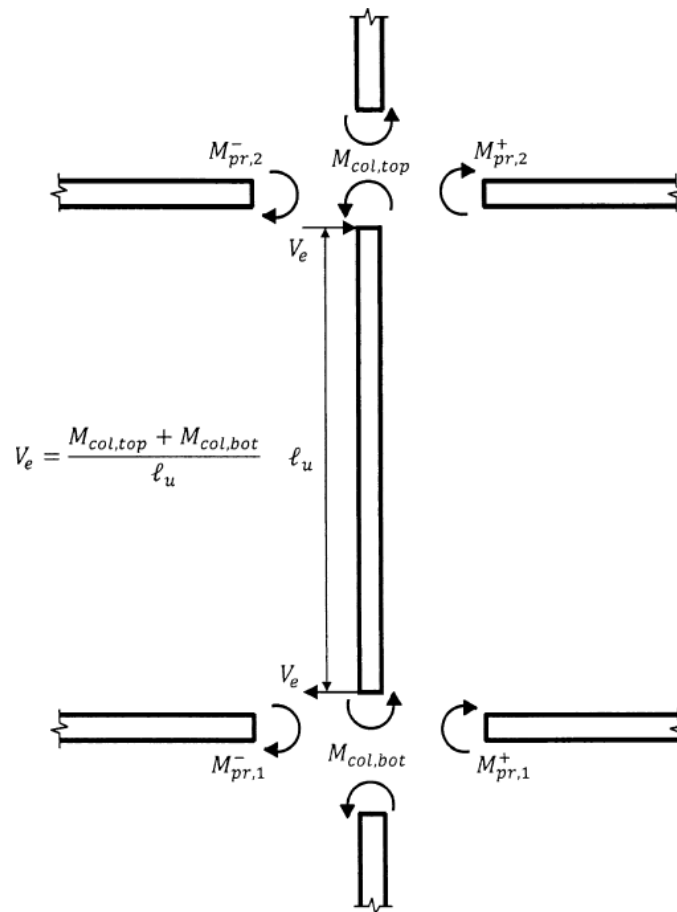
Figure 11.56 Nominal strength interaction diagrams for a column with  $f_y = 75$  ksi and  $\phi = 1.0$ .



Based on the discussion above, the maximum shear force that is obtained from  $M_{pr}$  at each end of the column is  $V_e = (M_{pr, top} + M_{pr, bot})/\ell_u$  where  $M_{pr, top}$  and  $M_{pr, bot}$  are the probable flexural strengths at the top and bottom of the column, respectively, and  $\ell_u$  is the unsupported length of the column.

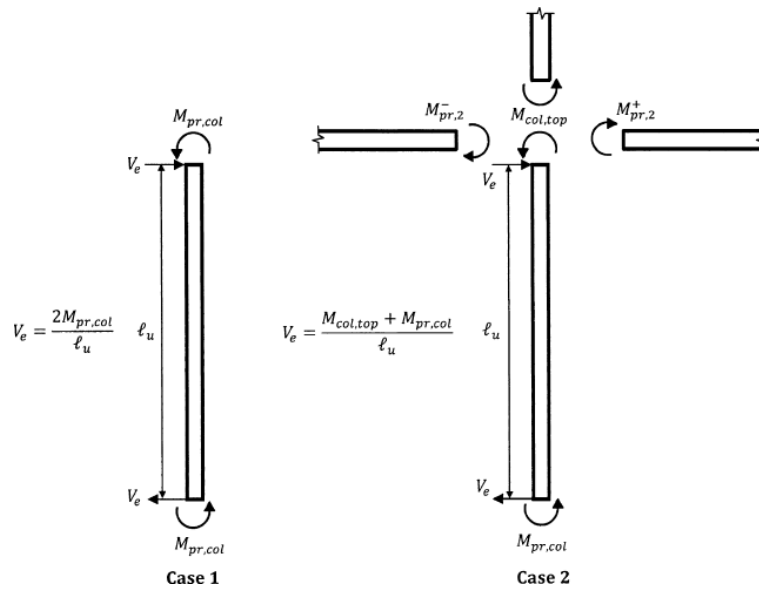
According to ACI 18.7.6.1.1, the maximum shear force in a column need not exceed those that are calculated from joint strengths based on the probable flexural strengths of the beams framing into the joint. Consider the column in Fig. 11.57 where beams frame into opposite sides of the joint. The combined probable flexural strength of the beams may be taken as the sum of the negative probable flexural strength of the beam  $M_n^-$  on one side of the joint and the positive probable flexural strength of the beam  $M_n^+$  on the other side. In general, an analysis must be performed to determine the moments at the top and bottom of the column assuming that the beams develop their probable flexural strengths. Note that the distribution of the combined probable flexural strength of the beams to the column is indeterminate. Once the column moments have been determined, the maximum shear forces can be obtained from statics (see Fig. 11.57).

**Figure 11.57** Maximum shear force in columns of special moment frames based on the probable flexural strengths of the beams framing into the joint.



For columns in the first story that support the first elevated floor, it is possible to develop the probable flexural strength of a column at its base. Thus, for a first-story column, shear forces are computed based on one of two ways: (1) the probable flexural strength of the column acting at both the top and bottom of the column and (2) the probable flexural strength of the column at the base and the moment in the column at the top based on the probable flexural strengths of the beams at the top (see Fig. 11.58). As noted previously, an analysis must be performed to determine the moment at the top of the column.

**Figure 11.58** Maximum shear force in columns located in the first story of special moment frames.

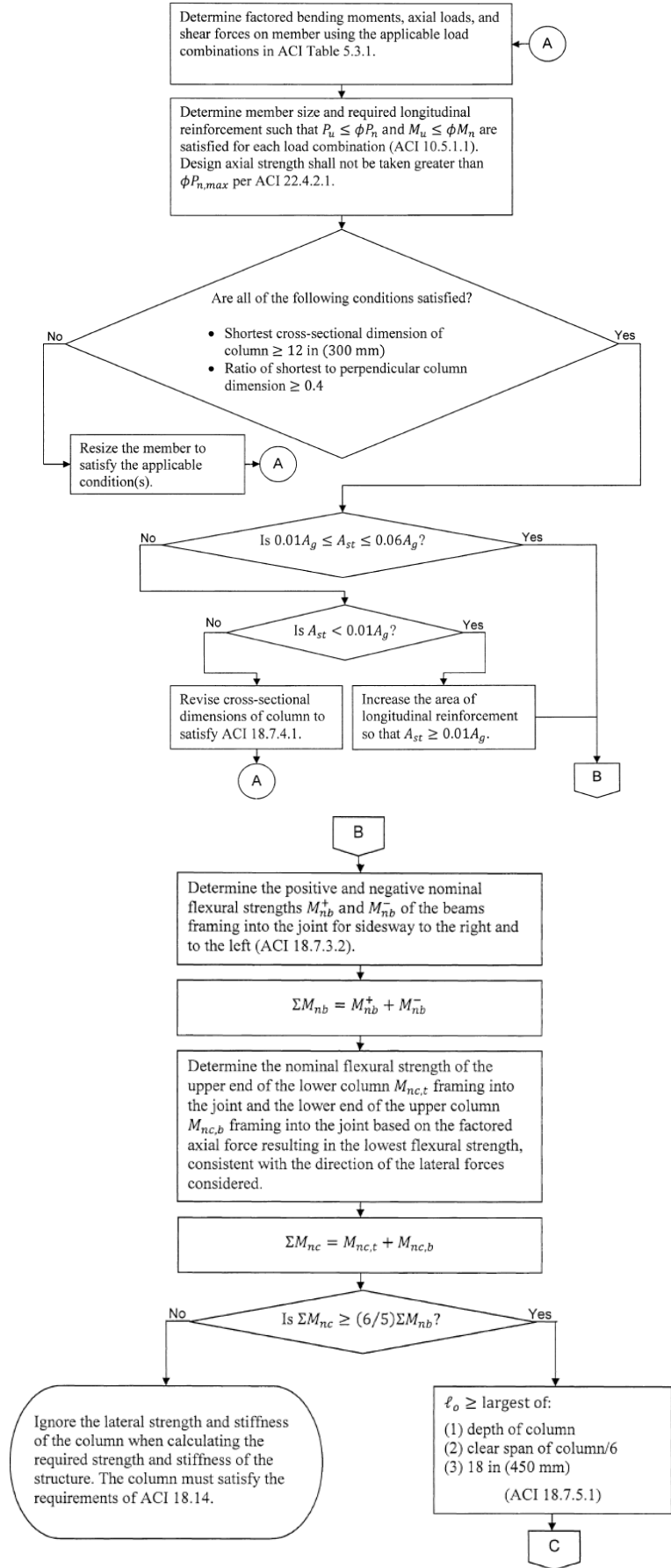


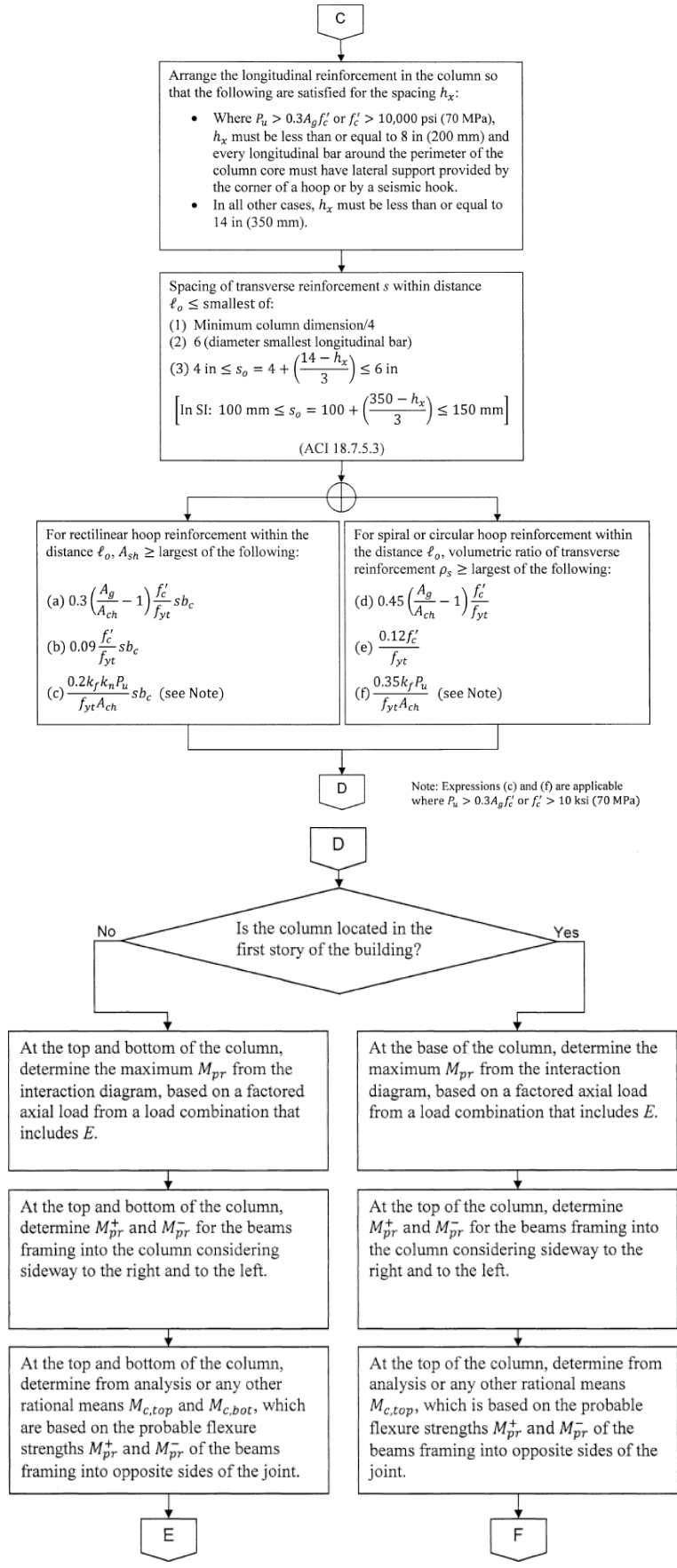
The design shear force must not be taken less than that determined from the structural analysis of the building.

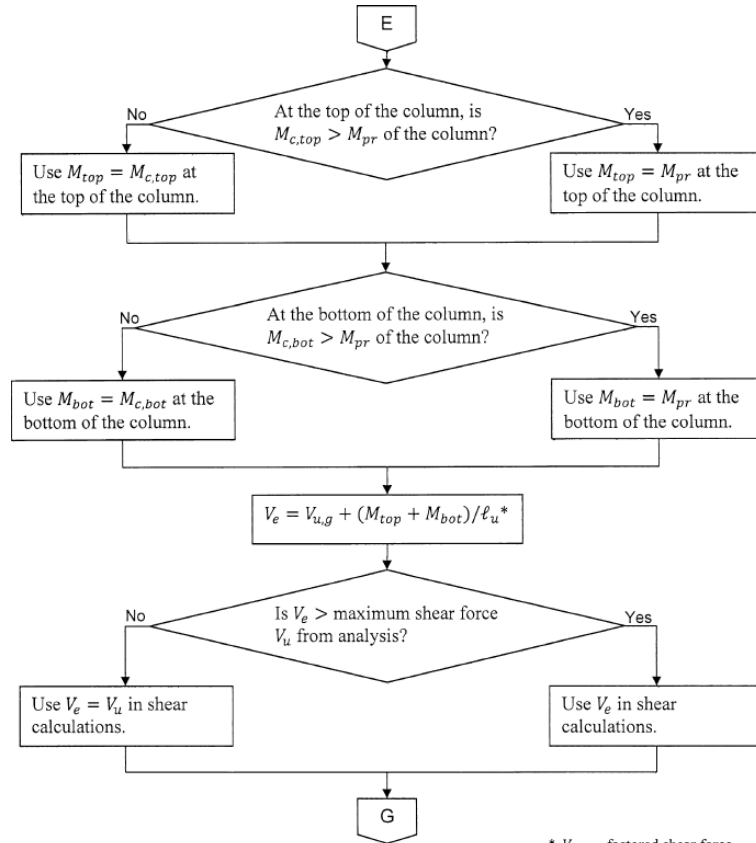
Like in the case of beams in special moment frames, the contribution of concrete shear strength  $V_c$  must be taken equal to zero for columns in special moment frames when the two conditions in ACI 18.7.6.2.1 are satisfied.

A summary of the overall design procedure for columns in special moment frames is given in [Fig. 11.59](#).

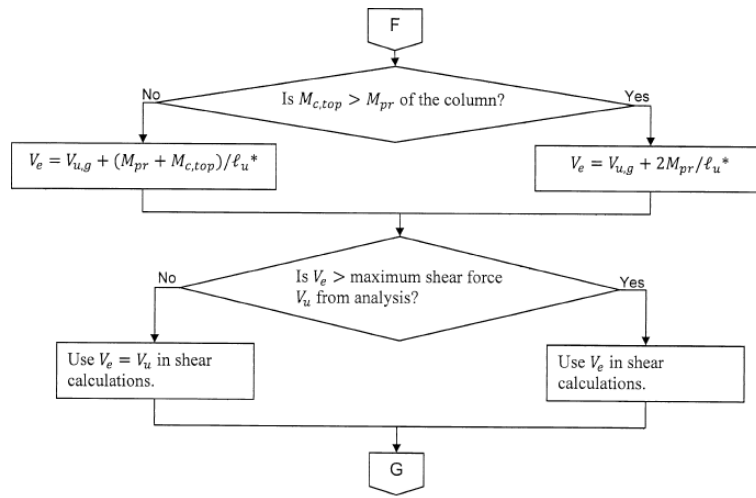
**Figure 11.59** Design procedure for columns in special moment frames.





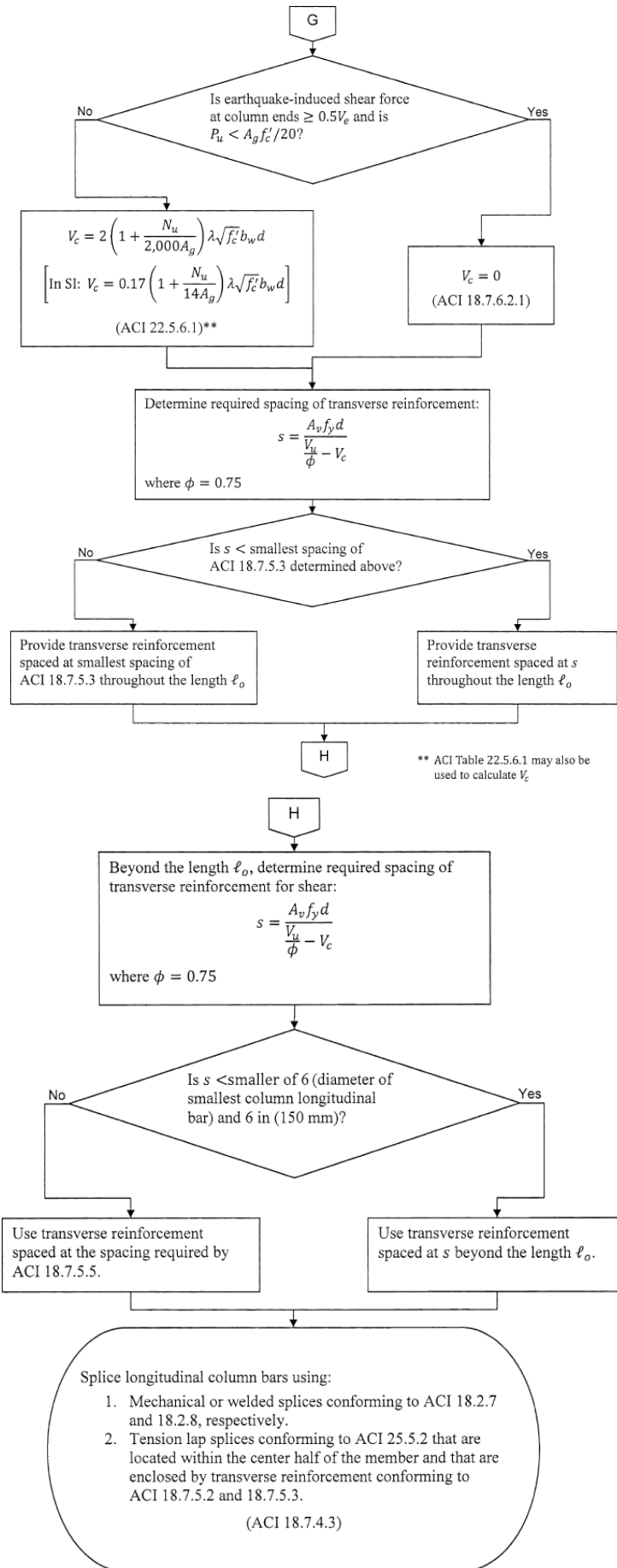


\*  $V_{u,g}$  = factored shear force due to gravity loads



\*  $V_{u,g}$  = factored shear force due to gravity loads



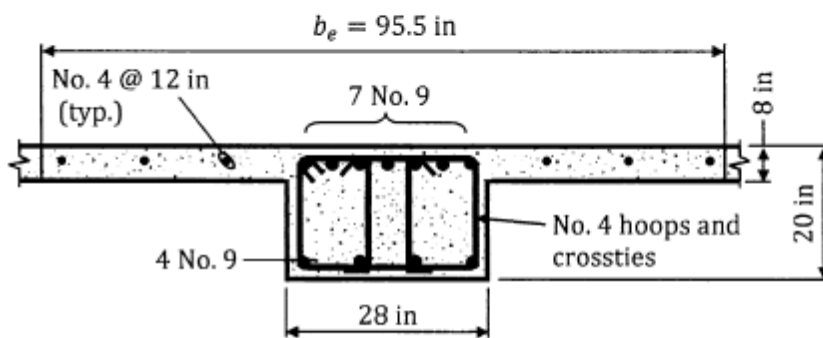
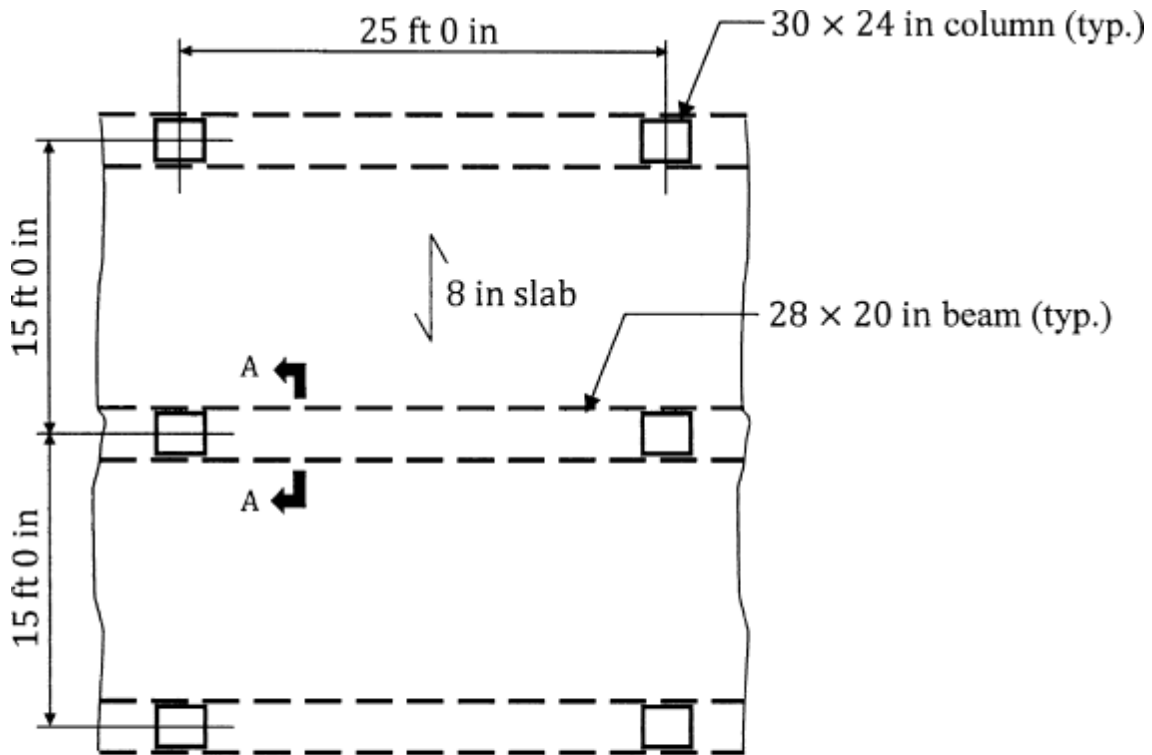


**Example 11.10** A partial plan of a special moment frame is depicted in Fig. 11.60. Determine whether or not the minimum flexural strength requirements of ACI 18.7.3 are satisfied at a typical interior joint. Assume normal-weight concrete with  $f'_c = 8,000$  psi for the columns and  $f'_c = 6,000$  psi for the beams. All reinforcement is Grade 60. The applicable load combinations for the column are given in Table 11.17.

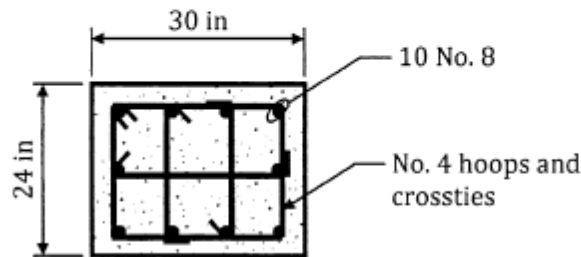
**Table 11.17** Load Combinations for the Column in the Special Moment Frame in Example 11.10

Load Combination		Axial Force (kips)	Bending Moment (ft kips)	Shear Force (kips)
1.4D	ACI Eq. (5.3.1a)	490	0	0
1.2D + 1.6L	ACI Eq. (5.3.1b)	540	0	0
1.3D + 0.5L + $Q_E$	ACI Eq. (5.3.1e)	493	375	73
0.8D + $Q_E$	ACI Eq. (5.3.1g)	280	375	73

Figure 11.60 Partial plan of special moment frame in Example 11.10.



Section A-A



Typ. Column

**Solution** ACI Eq. (18.7.3.2) is checked to determine if the requirements are satisfied or not. The nominal flexural strengths of the columns and beams framing into the joint are determined in accordance with ACI 18.7.3.2. Only the load combinations that include earthquake effects need to be considered when checking the relative strengths of columns and beams.

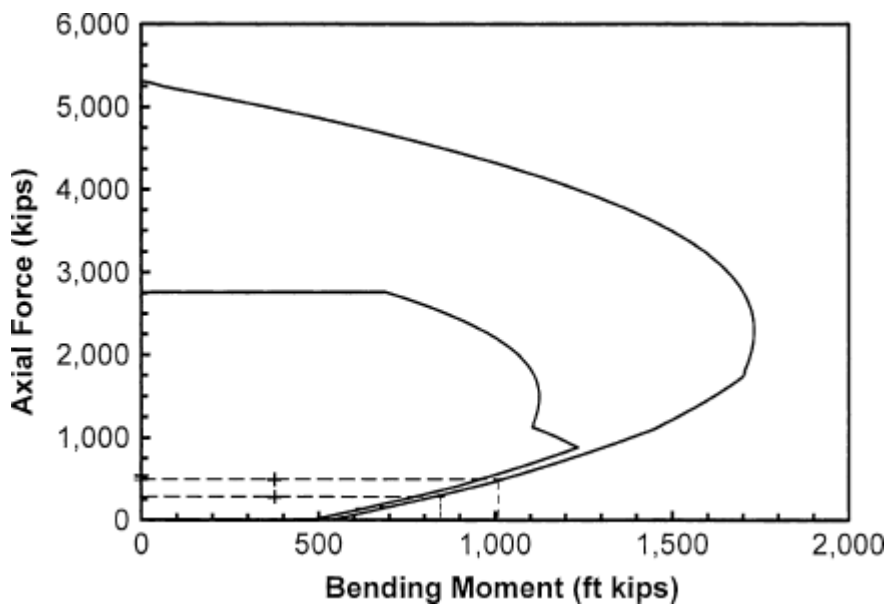
The negative moment strength of the beam at the face of the joint must include the contribution of the No. 4 bars shown in Section A-A in Fig. 11.60; these No. 4 bars are the required temperature and shrinkage reinforcement for the 8-in-thick one-way slab. The slab reinforcement within the effective slab width  $b_e$  defined in ACI 6.3.2 is assumed to contribute to  $M_{nb}$  where  $b_e$  is the smallest of the following:

$$b_e = \begin{cases} 16h + b_w = (16 \times 8) + 28 = 156 \text{ in} \\ s = 15 \times 12 = 180 \text{ in} \\ \ell_n/4 + b_w = [(22.5 \times 12)/4] + 28 = 95.5 \text{ in (governs)} \end{cases}$$

A strain compatibility analysis of the section with the 95.5-in effective width including the No. 9 top bars in the beam and the No. 4 bars in the slab yields  $M_{nb}^- = 649 \text{ ft kips}$ . Note that if the No. 4 bars are not included in the analysis, then  $M_{nb}^- = 560 \text{ ft kips}$ . Similarly, for the positive reinforcement,  $M_{nb}^+ = 396 \text{ ft kips}$ .

The nominal flexural strength of the column is determined for the factored axial compressive force resulting in the lowest flexural strength, consistent with the direction of analysis. For the column below the joint, the axial forces in Table 11.17 are applicable. It is evident from the table and the interaction diagram in Fig. 11.61 that the minimum  $M_n = 855 \text{ ft kips}$ , which corresponds to  $P_u = 280 \text{ kips}$  ( $P_n = 280/0.9 = 311 \text{ kips}$ ). Assuming the column above the joint is the same size and has the same longitudinal reinforcement as the column below the joint, the minimum  $M_n = 743 \text{ ft kips}$ , which corresponds to  $P_u = 179 \text{ kips}$  (this factored axial force was obtained from analysis of the structure).

Figure 11.61 Design and nominal strength interaction diagrams for the column in Example 11.10.



Check ACI Eq. (18.7.3.2):

$$\Sigma M_{nc} = 594 + 581 = 1,175 \text{ ft kips} > \frac{6}{5} \Sigma M_{nb} = \frac{6}{5} \times (378 + 314) = 830 \text{ ft kips}$$

Therefore, the minimum flexural strength requirements of ACI 18.7.3 are satisfied.

**Example 11.11** Given the information in Example 11.10, determine the required transverse reinforcement for a typical interior column in the special moment frame. Assume the column is located above the first story of the building and that the floor-to-floor height is 12 ft 0 in.

**Solution** The required transverse reinforcement is determined using the provisions of ACI 18.7.5 and 18.7.6. Rectilinear hoops and crossties are provided in the rectangular section.

Special transverse reinforcement for confinement is required over a distance of  $\ell_o$  from each joint face at both column ends where  $\ell_o$  is the greatest of the following (ACI 18.7.5.1):

- Depth of column = 30 in (governs)
- Clear span of column/6 =  $[(12 \times 12) - 20]/6 = 20.7 \text{ in}$
- 18 in

Check the applicability of the requirements of ACI 18.7.5.2(f). From Table 11.17, the largest factored axial compressive load including earthquake effects  $E$  is  $P_u = 493 \text{ kips}$ .

$$0.3A_g f'_c = 0.3 \times (22 \times 28) \times 4 = 739 \text{ kips} > P_u = 242 \text{ kips}$$

Therefore, the requirements of ACI 18.7.5.2(f) are not applicable in this example.

Transverse reinforcement within the distance  $\ell_o$  shall not be spaced greater than the smallest of the following (ACI 18.7.5.3):

- Minimum column dimension/4 =  $24/4 = 6$  in
- 6 (diameter of smallest longitudinal bar) =  $6 \times 1.0 = 6$  in
- $s_o = 4 + \left(\frac{14 - h_x}{3}\right) = 4 + \left(\frac{14 - 9.5}{3}\right) = 5.5$  in < 6.0 in (governs)

In the equation for  $s_o$ , the maximum center-to-center spacing of laterally supported longitudinal bars  $h_x$  occurs on the 24-in face of the column. Assuming No. 4 hoops with crossties around every longitudinal bar,  $h_x$  is the following (see Fig. 11.60):

$$h_x = \frac{22 - 2(1.5 + 0.5) - 1.0}{2} = 8.5 \text{ in} < 14 \text{ in}$$

Minimum required cross-sectional area of transverse reinforcement  $A_{sh}$  is the largest value obtained from Expressions (a) and (b) in ACI Table 18.7.5.4 [Expression (c) is not applicable because  $P_u < 0.3A_g f'_c$  and the compressive strength of the concrete in the column is less than 10,000 psi].

$$\text{Expression (a): } A_{sh} = 0.3s b_c \left(\frac{A_g}{A_{ch}} - 1\right) \frac{f'_c}{f_{yt}}$$

$$\text{Expression (b): } A_{sh} = 0.09s b_c \frac{f'_c}{f_{yt}}$$

In the direction parallel to the 24-in column face:

$$b_c = 22 - (2 \times 1.5) = 19 \text{ in}$$

In the direction parallel to the 30-in column face:

$$b_c = 28 - (2 \times 1.5) = 25 \text{ in}$$

Therefore, the area of the confined core  $A_{ch} = 21 \times 27 = 567 \text{ in}^2$ .

Minimum  $A_{sh}$  in the direction perpendicular to the 24-in column face:

$$A_{sh} = 0.3s b_c \left(\frac{A_g}{A_{ch}} - 1\right) \frac{f'_c}{f_{yt}} = 0.3 \times 5 \times 21 \times \left(\frac{24 \times 30}{567} - 1\right) \times \frac{8}{60} = 1.13 \text{ in}^2$$

$$A_{sh} = 0.09s b_c \frac{f'_c}{f_{yt}} = 0.09 \times 5 \times 21 \times \frac{8}{60} = 1.26 \text{ in}^2 \text{ (governs)}$$

Provide No. 6 hoops with one No. 6 crosstie ( $A_{sh,provided} = 1.32 \text{ in}^2$ ) spaced at 5 in on center within  $\ell_o$  (Note: Recalculating  $s_o$  using No. 6 hoops instead of the No. 4 hoops that were originally assumed results in a maximum spacing of 5.6 in, which is rounded down to 5 in).

Minimum  $A_{sh}$  in the direction perpendicular to the 30-in column face:

$$A_{sh} = 0.3s b_c \left(\frac{A_g}{A_{ch}} - 1\right) \frac{f'_c}{f_{yt}} = 0.3 \times 5 \times 27 \times \left(\frac{24 \times 30}{567} - 1\right) \times \frac{8}{60} = 1.46 \text{ in}^2$$

$$A_{sh} = 0.09s b_c \frac{f'_c}{f_{yt}} = 0.09 \times 5 \times 27 \times \frac{8}{60} = 1.62 \text{ in}^2 \text{ (governs)}$$

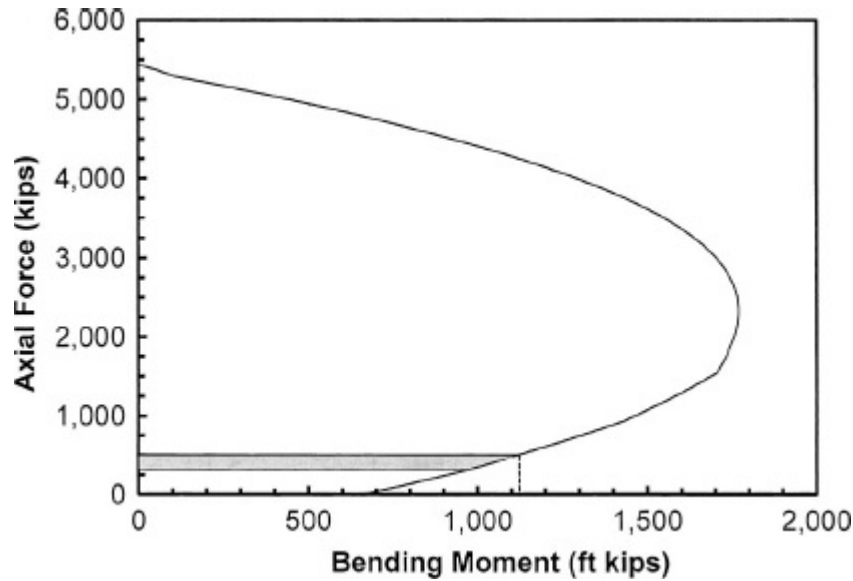
Provide No. 6 hoops with two No. 6 crossties ( $A_{sh,provided} = 1.76 \text{ in}^2$ ) spaced at 5 in on center within  $\ell_o$ .

Determine whether the transverse reinforcement determined above in accordance with ACI 18.7.5.4 is adequate to satisfy the shear requirements of ACI 18.7.6.

Design shear force  $V_e$  is determined using the maximum probable flexural strengths  $M_{pr}$  at each end of the column associated with the range of factored axial loads  $P_u$  acting on the column (ACI 18.7.6.1.1).

The design strength interaction diagram for this column with  $f_y = 1.25 \times 60 = 75$  ksi and  $\phi = 1.0$  is given in Fig. 11.62. The largest  $M_{pr}$  for the range of factored axial forces is equal to 1,113 ft kips, which corresponds to an axial load equal to 493 kips (see Table 11.17).

Figure 11.62 Nominal strength interaction diagram for the column in Example 11.11 with  $f_y = 75$  ksi and  $\phi = 1.0$ .



Therefore,  $V_e$  is equal to the following:

$$V_e = \frac{1,113 + 1,113}{12 - (20/12)} = 216 \text{ kips}$$

As expected, this shear force is greater than the maximum shear force of 73 kips obtained from the structural analysis of the building (see Table 11.17).

When determining the nominal shear strength of the column, the shear strength of the concrete  $V_c$  must be set equal to zero in this example because the earthquake-induced shear force is more than one-half of  $V_e$  (in this example, the earthquake-induced shear force is equal to  $V_e$ ) and the factored axial force including earthquake effects from ACI Eq. (5.3.1g), which is equal to 280 kips, is less than  $A_g f'_c / 20 = 288$  kips (see Table 11.17 and ACI 18.7.6.2.1).

Determine the required spacing of transverse reinforcement in the direction of analysis, which is parallel to the 30-in face of the column:

$$s = \frac{A_v f_y d}{\frac{V_u}{\phi} - V_c} = \frac{(3 \times 0.44) \times 60 \times 20.4}{\frac{216}{0.75} - 0} = 5.6 \text{ in}$$

where  $d$  was determined from a strain compatibility analysis of the section.

Thus, the 5-in transverse reinforcement spacing determined in accordance with ACI 18.7.5 is also adequate for shear strength requirements.

Use No.6 hoops at crossies spaced on 5 in on center over the distance of at least  $\ell_o$  from each end of the column.

The remainder of the column must contain hoop reinforcement with spacing less than or equal to the larger of six times the diameter of the longitudinal bars = 6 in or 6 in (ACI 18.7.5.5). Shear force  $V_e$  is constant over the entire length of the column. Outside the length  $\ell_o$ ,  $V_c$  is permitted to be used, and a spacing can be calculated based on the shear strength contributions of both  $V_c$  and  $V_s$ . However, because there is a maximum 6-in spacing permitted outside the length  $\ell_o$ , which is very close to the spacing permitted within  $\ell_o$ , use a 5-in spacing over the entire column length for simpler detailing. The general layout of the transverse reinforcement follows that shown in Fig. 11.52.

**Comments** If the concrete compressive strength of the column in this example was 12,000 psi instead of 8,000 psi, Expression (c) in ACI Table 18.7.5.4 would also be applicable in determining the minimum transverse reinforcement  $A_{sh}$ :

$$k_f = \frac{f'_c}{25,000} + 0.6 = \frac{12,000}{25,000} + 0.6 = 1.08$$

$$k_n = \frac{n_l}{n_l - 2} = \frac{10}{10 - 2} = 1.25$$

In the direction perpendicular to the 24-in column face:

$$A_{sh} = 0.2k_f k_n \frac{P_u}{f_{yt} A_{ch}} s b_c = 0.2 \times 1.08 \times 1.25 \times \frac{493}{60 \times 567} \times 5 \times 21 = 0.41 \text{ in}^2$$

In the direction perpendicular to the 30-in column face:

$$A_{sh} = 0.2k_f k_n \frac{P_u}{f_{yt} A_{ch}} s b_c = 0.2 \times 1.08 \times 1.25 \times \frac{493}{60 \times 567} \times 5 \times 27 = 0.53 \text{ in}^2$$

The calculations have been presented here for illustration purposes only. If a concrete strength of 12,000 psi was utilized, the axial forces, bending moments, and shear forces on the column would most likely be larger than those presented in this example.

**Example 11.12** Design column A2 in the building shown in Fig. 11.40, which is part of the special moment frame. Assume the column is located in the first story of the building and that the floor-to-floor height is 14 ft 0 in. Also assume normal-weight concrete with  $f'_c = 4,000$  psi and Grade 60 reinforcement. A summary of the applicable load combinations is given in Table 11.18.

**Solution** The flowchart in Fig. 11.59 is used to design this column.

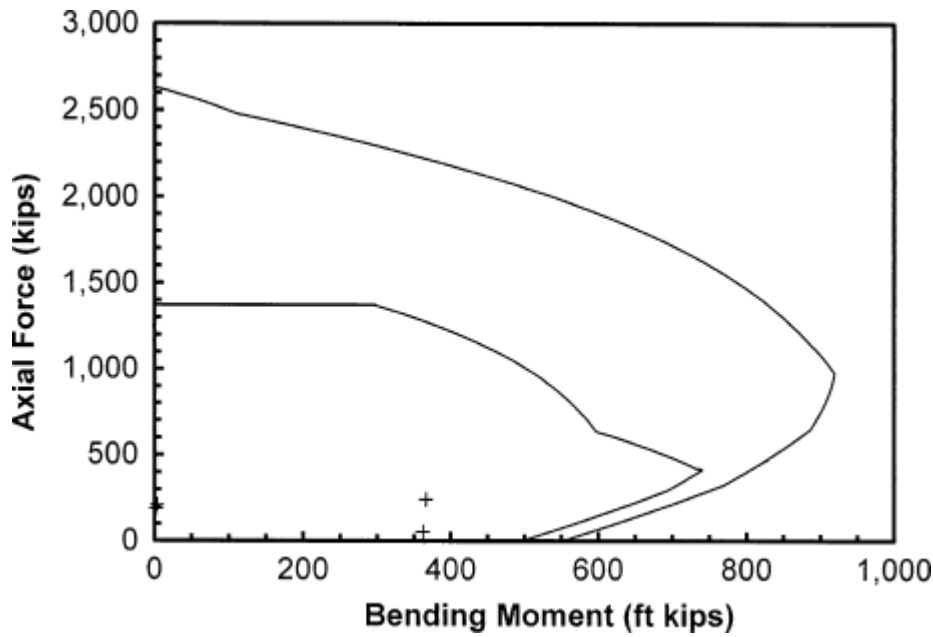
**Table 11.18** Load Combinations for Column in Special Moment Frame in Example 11.12

Load Combination		Axial Force (kips)	Bending Moment (ft kips)	Shear Force (kips)
1.4D	ACI Eq. (5.3.1a)	188	3	0.3
1.2D + 1.6L + 0.5L <sub>r</sub>	ACI Eq. (5.3.1b)	210	4	0.4
1.4D + 0.5L + Q <sub>E</sub>	ACI Eq. (5.3.1e)	242	366	42
0.7D + Q <sub>E</sub>	ACI Eq. (5.3.1g)	55	362	41

**Step 1. Determine the applicable load combinations.** The load combinations are given in Table 11.18.

**Step 2. Determine the column size and required longitudinal reinforcement.** The column size is given in Fig. 11.40 as 22 × 28 in. Based on the load combinations in Table 11.18, a 22 × 28 in column reinforced with 12 No. 8 bars ( $A_{st} = 0.015A_g$ ) is adequate for column A2 supporting the first floor level. The interaction diagram for the column is given in Fig. 11.63. Note that slenderness effects need not be considered for this column. The provided longitudinal reinforcement is within the allowable range specified in ACI 18.7.4.1.

Figure 11.63 Design and nominal strength interaction diagrams for the column in Example 11.12.



**Step 3. Check the dimensional limits of ACI 18.7.2.**

- Shortest cross-sectional dimension = 22 in > 12 in
- Ratio of shortest cross-sectional dimension to the perpendicular dimension = 22/28 = 0.8 > 0.4

Therefore, the dimensional limits are satisfied for this column.

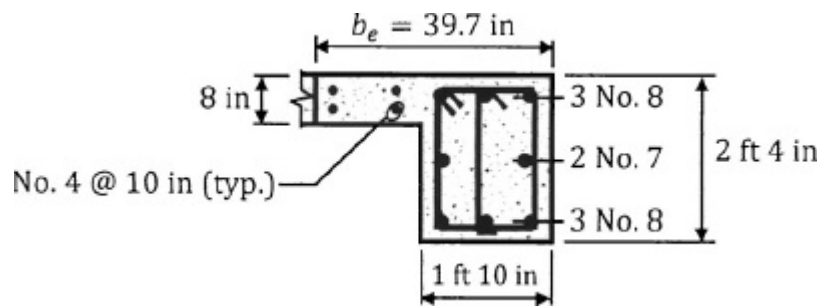
**Step 4. Check the minimum flexural strength requirements for the columns in accordance with ACI 18.7.3.** ACI Eq. (18.7.3.2) is checked to determine if the requirements are satisfied or not. The nominal flexural strengths of the columns and beams framing into the joint are determined in accordance with ACI 18.7.3.2. Only the load combinations that include earthquake effects need to be considered when checking the relative strengths of columns and beams.

The negative moment strength of the beam at the face of the joint must include the contribution of the reinforcement in the slab within the effective width defined in ACI 6.3.2 where  $b_e$  is the smallest of the following (Note: the overhanging flange width related to the clear distance to the adjacent web is not applicable in this case):

$$b_e = \begin{cases} 6h + b_w = (6 \times 8) + 22 = 70.0 \text{ in} \\ \ell_n/12 + b_w = [(17.67 \times 12)/12] + 22 = 39.7 \text{ in (governs)} \end{cases}$$

Design of the two-way slab for gravity loads results in two layers of No. 4 bars spaced at 10 in on center required for flexure at both the top and bottom of the slab as shown in Fig. 11.64. A strain compatibility analysis of this section yields a negative nominal flexural strength  $M_{nb}^- = 378 \text{ ft kips}$  at the face of the joint. Also, the positive nominal flexural strength  $M_{nb}^+ = 314 \text{ ft kips}$ .

Figure 11.64 Flexural reinforcement in the beam in Example 11.12.



The nominal flexural strength of the column is determined for the factored axial compressive force resulting in the lowest flexural strength, consistent with the direction of analysis. For the column below the joint, the axial forces in Table 11.18 are applicable. It is evident from the table and the interaction diagram in Fig. 11.63 that the minimum  $M_n = 594 \text{ ft kips}$ , which corresponds to  $P_u = 55 \text{ kips}$  ( $P_n = 55/0.9 = 61 \text{ kips}$ ). Assuming



the column above the joint is the same size and has the same longitudinal reinforcement as the column below the joint, the minimum  $M_n = 581$  ft kips, which corresponds to  $P_u = 39$  kips (this factored axial force was obtained from analysis of the structure).

Check ACI Eq. (18.7.3.2):

$$\Sigma M_{nc} = 594 + 581 = 1,175 \text{ ft kips} > \frac{6}{5} \Sigma M_{nb} = \frac{6}{5} \times (378 + 314) = 830 \text{ ft kips}$$

Therefore, the minimum flexural strength requirements of ACI 18.7.3 are satisfied.

**Step 5. Determine  $\ell_o$ .** Special transverse reinforcement for confinement is required over a distance of  $\ell_o$  from each joint face at both column ends where  $\ell_o$  is the greatest of the following (ACI 18.7.5.1):

- Depth of column = 28 in (governs)
- Clear span of column/6 =  $[(14 \times 12) - 28]/6 = 23.3$  in
- 18 in

**Step 6. Check if the requirements of ACI 18.7.5.2(f).** From Table 11.18, the largest factored axial compressive load including earthquake effects  $E$  is  $P_u = 242$  kips

$$0.3A_g f'_c = 0.3 \times (22 \times 28) \times 4 = 739 \text{ kips} > P_u = 242 \text{ kips}$$

Therefore, the requirements of ACI 18.7.5.2(f) are not applicable in this example.

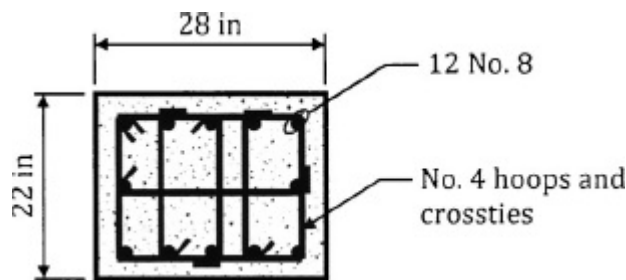
**Step 7. Determine the spacing of the transverse reinforcement within the distance  $\ell_o$ .** Transverse reinforcement within the distance  $\ell_o$  shall not be spaced greater than the smallest of the following (ACI 18.7.5.3):

- Minimum column dimension/4 =  $22/4 = 5.5$  in (governs)
- 6 (diameter of smallest longitudinal bar) =  $6 \times 1.0 = 6$  in
- $s_o = 4 + \left(\frac{14 - h_x}{3}\right) = 4 + \left(\frac{14 - 8.5}{3}\right) = 5.8 \text{ in} < 6 \text{ in}$

In the equation for  $s_o$ , the maximum center-to-center spacing of laterally supported longitudinal bars  $h_x$  occurs on the 22-in face of the column. Assuming No. 4 hoops with crossties around every longitudinal bar,  $h_x$  is the following (see Fig. 11.65):

$$h_x = \frac{22 - 2(1.5 + 0.5) - 1.0}{2} = 8.5 \text{ in} < 14 \text{ in}$$

Figure 11.65 Reinforcement layout for the column in Example 11.12.



**Step 8. Determine the minimum required cross-sectional area of transverse reinforcement  $A_{sh}$  within the distance  $\ell_o$ .** The minimum  $A_{sh}$  is the largest value obtained from Expressions (a) and (b) in ACI Table 18.7.5.4 [Expression (c) is not applicable because  $P_u < 0.3A_g f'_c$  and the compressive strength of the concrete in the column is less than 10,000 psi].

$$\text{Expression (a): } A_{sh} = 0.3s b_c \left( \frac{A_g}{A_{ch}} - 1 \right) \frac{f'_c}{f_{yt}}$$

$$\text{Expression (b): } A_{sh} = 0.09s b_c \frac{f'_c}{f_{yt}}$$

In the direction parallel to the 22-in column face:

$$b_c = 22 - (2 \times 1.5) = 19 \text{ in}$$

In the direction parallel to the 28-in column face:

$$b_c = 28 - (2 \times 1.5) = 25 \text{ in}$$

Therefore, the area of the confined core  $A_{ch} = 19 \times 25 = 475 \text{ in}^2$ .

Minimum  $A_{sh}$  in the direction perpendicular to the 22-in column face:

$$A_{sh} = 0.3sb_c \left( \frac{A_g}{A_{ch}} - 1 \right) \frac{f'_c}{f_{yt}} = 0.3 \times 5 \times 21 \times \left( \frac{24 \times 30}{567} - 1 \right) \times \frac{8}{60} = 1.13 \text{ in}^2$$

$$A_{sh} = 0.09sb_c \frac{f'_c}{f_{yt}} = 0.09 \times 5 \times 21 \times \frac{8}{60} = 1.26 \text{ in}^2 \text{ (governs)}$$

Provide No. 4 hoops with one No. 4 crosstie ( $A_{sh,provided} = 0.60 \text{ in}^2$ ) spaced at 5 in on center within  $\ell_o$ .

Minimum  $A_{sh}$  in the direction perpendicular to the 28-in column face:

$$A_{sh} = 0.3sb_c \left( \frac{A_g}{A_{ch}} - 1 \right) \frac{f'_c}{f_{yt}} = 0.3 \times 5 \times 27 \times \left( \frac{24 \times 30}{567} - 1 \right) \times \frac{8}{60} = 1.46 \text{ in}^2$$

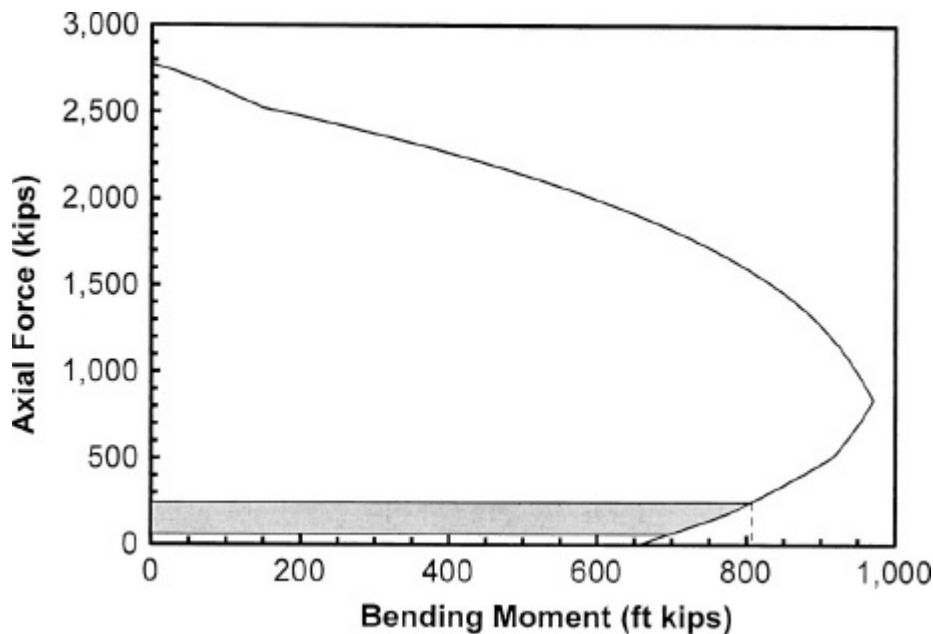
$$A_{sh} = 0.09sb_c \frac{f'_c}{f_{yt}} = 0.09 \times 5 \times 27 \times \frac{8}{60} = 1.62 \text{ in}^2 \text{ (governs)}$$

Provide No. 4 hoops with three No. 4 crossties ( $A_{sh,provided} = 1.0 \text{ in}^2$ ) spaced at 5 in on center within  $\ell_o$ .

**Step 9. Determine the design shear force  $V_e$ .** Design shear force  $V_e$  is determined using the maximum probable flexural strengths  $M_{pr}$  at each end of the column associated with the range of factored axial loads  $P_u$  acting on the column (ACI 18.7.6.1.1).

The design strength interaction diagram for this column with  $f_y = 1.25 \times 60 = 75 \text{ ksi}$  and  $\phi = 1.0$  is shown in Fig. 11.66. The largest  $M_{pr}$  for the range of factored axial forces is equal to 804 ft kips, which corresponds to an axial load equal to 242 kips (see Table 11.18).

Figure 11.66 Nominal strength interaction diagram for the column in Example 11.12 with  $f_y = 75 \text{ ksi}$  and  $\phi = 1.0$ .



Therefore,  $V_e$  is equal to the following:

$$V_e = \frac{804 + 804}{14 - (28/12)} = 138 \text{ kips}$$

As expected, this shear force is greater than the maximum shear force of 42 kips obtained from the structural analysis of the building (see Table 11.18).

**Step 10. Determine if the nominal shear strength of the concrete  $V_c$  can be utilized.** When determining the nominal shear strength of the column, the shear strength of the concrete  $V_c$  must be set equal to zero in this example because the earthquake-induced shear force is more than one-half of  $V_e$  and the factored axial force including earthquake effects from ACI Eq. (5.3.1g), which is equal to 55 kips, is less than  $A_g f'_c / 20 = 123$  kips (see Table 11.18 and ACI 18.7.6.2.1).

**Step 11. Determine the required spacing of the transverse reinforcement over the length  $\ell_o$ .**

$$s = \frac{A_v f_y d}{\frac{V_u}{\phi} - V_c} = \frac{(3 \times 0.2) \times 60 \times 17.8}{\frac{138}{0.75} - 0} = 3.5 \text{ in}$$

where  $d$  was determined from a strain compatibility analysis of the section.

Use No. 4 hoops and crossties spaced at 3.5 in at the column ends over the length  $\ell_o$ .

**Step 12. Determine the required spacing of the transverse reinforcement outside the length  $\ell_o$ .** The remainder of the column must contain hoop reinforcement with spacing less than or equal to the larger of six times the diameter of the longitudinal bars = 6 in or 6 in (ACI 18.7.5.5). Shear force  $V_e$  is constant over the entire length of the column. Outside the length  $\ell_o$ ,  $V_c$  is permitted to be used, and a spacing can be calculated based on the contributions of both  $V_c$  and  $V_s$ . The required spacing is 7.9 in, which is greater than 6 in.

Use No. 4 hoops and crossties spaced at 6 in outside the length  $\ell_o$ .

**Step 13. Determine the lap splice length of the longitudinal reinforcement.** Because all of the bars are to be spliced within the center portion of the column, a Class B splice is required (ACI 25.5.2.1):

$$\ell_{st} = \text{Class B splice length} = 1.3\ell_d$$

The development length  $\ell_d$  is determined by ACI Eq. (25.4.2.3a):

$$\ell_d = \left( \frac{3}{40} \frac{f_y}{\lambda \sqrt{f'_c}} \frac{\psi_t \psi_e \psi_s}{\left( \frac{c_b + K_{tr}}{d_b} \right)} \right) d_b$$

where  $\psi_t$  = modification factor for casting position = 1.0 for other than top bars

$\psi_e$  = modification factor for reinforcement coating = 1.0 for uncoated bars

$\psi_s$  = modification factor for reinforcement size = 1.0 for No. 8 bars

$\lambda$  = modification factor for lightweight concrete 1.0 for normal-weight concrete

$c_b$  = spacing or cover dimension

$$= 1.5 + 0.5 + \frac{1.0}{2} = 2.5 \text{ in (governs)}$$

$$= \frac{22 - 2(1.5 + 0.5) - 1.0}{2 \times 2} = 4.3 \text{ in}$$

$K_{tr}$  = transverse reinforcement index

$$= \frac{40 A_{tr}}{s n} = \frac{40 \times 3 \times 0.2}{4 \times 3} = 2.0$$

$$\frac{c_b + K_{tr}}{d_b} = \frac{2.5 + 2.0}{1.0} = 4.5 > 2.5, \text{ use } 2.5$$

Therefore,

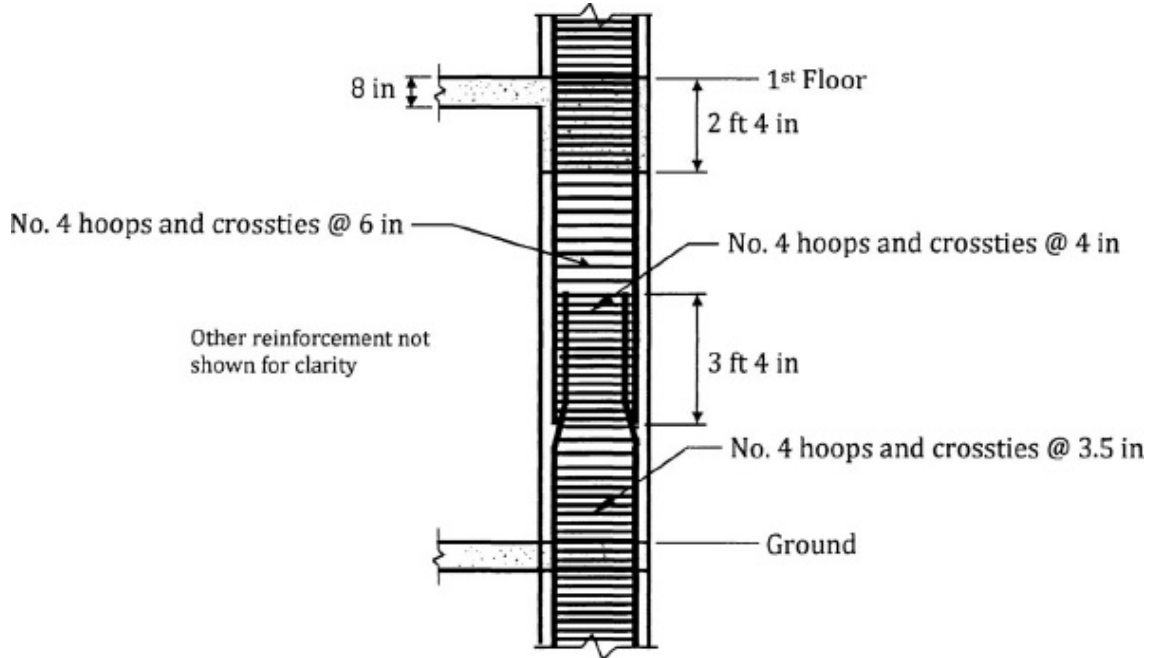
$$\ell_d = \left( \frac{3}{40} \frac{60,000}{1.0 \sqrt{4,000}} \frac{1.0 \times 1.0 \times 1.0}{2.5} \right) \times 1.0 = 28.5 \text{ in} = 2.4 \text{ ft}$$

Class B splice length =  $1.3 \times 2.4 = 3.1$  ft

Use a 3 ft 4 in splice length with No. 4 hoops spaced at the smaller of  $d/4 = 4.5$  in or 4 in (governs) on center over the entire splice length (ACI 18.7.4.3).

Figure 11.67 shows the reinforcement details for this column.

Figure 11.67 Reinforcement details for the column in Example 11.12.



**Comments** It may be desirable from a construction perspective to increase the size of the hoops and crossties so that the spacing at the ends of the column is more than 3.5 in.

## 11.7.4. Joints

Table 11.19 contains the requirements for beam–column joints of special moment frames that are part of the SFRS of the structure (see ACI 18.8).

Table 11.19 Design and Detailing Requirements for Joints in Special Moment Frames General Requirements

	Requirement	ACI Section Number(s)
General	Forces in longitudinal beam reinforcement at joint faces shall be determined assuming the stress in the flexural tensile reinforcement = $1.25f_y$ .	18.8.2.1
	Beam longitudinal reinforcement terminated in a column is to be extended to the far face of the confined column core and is to be anchored in tension in accordance with ACI 18.8.5 and in compression in accordance with ACI 25.4.9.	18.8.2.2
	Where longitudinal beam reinforcement extends through a beam–column joint, the column dimension parallel to the beam reinforcement must be greater than or equal to 20 times the diameter of the largest longitudinal bar in the beam for normal-weight concrete. For lightweight concrete, the column dimension must be greater than or equal to 26 times the bar diameter.	18.8.2.3
	The depth of the joint must be greater than or equal to one-half of the depth of any beam framing into the joint that generates joint shear as part of the SFRS.	18.8.2.4

Requirement		ACI Section Number(s)
Transverse Reinforcement	Joint transverse reinforcement must satisfy the provisions in ACI 18.7.5.2, 18.7.5.3, 18.7.5.4, and 18.7.5.7, except as permitted in ACI 18.8.3.2.	18.8.3.1
	Where beams frame into all four sides of a joint and the width of the beams are greater than or equal to three-quarters the width of the column, the amount of transverse reinforcement through the joint can be reduced to 50% of that required by ACI 18.7.5.4. The required spacing of the transverse reinforcement per ACI 18.7.5.3 is permitted to be increased to 6 in (150 mm) within the overall depth $h$ of the shallowest framing beam.	18.8.3.2
	Longitudinal beam reinforcement outside of the column core shall be confined by transverse reinforcement passing through the column that satisfies the spacing requirements of ACI 18.6.4.4 and the requirements of ACI 18.6.4.2 and 18.6.4.3, if such confinement is not provided by a beam framing into the joint.	18.8.3.3
	Where beam negative moment reinforcement is provided by headed deformed bars that terminate in the joint, the column must extend above the top of the joint a distance greater than or equal to the depth of the joint. Alternatively, the beam reinforcement must be enclosed by additional vertical joint reinforcement providing equivalent confinement to the top face of the joint.	18.8.3.4
Shear Strength	<p>The nominal shear strength of the joint <math>V_n</math> shall not exceed the following values:</p> <ul style="list-style-type: none"> <li>For joints confined by beams on all four faces: <math>20\lambda\sqrt{F'_c}A_j</math> [In SI: <math>1.7\lambda\sqrt{F'_c}A_j</math>]</li> <li>For joints confined by beams on three faces or on two opposite faces: <math>15\lambda\sqrt{F'_c}A_j</math> [In SI: <math>1.2\lambda\sqrt{F'_c}A_j</math>]</li> <li>For all other cases: <math>12\lambda\sqrt{F'_c}A_j</math> [In SI: <math>1.0\lambda\sqrt{F'_c}A_j</math>]</li> </ul> <p><math>\lambda = 0.75</math> for lightweight concrete and 1.0 for normal-weight concrete</p>	18.8.4.1
Shear Strength	A joint face is considered to be confined by a beam if the beam width is at least three-quarters of the effective joint width.	18.8.4.2
	Extensions of beams that are at least one overall beam depth $h$ beyond the joint face are considered adequate for confining that joint face. Extensions of beams shall satisfy ACI 18.6.2.1(b), 18.6.3.1, 18.6.4.2, 18.6.4.3, and 18.6.4.4.	18.8.4.2
	<p><math>A_j</math> = effective cross-sectional area within a joint = joint depth <math>\times</math> effective joint width</p> <p>Joint depth = overall depth of the column</p> <p>Effective joint width = overall width of the column, except where a beam frames into a wider column, effective joint width shall not exceed the lesser of the following:</p> <ol style="list-style-type: none"> <li>beam width + joint depth</li> <li>2 (smaller perpendicular distance from longitudinal axis of beam to column side)</li> </ol>	18.8.4.3

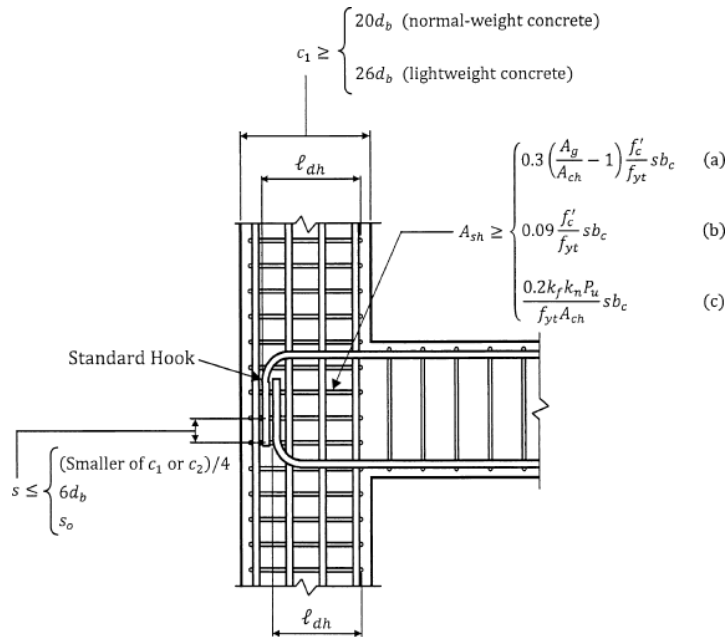
Requirement		ACI Section Number(s)
Development Length of Bars in Tension	<p>For Nos. 3 through 11 (Nos. 10 through 36) bars terminating in a standard 90-degree hook in normal-weight concrete located within the confined core of a column or boundary element with the hook bent into the joint, the development length <math>\ell_{dh}</math> must be greater than or equal to the greater of the following:</p> <ol style="list-style-type: none"> <li><math>f_y d_b / (65 \sqrt{f'_c})</math> [In SI: <math>f_y d_b / (5.4 \sqrt{f'_c})</math>]</li> <li><math>8d_b</math></li> <li>6 in (150 mm)</li> </ol> <p>For lightweight concrete, <math>\ell_{dh}</math> is the greater of the following:</p> <ol style="list-style-type: none"> <li><math>f_y d_b / [(65 \times 0.75) \sqrt{f'_c}]</math> [In SI: <math>f_y d_b / [(5.4 \times 0.75) \sqrt{f'_c}]</math>]</li> <li><math>10d_b</math></li> <li>7.5 in (190 mm)</li> </ol>	18.8.5.1
	For headed deformed bars satisfying ACI 20.2.1.6, the development in tension is to be in accordance with ACI 25.4.4, except the clear spacing between the bars is permitted to be at least $3d_b$ or greater.	18.8.5.2
	For Nos. 3 through 11 (Nos. 10 through 36) bars, the development length $\ell_d$ in tension for a straight bar is the greater of the following: <ol style="list-style-type: none"> <li>2.5 times the length required by ACI 18.8.5.1 if the depth of the concrete cast in one lift beneath the bar <math>\leq</math> 12 in (300 mm)</li> <li>3.25 times the length required by ACI 18.8.5.1 if the depth of the concrete cast in one lift beneath the bar <math>&gt;</math> 12 in (300 mm)</li> </ol>	18.8.5.3
	Straight bars terminated at a joint shall pass through the confined core of a column or boundary element. Any portion of $\ell_d$ not within the confined core shall be increased by a factor of 1.6.	18.8.5.4
	Where epoxy-coated reinforcing bars are used, the development lengths prescribed in ACI 18.8.5.1, 18.8.5.3, and 18.8.5.4 are to be multiplied by the applicable factors in ACI 25.4.2.4 or 25.4.3.2.	18.8.5.5

The overall integrity and performance of a special moment frame is dependent on the behavior of the beam–column joints in the frames. The inelastic rotations at the faces of the joints produce strains in the reinforcement well in excess of the yield strain. Thus, the joint shear force is calculated using a stress of  $1.25f_y$  in the beam longitudinal reinforcement passing through the joint. Ensuring that the joints in a special moment frame are adequate should occur early in the design phase because column and/or beam sizes or the concrete strength may need to change to satisfy joint shear strength requirements.

Slippage of the longitudinal reinforcement through a joint can lead to an increase in joint rotation. Longitudinal bars must be continued through the joint or must be properly developed for tension in accordance with ACI 18.8.5 and for compression according to ACI 25.4.9 in the confined column core. Standard 90-degree hooks are usually used instead of 180-degree hooks because the design provisions are based mainly on research and experience for joints with standard 90-degree hooks. Headed bars are also an option, especially in congested joints.

The minimum column size requirements of ACI 18.8.2.3 for both normal-weight and lightweight concrete are related to the longitudinal bars from the adjacent beams that pass through the joint. The bond stresses on these bars can be very large, so the purpose of these requirements is to help reduce the possibility of bond failure and bar slippage during load reversals that take the longitudinal reinforcement well beyond its yield point. The general requirements of this section are illustrated in Fig. 11.68.

**Figure 11.68** General requirements and transverse reinforcement requirements for rectilinear hoops in joints not confined by beams.



**Notes**

1. For normal-weight concrete:  $\ell_{dh} = \text{Larger of } \begin{cases} f_y d_b / 65 \sqrt{f'_c} \\ 8d_b \\ 6 \text{ in} \end{cases}$
2. For lightweight concrete:  $\ell_{dh} = \text{Larger of } \begin{cases} f_y d_b / (65 \times 0.75) \sqrt{f'_c} \\ 10d_b \\ 7.5 \text{ in} \end{cases}$
3. For epoxy-coated or zinc and epoxy dual-coated bars, multiply  $\ell_{dh}$  by 1.2.
4. Expression (c) for  $A_{sh}$  is applicable only where  $P_u > 0.3A_g f'_c$  or  $f'_c > 10,000$  psi.

ACI 18.8.2.4 contains a dimensional limit that needs to be satisfied at any joint in a special moment frame: The depth of the joint, which is equal to the overall depth of the column in the direction of analysis, must be greater than or equal to one-half of the overall depth of the beam framing into the joint. Tests have demonstrated that joints that are not as deep as the minimum prescribed in this section are less effective in resisting joint shear.

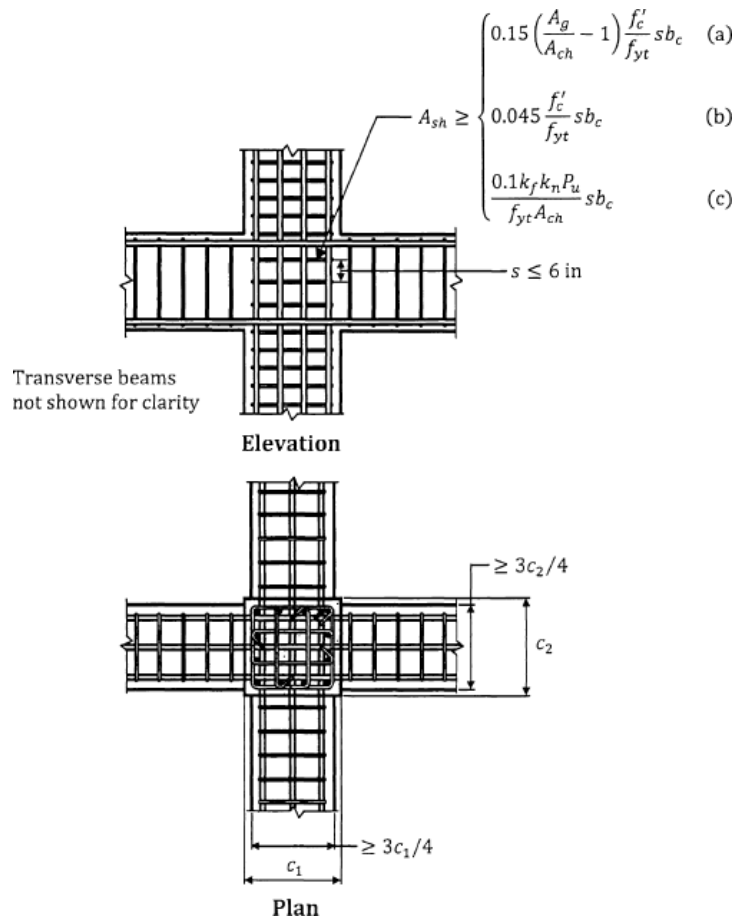
### 11.7.4.1. Transverse Reinforcement

Transverse reinforcement in a beam–column joint is required to adequately confine the concrete to ensure its ductile behavior and to allow it to maintain its vertical load-carrying capacity even after the outer shell were to spall off. A minimum amount of transverse reinforcement is required in a joint regardless of the magnitude of the shear force. The minimum amount is equal to that which is required for potential hinging regions in columns of special moment frames, unless the joint is confined by beams in accordance with ACI 18.8.3.2. Figure 11.68 illustrates the transverse reinforcement requirements for rectilinear hoops when less than four beams frame into the beam–column joint.

Fifty percent of the transverse reinforcement required by ACI 18.7.5.4 may be used within a joint when beams frame into all four sides, provided the widths of the beams are at least three-fourths the corresponding column widths. Beams help resist bursting pressures that can be generated within a joint and this provision recognizes that. The requirements of ACI 18.8.3.2 are shown in Fig. 11.69 for the case of rectilinear hoops. For simpler detailing, the transverse reinforcement at the ends of the columns is usually continued through the joint regardless of the amount of confinement provided by beams that frame into the joint.



**Figure 11.69** Transverse reinforcement requirements for joints with rectilinear hoops that are confined by beams.

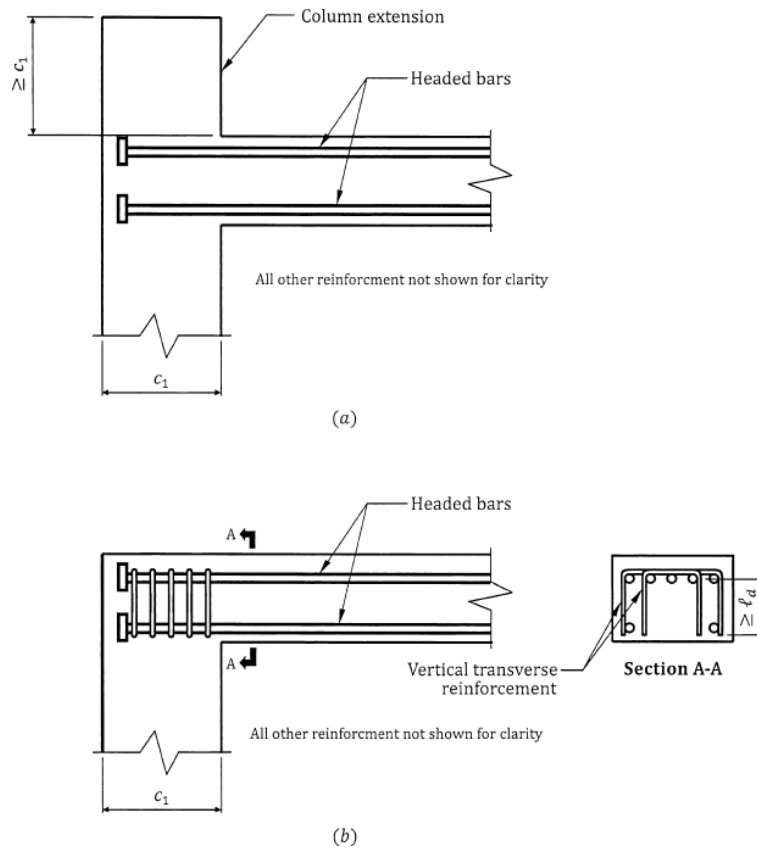


ACI 18.8.3.3 contains provisions for joints where the beam width is greater than the corresponding column width. Beam reinforcement that is not confined by column reinforcement must be confined by transverse reinforcement that satisfies the spacing and detailing requirements at the ends of beams in special moment frames (ACI 18.6.4.2 through 18.6.4.4), unless a beam framing into the joint provides confinement. ACI Fig. R18.6.2 shows an example of the transverse reinforcement that is required to confine the beam longitudinal reinforcement that passes outside of the column core.

Headed bars used as negative flexural reinforcement in beams that are terminated in a joint where there is no column above require confinement along the top face of the joint. ACI 18.8.3.4 provides two options to achieve this confinement. In the first option, the column below the joint is extended above the top of the joint a distance equal to at least the depth of the joint in the direction of analysis (see Fig. 11.70a). Extending the column in this manner may not always be possible due to architectural or other constraints. In the second option, additional vertical joint reinforcement is provided that hooks around the headed bars and extends downward into the joint. Inverted U-shaped stirrups like those illustrated in Fig. 11.70b may be used in such cases.



**Figure 11.70** Confinement for headed bars in joints of special moment frames (a) column extension (b) vertical transverse reinforcement.

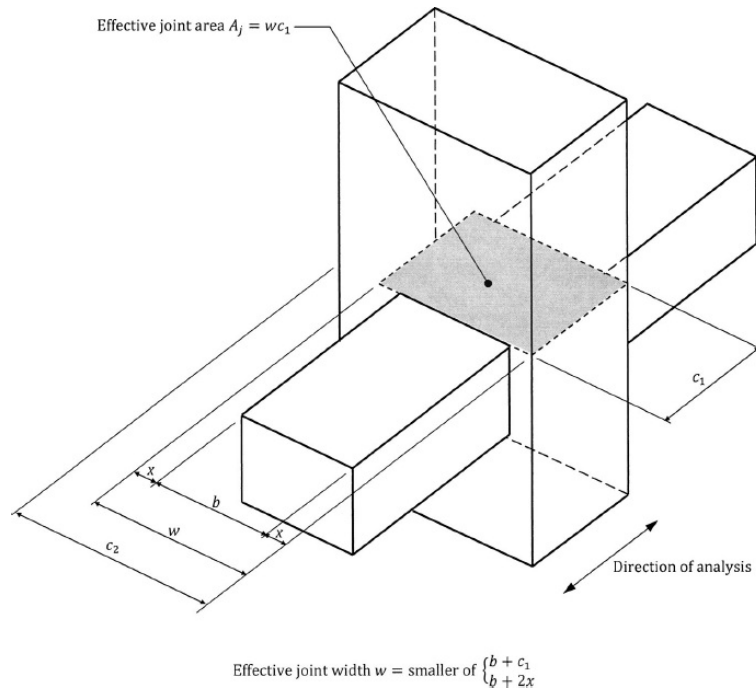


As noted previously, the minimum amount of transverse reinforcement for all of the cases noted above must be provided through the joint regardless of the magnitude of the calculated shear force in the joint.

### 11.7.4.2. Shear Requirements

Shear strength of a joint is a function of the concrete strength and the cross-sectional area of the joint  $A_j$  only. Tests have shown that shear strength is not altered significantly with changes in transverse reinforcement, provided a minimum amount of such reinforcement is present. **Figure 11.71** illustrates the effective joint area that is to be used in the calculation of the nominal shear strength  $V_n$ . The depth of the joint is always the cross-sectional dimension of the column parallel to the direction of analysis, which is denoted as  $c_1$  in the figure. Where the beams are as wide as or wider than the column, the effective joint width is the width of the column perpendicular to the direction of analysis, which is  $c_2$ . The effective joint area in such cases is  $A_j = c_1 \times c_2$ . For beams that are not as wide as the column,  $A_j = w \times c_1$  where the effective joint width  $w$  is defined in ACI 18.8.4.3 and **Fig. 11.71**. In the case of circular columns, a square section that has the same area as the circular section should be used to determine  $A_j$  (ACI R18.8.4).

Figure 11.71 Effective joint area  $A_j$ .



Once  $A_j$  has been determined,  $V_n$  is obtained from the appropriate equation in ACI Table 18.8.4.1. Larger values of  $V_n$  are permitted where more faces of the joints are confined by beams. As noted previously, a beam is assumed to provide confinement to a joint face if its width is greater than or equal to three-quarters of the effective joint width (ACI 18.8.4.2).

The following equation must be satisfied for joint shear strength:

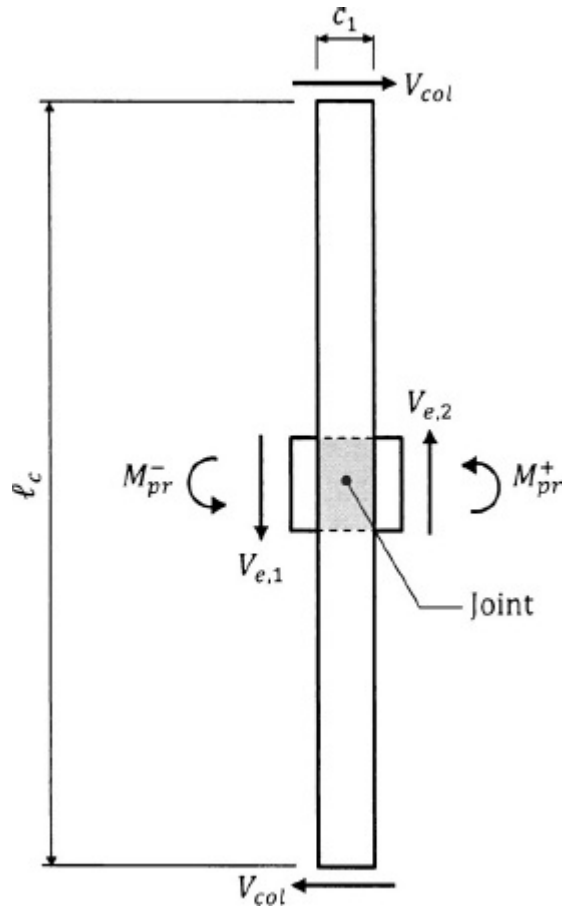
$$V_j \leq \phi V_n = \phi f_v \lambda \sqrt{f'_c} A_j$$

(11.9)

In this equation,  $V_j$  is the required horizontal joint shear;  $\phi$  is the strength reduction factor, which is equal to 0.85 for shear in beam–column joints (ACI 21.2.4.3);  $f_v$  is the strength coefficient defined in ACI Table 18.8.4.1, which is equal to 20, 15, or 12 (In SI: 1.7, 1.2, or 1.0) depending on the level of joint confinement by beams; and,  $\lambda$  is the modification factor for lightweight concrete, which is equal to 1.0 for normal-weight concrete and 0.75 for lightweight concrete.

The required horizontal joint shear  $V_j$  is determined from statics assuming that flexural yielding occurs at the ends of the beams that frame into the joint, that is, the probable flexural strengths  $M_{pr}$  of the beams are developed at the faces of the column. Figure 11.72 shows a free-body diagram of a column including portions of the plastic hinges in the beams that frame into it for the case of sidesway to the right. It is reasonable to assume that points of inflection occur at the midheight of columns that are located above the first story level in a typical moment frame. Therefore, the length  $\ell_c$  in Fig. 11.72 is equal to the depth of the beams plus one-half the clear story height above and below the joint.

Figure 11.72 Shear force in a column of a special moment frame.



The shear force in the column  $V_{col}$  can be obtained by summing moments about the center point at the top or bottom of the column:

$$V_{col} = \frac{M_{pr}^+ + M_{pr}^-}{\ell_c} + \frac{(V_{e,1} + V_{e,2}) \times (c_1/2)}{\ell_c}$$

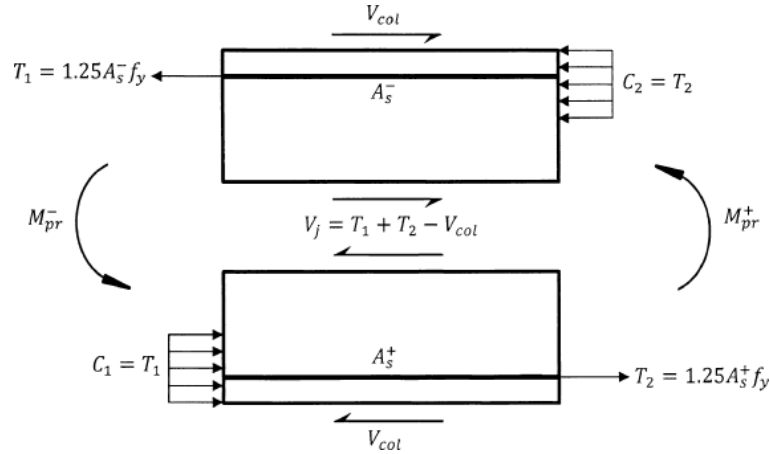
(11.10)

In this equation,  $M_{pr}^+$  and  $M_{pr}^-$  are the positive and negative probable flexure strengths, respectively, of the beams framing into the joint and  $V_{e,1}$  and  $V_{e,2}$  are the design shear forces in the beams due to the factored gravity loads and the development of the probable flexural strengths at both ends of the beam (see Section 11.7.2). It has been assumed in this derivation that the shear forces due to the factored gravity loads are less than the shear forces due to the earthquake effects  $M_{pr}^+$  and  $M_{pr}^-$ ; thus, the net shear force  $V_{e,2}$  on the beam is downward which makes the shear force on the joint upward, as shown in Fig. 11.72. Therefore, its contribution to the net shear force in the column  $V_{col}$  is additive. Where the shear forces from the factored gravity loads are greater than those due to  $M_{pr}^+$  and  $M_{pr}^-$ , the direction of  $V_{e,2}$  on the beam and joint are opposite to those noted above, and  $V_{e,2}$  is subtracted from  $V_{e,1}$  in Eq. (11.10).

Once  $V_{col}$  is determined,  $V_j$  is obtained by considering equilibrium of horizontal forces on the joint. Figure 11.73 shows a free-body diagram of the joint in Fig. 11.72 where it is assumed that the beam is not subjected to any axial forces. The flexural compressive force in the beam on one side of the joint must be equal to the flexural tension force on the same side of the joint to satisfy equilibrium. When determining the force in the longitudinal reinforcement in the beams, the stress is conservatively taken as  $1.25f_y$ . The multiplier of 1.25 takes into account the likelihood that due to strain hardening and actual

strengths higher than the specified yield strengths, larger tensile forces may develop in the bars, which would result in a larger shear force in the joint.

Figure 11.73 Shear force in an interior joint of a special moment frame.



Summing forces in the horizontal direction results in the following expression for  $V_j$ :

$$V_j = 1.25A_s^- f_y + 1.25A_s^+ f_y - V_{col}$$

(11.11)

where  $V_{col}$  is determined by Eq. (11.10).

As noted above, the above derivation is based on the case of sidesway to the right. A similar set of equations can be derived for sidesway to the left. Where the same top reinforcement and the same bottom reinforcement are used in the beams that frame into a joint, the above derivation is also valid for sidesway to the left.

Equations (11.10) and (11.11) are applicable for interior joints with beams on both sides of the joint in the direction of analysis. For edge columns with one beam framing into the joint in the direction of analysis, the following equations can be utilized to determine  $V_{col}$  and  $V_j$ :

For sidesway to the right:

$$V_{col} = \frac{M_{pr}^+}{\ell_c} + \frac{V_e \times (c_1/2)}{\ell_c}$$

(11.12a)

$$V_j = 1.25A_s^+ f_y - V_{col}$$

(11.12b)

For sidesway to the left:

$$V_{col} = \frac{M_{pr}^-}{\ell_c} + \frac{V_e \times (c_1/2)}{\ell_c}$$

(11.13a)

$$V_j = 1.25A_s^- f_y - V_{col}$$

(11.13b)

Because the negative reinforcement in a beam is usually greater than the positive reinforcement, the largest horizontal joint shear  $V_j$  at an edge column is usually obtained from Eq. (11.13b).

The slab reinforcement within the effective slab width defined in ACI 6.3.2 was not included in the above analysis because it is not required by the Code to be included like it is for minimum flexural strength of columns. [Reference 4](#) recommends including the forces in the slab reinforcement when calculating the design horizontal joint shear.

For columns in the first story of a moment frame or for moment frames where the above assumption regarding the location of the point of inflection is not valid, similar analyses can be performed to determine the design horizontal joint shear using appropriate assumptions for the case at hand. Overall analysis results of the moment frame can be used as a guide to locate points of inflection in the columns.

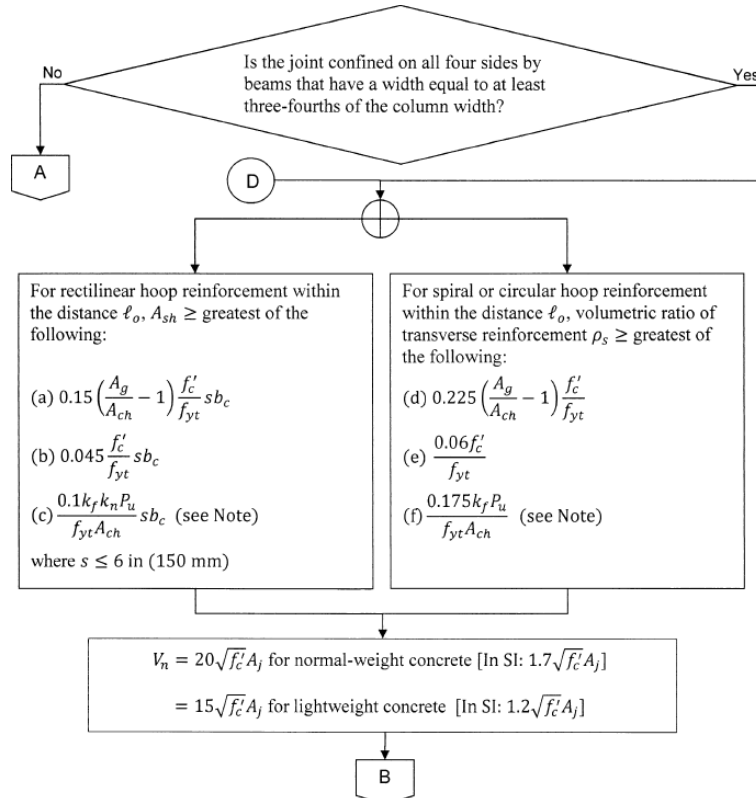
It is evident from the above discussion that the amount of longitudinal reinforcement in the beams has a direct impact on the magnitude of the design horizontal joint shear. That is why it is important not to needlessly provide any more flexural reinforcement in the beams than necessary. When Eq. (11.9) is not satisfied, increasing the column size is the most effective option where there is a large difference between the design and required strengths. Increasing the compressive strength of the concrete may be a viable option where the required joint shear is slightly larger than the design joint shear strength. Increasing the amount of transverse reinforcement in the joint has no effect on its design strength according to the provisions in the Code.

### 11.7.4.3. Development Length of Bars in Tension

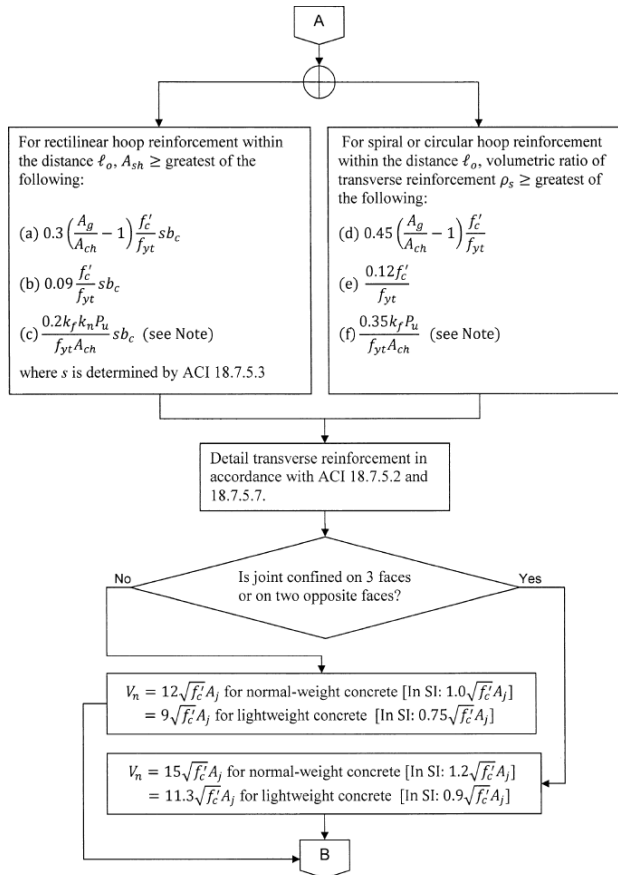
Provisions for the development length of longitudinal beam bars in joints of special moment frames are given in ACI 18.8.5. These requirements are discussed in [Section 11.7.2](#) for beams.

A summary of the overall design procedure for joints in special moment frames is given in [Fig. 11.74](#).

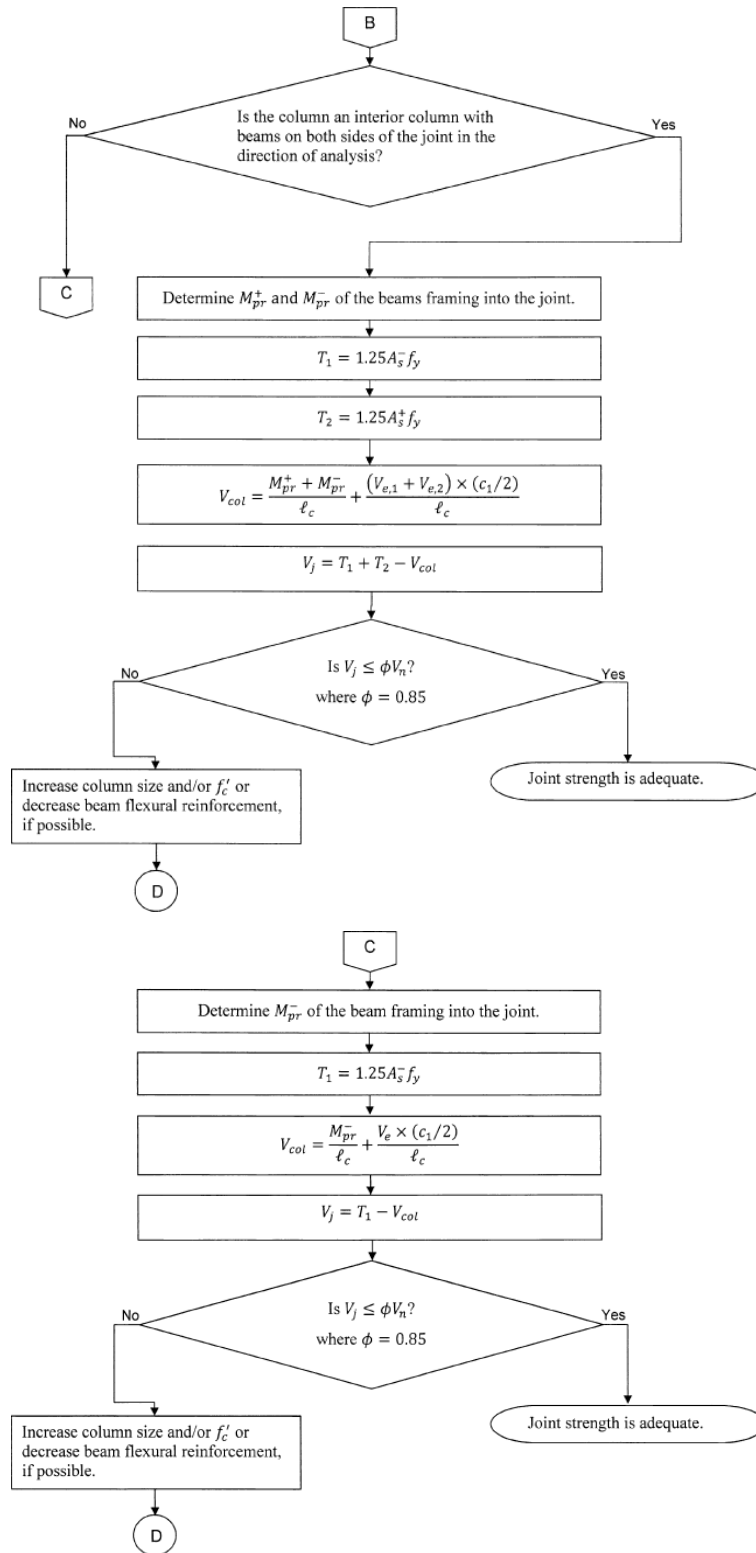
*Figure 11.74 Design procedure for joints in special moment frames.*



Note: Expressions (c) and (f) are applicable where  $P_u > 0.3A_g f'_c$  or  $f'_c > 10$  ksi (70 MPa)



Note: Expressions (c) and (f) are applicable where  $P_u > 0.3A_g f'_c$  or  $f'_c > 10$  ksi (70 MPa)



**Example 11.13** Check the shear strength requirements at the joint located at column lines A and 3 in Fig. 11.40 in the east-west direction. Assume the floor-to-floor heights are equal to 14 ft 0 in.

**Solution** The shear strength requirements at this interior joint are checked using Eq. (11.9).

**Step 1. Determine the required horizontal joint shear  $V_j$ .** Equation (11.11) is used to determine  $V_j$ . It was determined in Example 11.8 that three No. 8 top bars and three No. 8 bottom bars are required as flexural reinforcement in the beam. Therefore,  $A_s^- = A_s^+ = 2.37 \text{ in}^2$ .

The shear force in the column  $V_{col}$  is determined by Eq. (11.10):

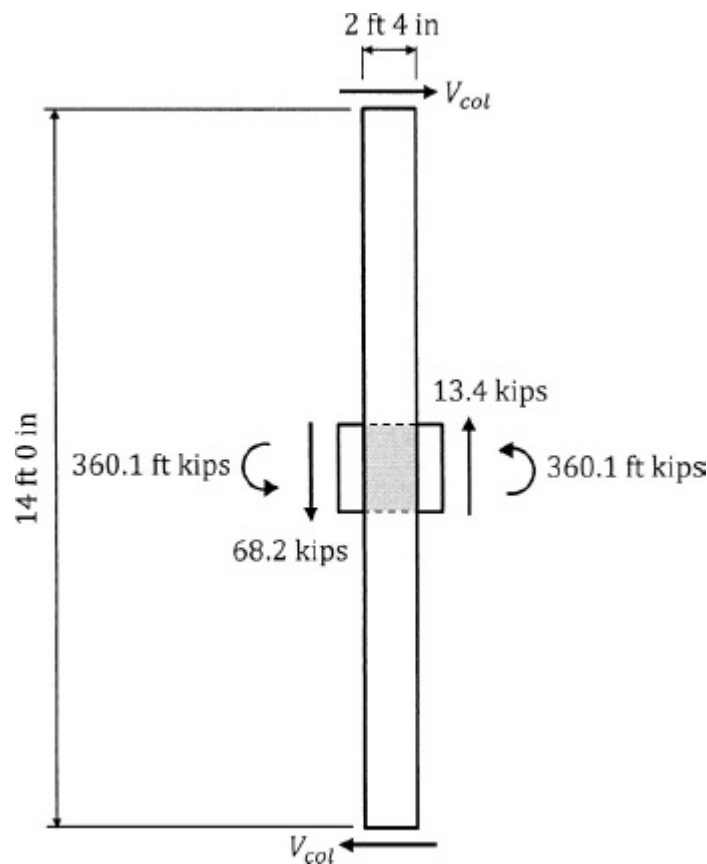
$$V_{col} = \frac{M_{pr}^+ + M_{pr}^-}{\ell_c} + \frac{(V_{e,1} + V_{e,2}) \times (c_1/2)}{\ell_c}$$

It was determined in Example 11.8 that  $M_{pr}^+ = M_{pr}^- = 360.1$  ft kips. The shear forces  $V_{e,1}$  and  $V_{e,2}$  were also determined in Example 11.8 and are given in Fig. 11.41. A free-body diagram of the column is given in Fig. 11.75 where  $\ell_c = 14.0$  ft. Therefore,

$$V_{col} = \frac{360.1 + 360.1}{14.0} + \frac{(68.2 + 13.4) \times (2.33/2)}{14.0} = 58.2 \text{ kips}$$

$$V_j = 1.25A_s^- f_y + 1.25A_s^+ f_y - V_{col} = (2 \times 1.25 \times 2.37 \times 60) - 58.2 = 297.3 \text{ kips}$$

Figure 11.75 Free-body diagram of the column in Example 11.13.



**Step 2. Determine the design horizontal joint shear strength  $\phi V_n$ .** The nominal shear strength of the joint is determined by the appropriate equation in ACI Table 18.8.4.1, which depends on the joint configuration. There are three beams that frame into column A3. In the east-west direction, the beams are the same width as the column, so the joint is confined on these opposite faces. In the north-south direction, the 20-in-wide beam is less than three-quarters of the width of the column, which is equal to 21 in, so that face of the joint is not confined. Therefore, for joints confined on two opposite faces, the nominal and design shear strengths are the following:

$$V_n = 15\lambda\sqrt{f'_c}A_j = 15 \times 1.0\sqrt{4,000} \times (22 \times 28)/1,000 = 584.4 \text{ kips}$$

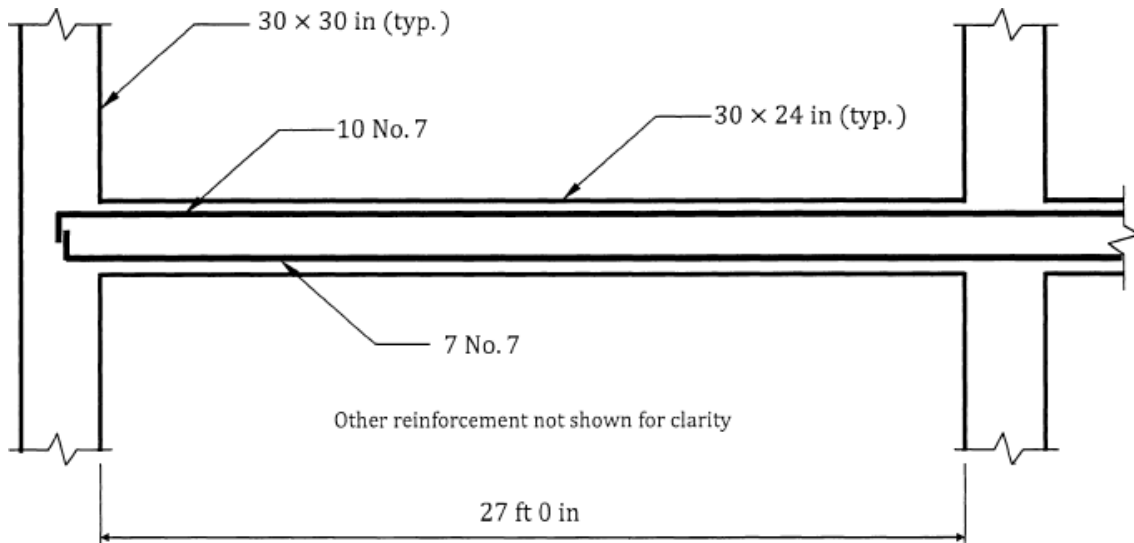
$$\phi V_n = 0.85 \times 584.4 = 496.7 \text{ kips} > 297.3 \text{ kips}$$

Thus, the shear strength of this joint is adequate.

**Example 11.14** Check the shear strength requirements at the joint located at the corner column of the special moment frame depicted in Fig. 11.76. The total factored uniformly distributed gravity load on the beam is 5.5 kips/ft and the floor-to-floor height is 11 ft. Assume normal-weight concrete with  $f'_c = 4,000$  psi and Grade 60 reinforcement.



Figure 11.76 The special moment frame in Example 11.14.



**Solution** The shear strength requirements at this exterior joint are checked using Eq. ( 11.9).

**Step 1. Determine the required horizontal joint shear  $V_j$ .** Equations (11.13a) and (11.13b) are used to determine  $V_j$  for sidesway to the right and to the left, respectively.

$$M_{pr}^+ = A_s^+(1.25f_y) \left[ d - \frac{A_s^+(1.25f_y)}{1.7f_c'b_w} \right]$$

$$= 4.2 \times (1.25 \times 60) \times \left[ 21.5 - \frac{4.2 \times (1.25 \times 60)}{1.7 \times 4 \times 30} \right] / 12 = 523.8 \text{ ft kips}$$

$$M_{pr}^- = A_s^-(1.25f_y) \left[ d - \frac{A_s^-(1.25f_y)}{1.7f_c'b_w} \right]$$

$$= 6.0 \times (1.25 \times 60) \times \left[ 21.5 - \frac{6.0 \times (1.25 \times 60)}{1.7 \times 4 \times 30} \right] / 12 = 723.5 \text{ ft kips}$$

For sidesway to the right:

$$V_e = \frac{5.5 \times 27}{2} - \frac{723.5 + 523.8}{27} = 28.1 \text{ kips}$$

Note that the gravity load reaction at the end of the beam is larger than the reaction due to the probable flexural strengths; thus, the net reaction  $V_e$  is upward on the beam and downward on the joint. The moment on the joint due to  $V_e$  is opposite in direction to  $M_{pr}^+$ .

$$V_{col} = \frac{523.8}{11} - \frac{28.1 \times (2.5/2)}{11} = 44.4 \text{ kips}$$

$$V_j = 1.25A_s^+f_y - V_{col} = (1.25 \times 4.2 \times 60) - 44.4 = 270.6 \text{ kips}$$

For sidesway to the left:

$$V_e = \frac{5.5 \times 27}{2} + \frac{723.5 + 523.8}{27} = 120.5 \text{ kips}$$

The moment on the joint due to  $V_e$  is in the same direction as  $M_{pr}^-$ .

$$V_{col} = \frac{723.5}{11} + \frac{120.5 \times (2.5/2)}{11} = 79.5 \text{ kips}$$

$$V_j = 1.25A_s f_y - V_{col} = (1.25 \times 6.0 \times 60) - 79.5 = 370.5 \text{ kips (governs)}$$

**Step 2. Determine the design horizontal joint shear strength  $\phi V_n$ .** The nominal shear strength of the joint is determined by the appropriate equation in ACI Table 18.8.4.1, which depends on the joint configuration. There is one beam that frames into the corner column in the direction of analysis and one beam that frames into it in the perpendicular direction. Therefore, the nominal and design shear strengths are the following:

$$V_n = 12\lambda\sqrt{f'_c}A_j = 12 \times 1.0\sqrt{4,000} \times (30 \times 30)/1,000 = 683.1 \text{ kips}$$

$$\phi V_n = 0.85 \times 683.1 = 580.6 \text{ kips} > 370.5 \text{ kips}$$

Thus, the shear strength of this joint is adequate.

**Comments** Check if the longitudinal bars in the beam can adequately be developed in the column using the provisions of ACI 18.8.5.1. The development length  $\ell_{dh}$  for bar with a standard 90-degree hook in normal-weight concrete is the greatest of the following:

$$\ell_{dh} = \begin{cases} \frac{f_y d_b}{65\lambda\sqrt{f'_c}} = \frac{60,000 \times 0.875}{65 \times 1.0\sqrt{4,000}} = 12.8 \text{ in (governs)} \\ 8d_b = 8 \times 0.875 = 7.0 \text{ in} \\ 6.0 \text{ in} \end{cases}$$

The required development length can be accommodated within the 30-in deep column. The 90-degree hook is located within the confined core of the column with the hook bent into the joint.

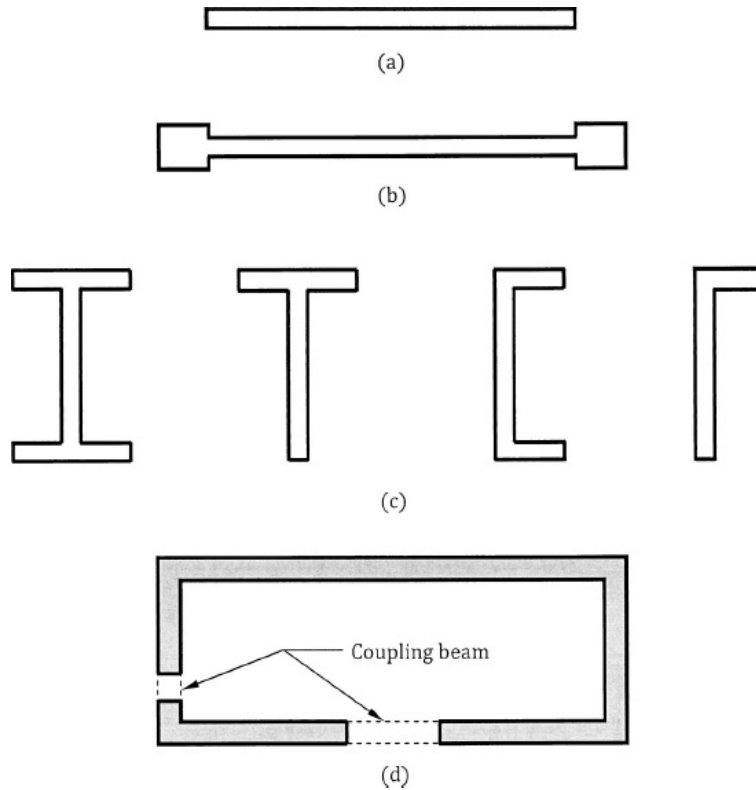
## 11.8. Special Structural Walls

### 11.8.1. Overview

Special structural walls are required in buildings assigned to SDC D, E and F that utilize bearing wall systems, building frame systems, and dual systems. Limitations on height for the different systems can be found in ASCE/SEI Table 12.2-1 (also see [Table 11.6](#)). Note that a dual system consisting of special structural walls (which are referred to as special reinforced concrete shear walls in ASCE/SEI Table 12.2-1) and special moment frames capable of independently resisting at least 25% of the code-prescribed seismic forces can be used in SDC D, E, and F with no limitations. Special structural walls can also be used in a building assigned to the lower seismic design categories, provided all of the design and detailing requirements of ACI 18.10 are satisfied.

Special structural walls can be configured in a variety of ways. Walls with rectangular cross-sections are commonly used and are relatively easy to construct (see [Fig. 11.77a](#)). These walls can perform well during an earthquake as long as they have adequate thickness (the reasons for this will become evident later). In order to achieve greater flexural resistance, columns are sometimes incorporated at the ends of walls, as depicted in [Fig. 11.77b](#). This configuration, which is commonly referred to as a barbell wall because of its shape in plan is generally more expensive to form than a straight rectangular wall and may not be suitable considering the architectural requirements. Walls that intersect at 90-degree angles to each other are shown in [Fig. 11.77c](#). An effective flange width in the direction perpendicular to the direction of analysis can usually be utilized to enhance flexural resistance. Walls that enclose elevators and stairs that are located in the core area of a building can be connected together over the openings in the walls by coupling beams (see [Fig. 11.77d](#)). When properly designed and detailed, these beams can significantly improve the performance of the walls.

Figure 11.77 Structural wall configurations (a) rectangular wall (b) barbell wall (c) intersecting walls (d) core walls.



Provisions for design and detailing of special structural walls are given in ACI 18.10 and are summarized in Table 11.20. Included are the requirements for all of the components of special structural walls including coupling beams and wall piers.

Table 11.20 Design and Detailing Requirements for Special Structural Walls, Coupling Beams, and Wall Piers

	Requirement	ACI Section Number(s)
Reinforcement	<p>Where <math>V_u &gt; A_{cv}\lambda\sqrt{F'_c}</math> [In SI: <math>V_u &gt; 0.083A_{cv}\lambda\sqrt{F'_c}</math>], distributed web reinforcement ratios <math>\rho_e</math> and <math>\rho_t</math> must be greater than or equal to 0.0025.</p> <p>Where <math>V_u \leq A_{cv}\lambda\sqrt{F'_c}</math> [In SI: <math>V_u \leq 0.083A_{cv}\lambda\sqrt{F'_c}</math>] the minimum reinforcement ratio values of ACI 11.6 may be used.</p> <p>The spacing of reinforcement shall be less than or equal to 18 in (450 mm).</p> <p>Reinforcement shall be continuous and shall be distributed across the shear plane.</p>	18.10.2.1
	<p>At least two curtains of reinforcement are required where <math>V_u &gt; 2A_{cv}\lambda\sqrt{F'_c}</math> [In SI: <math>V_u &gt; 0.17A_{cv}\lambda\sqrt{F'_c}</math>] or <math>h_w/l_w \geq 2.0</math></p>	18.10.2.2

	Requirement	ACI Section Number(s)
	<p>Reinforcement shall be developed or spliced for <math>f_y</math> in tension in accordance with ACI 25.4, 25.5, and the following:</p> <p>(a) Longitudinal reinforcement shall extend beyond the point it is no longer required to resist flexure by at least <math>0.8i_w</math> except at the top of the wall.</p> <p>(b) Development lengths of longitudinal reinforcement shall be 1.25 times the values calculated for <math>f_y</math> in tension at locations where yielding of longitudinal reinforcement is likely to occur as a result of lateral displacements.</p> <p>(c) Mechanical and welded splices shall conform to ACI 18.2.7 and 18.2.8, respectively.</p>	18.10.2.3
<b>Shear Strength</b>	<p>The nominal shear strength shall not exceed the following:</p> $V_n = A_{cv}(\alpha_c \lambda \sqrt{f'_c} + \rho_t f_y)$ <p>where <math>\alpha_c = 3.0</math> for <math>h_w/\ell_w \leq 1.5</math> [In SI: <math>\alpha_c = 0.25</math>]</p> <p><math>\alpha_c = 2.0</math> for <math>h_w/\ell_w \geq 2.0</math> [In SI: <math>\alpha_c = 0.17</math>]</p> <p><math>\alpha_c</math> varies linearly between 3.0 [0.25] and 2.0 [0.17] for <math>h_w/\ell_w</math> between 1.5 and 2.0</p>	18.10.4.1
	<p>The value of <math>h_w/\ell_w</math> to be used when determining <math>V_n</math> by ACI 18.10.4.1 for segments of a wall shall be the greater of the ratios for the entire wall and the segment of the wall considered.</p>	18.10.4.2
	<p>Walls shall have distributed shear reinforcement in two orthogonal directions in the plane of the wall.</p> <p>Reinforcement ratio <math>\rho_t</math> must be greater than or equal to reinforcement ratio <math>\rho_t</math> where <math>h_w/\ell_w \leq 2.0</math>.</p>	18.10.4.3
	<p>For all vertical wall segments sharing a common lateral force,</p> $V_n \leq 8A_{cv}\sqrt{f'_c} \text{ [In SI: } 0.66A_{cv}\sqrt{f'_c}\text{]}$ <p>For any of the individual vertical wall segments, <math>V_n \leq 10A_{cw}\sqrt{f'_c}</math> [In SI: <math>0.83A_{cw}\sqrt{f'_c}</math>].</p>	18.10.4.4
	<p>For horizontal wall segments and coupling beams, <math>V_n \leq 10A_{cw}\sqrt{f'_c}</math> [In SI: <math>0.83A_{cw}\sqrt{f'_c}</math>].</p>	18.10.4.5
<b>Flexure and Axial Force</b>	<p>Structural walls and portions of structural walls subjected to combined flexure and axial loads shall be designed in accordance with ACI 22.4. Concrete and developed longitudinal reinforcement within effective flange widths, boundary elements, and the wall web shall be considered effective. The effects of openings shall be considered.</p>	18.10.5.1
	<p>Effective flange widths of flanged sections shall extend from the face of the web a distance equal to the lesser of one-half the distance to an adjacent wall web and 25% of the total wall height, unless a more detailed analysis is performed.</p>	18.10.5.2
<b>Boundary Elements</b>	<p>The need for special boundary elements at the edges of structural walls shall be evaluated by ACI 18.10.6.2 or 18.10.6.3. The requirements of ACI 18.10.6.4 and 18.10.6.5 must also be satisfied.</p>	18.10.6.1

	Requirement	ACI Section Number(s)
	<p>For walls or wall piers with <math>h_w/\ell_w \geq 2.0</math> that are effectively continuous from the base of the structure to the top of the wall and that are designed to have a single critical section for flexure and axial loads, compression zones shall be reinforced with special boundary elements where</p> $c \geq \frac{\ell_w}{600(1.5\delta_u/h_w)}$ <p>where <math>\delta_u/h_w \geq 0.005</math>.</p> <p>The special boundary element transverse reinforcement shall extend vertically above and below the critical section a distance <math>\geq</math> the greater of <math>\ell_w</math> or <math>M_u/4V_u</math> except as permitted by ACI 18.10.6.4(g).</p>	18.10.6.2
	<p>Where walls are not designed to the provisions of ACI 18.10.6.2, special boundary elements are required at boundaries and edges around openings where the maximum extreme fiber compressive stress corresponding to load combinations including <math>E</math> exceeds <math>0.2 f'_c</math>. Special boundary elements may be discontinued where the compressive stress is less than <math>0.15 f'_c</math>. Stresses shall be calculated using a linearly elastic model and gross section properties. For walls with flanges, an effective flange width determined in accordance with ACI 18.10.5.2 is to be used.</p>	18.10.6.3
	<p>Where special boundary elements are required by ACI 18.10.6.2 or 18.10.6.3, the following shall be satisfied:</p> <p>(a) Boundary elements shall extend horizontally from the extreme compression fiber a distance greater than or equal to the larger of <math>c - 0.1 \ell_w</math> and <math>c/2</math>.</p> <p>(b) Width of the flexural compression zone <math>b</math> over the horizontal distance calculated by ACI 18.10.6.4(a), including flange if present, must be greater than or equal to <math>h_u/16</math>.</p> <p>(c) For walls or wall piers with <math>h_w/\ell_w \geq 2.0</math> and <math>c/\ell_w \geq 3/8</math> that are effectively continuous from the base of the structure to the top of the wall and that have been designed to have a single critical section for flexure and axial loads, the width of the flexural compression zone <math>b</math> over the length calculated by ACI 18.10.6.4(a) shall be greater than or equal to 12 in (300 mm).</p>	18.10.6.4
	<p>(d) In flanged sections, the boundary element shall include the effective flange width in compression and shall extend at least 12 in (300 mm) into the web.</p> <p>(e) The boundary element transverse reinforcement shall satisfy ACI 18.7.5.2(a) through 18.7.5.2(e) and ACI 18.7.5.3, except (i) the value of <math>h_x</math> in ACI 18.7.5.2 shall not exceed the lesser of 14 in (350 mm) and two-thirds of the boundary element thickness and (ii) the transverse reinforcement spacing limit of ACI 18.7.5.3(a) shall be one-third of the least dimension of the boundary element.</p> <p>(f) The amount of transverse reinforcement shall be in accordance with ACI Table 18.10.6.4(f).</p> <p>(g) Where the critical section occurs at the base of the wall, boundary element transverse reinforcement at the base of the wall shall extend into the support at least the tension development length <math>l_d</math> determined by ACI 18.10.2.3 of the largest longitudinal bar in the special boundary element. Where special boundary elements terminate on a footing, mat, or pile cap, special boundary element transverse reinforcement shall extend at least 12 in (300 mm) into the footing, mat, or pile cap unless a greater extension is required by ACI 18.13.2.3.</p> <p>(h) Horizontal reinforcement in the web of the wall shall extend to within 6 in (150 mm) of the end of the wall.</p> <p>Reinforcement shall be anchored to develop <math>f_y</math> within the confined core of the boundary element using standard hooks or heads.</p> <p>Where the confined boundary element has sufficient length to develop the horizontal web reinforcement and <math>A_s f_y/s</math> of the horizontal web reinforcement <math>\leq A_s f_y/s</math> of the boundary element transverse reinforcement parallel to the horizontal web reinforcement, it is permitted to terminate the horizontal web reinforcement without a standard hook or head.</p>	

	Requirement	ACI Section Number(s)
	<p>Where special boundary elements are not required by ACI 18.10.6.2 or 18.10.6.3, the following shall be satisfied:</p> <p>(a) Boundary transverse reinforcement shall satisfy ACI 18.7.5.2(a) through (e) over the distance calculated in accordance with ACI 18.10.6.4(a) where the longitudinal reinforcement ratio at the wall boundary is greater than <math>400/f_y</math>. Longitudinal spacing of transverse reinforcement at the wall boundary must conform to the following: (i) spacing shall be less than or equal to the lesser of 6 in (150 mm) and <math>6db</math> of the smallest primary flexural reinforcing bars within a distance equal to the greater of <math>\ell_w</math> and <math>M_u/4V_u</math> above and below the critical sections where yielding of longitudinal reinforcement is likely to occur as a result of inelastic lateral displacements and (ii) spacing shall be less than or equal to the lesser of 8 in (200 mm) and <math>8db</math> of the smallest primary flexural reinforcing bars elsewhere.</p> <p>(b) Where <math>V_u \geq A_{cv}\lambda\sqrt{f'_c}</math> [In SI: <math>V_u \geq 0.083A_{cv}\lambda\sqrt{f'_c}</math>], horizontal reinforcement terminating at the edges of structural walls without boundary elements shall have a standard hook engaging the edge reinforcement or the edge reinforcement shall be enclosed in U-stirrups having the same size and spacing as, and spliced to, the horizontal reinforcement.</p>	18.10.6.5
<b>Coupling Beams</b>	Coupling beams with $\ell_r/h \geq 4$ shall satisfy the requirements of ACI 18.6 with the wall boundary interpreted as being a column. If it can be shown by analysis that the beam has adequate lateral stability, the provisions of ACI 18.6.2.1(b) and (c) need not be satisfied.	18.10.7.1
	Two intersecting groups of diagonally placed bars symmetrical about the midspan of the coupling beam must be provided where $\ell_r/h < 2$ and $V_u \geq 4\lambda\sqrt{f'_c}A_{cw}$ [In SI: $V_u \geq 0.33\lambda\sqrt{f'_c}A_{cw}$ ]. This provision need not be satisfied where it can be shown that the loss of stiffness and strength of the coupling beam does not impair the vertical load-carrying ability of the structure, the egress of the structure, or the integrity of nonstructural components and their connections to the structure.	18.10.7.2
	Two intersecting groups of diagonally placed bars symmetrical about the midspan of the coupling beam or reinforcement in accordance with ACI 18.6.3 through 18.6.5 is permitted where coupling beams are not governed by ACI 18.10.7.1 or 18.10.7.2.	18.10.7.3

	Requirement	ACI Section Number(s)
	<p>Coupling beams reinforced with two intersecting groups of diagonally placed bars symmetrical about the midspan shall satisfy (a), (b), and either (c) or (d), and the requirements of ACI 9.9 need not be satisfied:</p> <p>(a) <math>V_n = 2 A_{vd} f_y \sin \alpha \leq 10 \sqrt{f'_c} A_{cw}</math>.</p> <p>[In SI: <math>V_n = 2 A_{vd} f_y \sin \alpha \leq 0.83 \sqrt{f'_c} A_{cw}</math>].</p> <p>(b) Each group of diagonal bars shall consist of a minimum of four bars provided in two or more layers. The embedment length of the diagonal bars into the wall must be greater than or equal to 1.25 times the development length for <math>f_y</math> in tension.</p> <p>(c) Each group of diagonal bars shall be enclosed by rectilinear reinforcement having out-to-out dimensions of at least <math>b_w/2</math> in the direction parallel to <math>b_w</math> and <math>b_w/5</math> along the other sides where <math>b_w</math> is the web width of the coupling beam.</p> <p>Transverse reinforcement <math>A_{sh}</math> is to be determined in accordance with ACI 18.7.5.2(a) through (e) with <math>A_{sh}</math> greater than or equal to the greater of the following:</p> <p>(i) <math>0.09 s b_c f'_c / f_{yt}</math></p> <p>(ii) <math>0.3 s b_c [(A_g / A_{ch}) - 1] f'_c / f_{yt}</math></p> <p>(d) Transverse reinforcement shall be provided for the entire beam cross-section in accordance with ACI 18.7.5.2(a) through (e) with <math>A_{sh}</math> greater than or equal to the greater of the following:</p> <p>(i) <math>0.09 s b_c f'_c / f_{yt}</math></p> <p>(ii) <math>0.3 s b_c [(A_g / A_{ch}) - 1] f'_c / f_{yt}</math></p>	
	<p>Longitudinal spacing of transverse reinforcement must be less than or equal to the lesser of 6 in (150 mm) and <math>6d_b</math> of the smallest diagonal bars. Spacing of crossties or legs of hoops both vertically and horizontally in the plane of the beam cross-section must be less than or equal to 8 in (200 mm). Each crosstie and each hoop leg must engage a longitudinal bar of equal or greater diameter, and the hoop configuration specified in ACI 18.6.4.3 is permitted.</p>	18.10.7.4
<b>Wall Piers</b>	<p>Wall piers must satisfy the requirements in ACI 18.7.4, 18.7.5, and 18.7.6 for columns in special moment frames. The faces of the joint are to be taken as the top and bottom of the clear height of the wall pier.</p>	18.10.8.1
	<p>As an alternate to satisfying ACI 18.7.4, 18.7.5, and 18.7.6, wall piers with <math>\ell_w/b_w \geq 2.5</math> shall satisfy the following:</p> <p>(a) Design shear forces shall be calculated in accordance with ACI 18.7.6.1 with joint faces taken as the top and bottom of the clear height of the pier. The design shear need not exceed the overstrength factor <math>\Omega_o</math> times the factored shear force obtained from the analysis of the structure for earthquake effects.</p> <p>(b) Nominal shear strength <math>V_n</math> and distributed shear reinforcement shall satisfy ACI 18.10.4.</p> <p>(c) Transverse reinforcement shall consist of hoops except single-leg horizontal reinforcement parallel to <math>\ell_w</math> is permitted where only one curtain of distributed shear reinforcement is provided. The single-leg horizontal bars must have 180-degree bends at each end that engage the boundary longitudinal reinforcement of the pier.</p> <p>(d) Vertical spacing of transverse reinforcement must be less than or equal to 6 in (150 mm).</p> <p>(e) Transverse reinforcement shall extend at least 12 in (300 mm) above and below the clear height of the wall pier.</p> <p>(f) Special boundary elements shall be provided if required by ACI 18.10.6.3.</p>	18.10.8.1

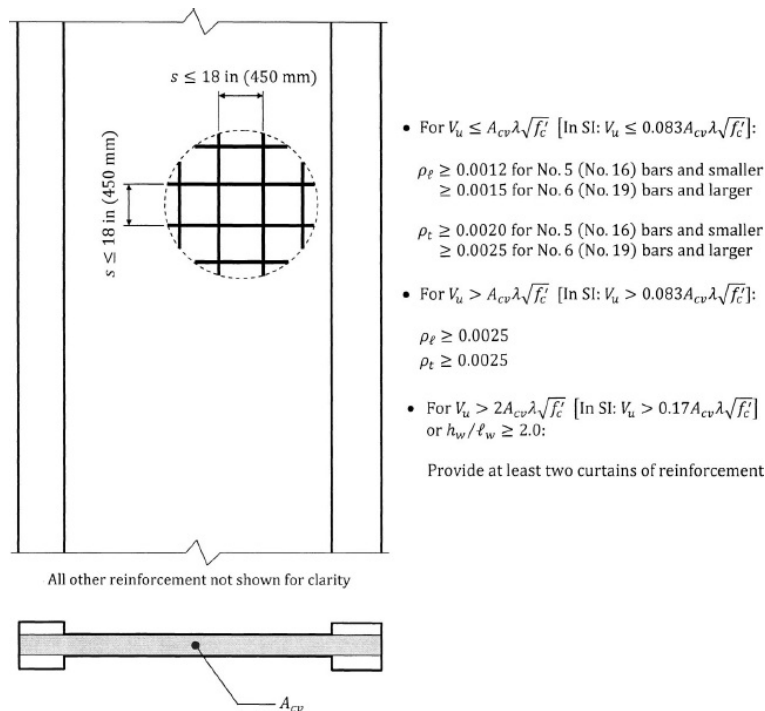


Requirement		ACI Section Number(s)
	For wall piers at the edge of a wall, horizontal reinforcement shall be provided in adjacent wall segments above and below the wall pier and be designed to transfer the design shear force from the wall pier into the adjacent wall segments.	18.10.8.2
	Construction joints in structural walls shall be specified according to ACI 26.5.6, and contact surfaces shall be roughened consistent with condition (b) in ACI Table 22.9.4.2.	18.10.9
	Columns supporting discontinuous structural walls shall be reinforced in accordance with ACI 18.7.5.6.	18.10.10

## 11.8.2. Reinforcement

Special structural walls must have reinforcement in two orthogonal directions in the plane of the wall. The minimum reinforcement requirements of ACI 18.10.2.1 for longitudinal and transverse reinforcement follow from preceding codes. The provisions of ACI 18.10.2.1 for required web reinforcement are summarized in Fig 11.78 where  $\rho_\ell$  is the ratio of the area of the distributed longitudinal reinforcement in the wall to the gross concrete area perpendicular to that reinforcement and  $\rho_t$  is the ratio of the area of the distributed transverse reinforcement in the wall to the gross concrete area perpendicular to that reinforcement.

Figure 11.78 Web reinforcement requirements for Grade 60 (Grade 420) bars.



Reinforcement provided for shear strength must be continuous and uniformly distributed across the shear plane. Uniform distribution of reinforcement across the height and horizontal length of the wall helps control the width of inclined cracks. For walls subjected to substantial in-plane design shear forces (i.e., where  $V_u > 2A_{cv}\lambda\sqrt{f'_c}$  [In SI:  $V_u > 0.17A_{cv}\lambda\sqrt{f'_c}$ ]), at least two layers of reinforcement must be provided (ACI 18.10.2.2). Among other things, this serves to reduce fragmentation and premature deterioration of the concrete under load reversals into the inelastic range.



Because the actual forces in the longitudinal reinforcing bars of walls may exceed the calculated forces, ACI 18.10.2.3 requires that all reinforcement in walls be fully developed or spliced to reach the yield strength of the bar  $f_y$  in tension. At locations where plastic hinges are likely to form, such as at the base of a cantilever wall, yielding of the longitudinal reinforcement is expected and the Code requires that development lengths be multiplied by 1.25. The purpose of this multiplier is to account for the likelihood that the actual  $f_y$  exceeds the specified yield strength as well as for strain hardening and cyclic load reversals.

No requirements are given in the Code regarding whether the longitudinal (vertical) or transverse (horizontal) reinforcement should be in the outer layer. Generally, lap splices of the longitudinal reinforcement will perform better if the transverse bars are placed in the outer layer.

### 11.8.3. Shear Strength

The nominal shear strength of structural walls is determined by ACI Eq. (18.10.4.1):

$$V_n = A_{cv}(\alpha_c \lambda \sqrt{f'_c} + \rho_t f_y)$$

(11.14)

where  $\alpha_c = 3.0$  for  $h_w/\ell_w \leq 1.5$  [In SI:  $\alpha_c = 0.25$ ]

$\alpha_c = 2.0$  for  $h_w/\ell_w \geq 2.0$  [In SI:  $\alpha_c = 0.17$ ]

$\alpha_c$  varies linearly between 3.0 [0.25] and 2.0 [0.17] for  $h_w/\ell_w$  between 1.5 and 2.0

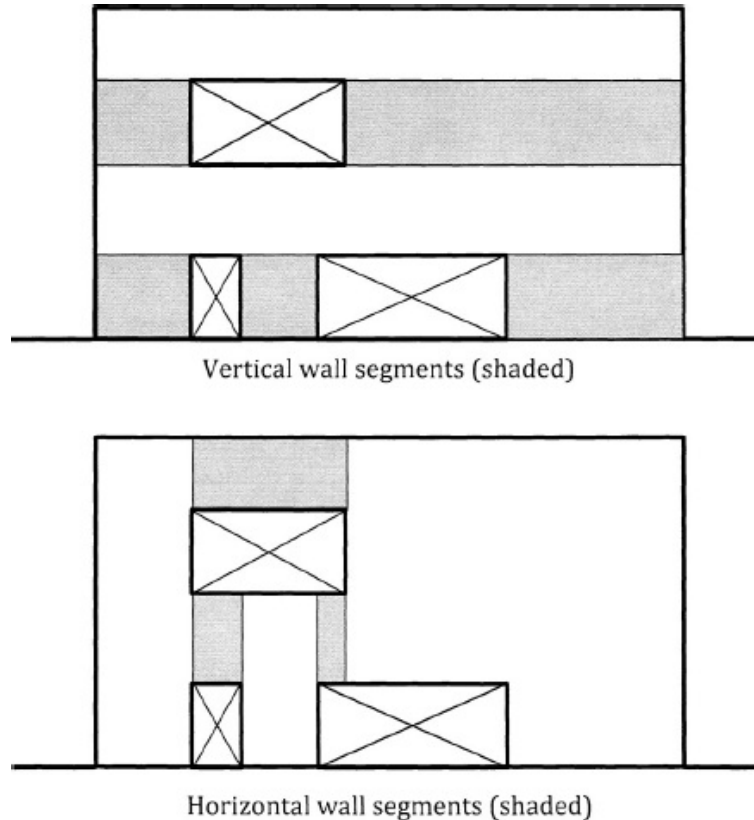
The area  $A_{cv}$  is the gross area of the cross-section of the wall that resists the shear force, which for a rectangular wall with no openings is equal to the product of the thickness of the web and the length of the web in the direction of analysis. This is different than the net shear area for beams, which is equal to the width of the section times the effective depth  $d$ . Shown in [Fig 11.78](#) is  $A_{cv}$  for the case of a barbell wall. If a wall section has openings, the area of the openings is not included in  $A_{cv}$ .

The maximum factored shear force  $V_u$  is obtained from the appropriate load combinations and it must be less than or equal to  $\phi V_n$ . Equation (11.14) recognizes the experimental findings that short, squat walls have higher shear strength capacity than taller, more slender walls.<sup>13,14</sup>

To effectively restrain inclined cracks, horizontal and vertical shear reinforcement must be appropriately distributed along the height and length of a wall. Within practical limits, shear reinforcement should be uniformly distributed and at a relatively small spacing. The reinforcement ratio  $\rho_\ell$  must be greater than or equal to the reinforcement ratio  $\rho_t$  when the overall wall height-to-length ratio  $h_w/\ell_w$  is less than or equal to 2.0. Any concentrated reinforcement near wall edges provided primarily for resisting bending moments is not to be included when computing  $\rho_\ell$  and  $\rho_t$ .

ACI 18.10.4 also contains nominal shear strength limits for vertical and horizontal wall segments. Such segments are present where there are openings in a wall (see [Fig. 11.79](#)). A vertical wall segment is the part of a wall that is bounded horizontally by openings in the wall or by an opening and an edge of the wall. As in isolated walls,  $\rho_\ell$  refers to vertical reinforcement and  $\rho_t$  refers to horizontal reinforcement in vertical wall segments. Horizontal wall segments are sections of wall between two vertically aligned openings in the wall. When the wall openings are aligned vertically over the height of the building, the horizontal wall segments are referred to as coupling beams, which are covered in [Section 11.8.6](#). In horizontal wall segments and coupling beams,  $\rho_t$  refers to vertical reinforcement and  $\rho_\ell$  refers to horizontal reinforcement.

Figure 11.79 Vertical and horizontal wall segments in a structural wall.



The nominal shear strength  $V_n$  is limited to  $8A_{cv}\sqrt{f'_c}$  [In SI:  $0.66A_{cv}\sqrt{f'_c}$ ] where several vertical wall segments resist the factored shear force. When designing a vertical wall segment in a wall, Eq. (11.14) is used to determine  $V_n$ ; the value of  $\alpha_c$  in this equation is obtained using the larger of the following two ratios: (1)  $h_w/\ell_w$  based on the dimensions of the entire wall and (2)  $h_w/\ell_w$  based on the dimensions of the wall segment. The intent of this requirement is that a vertical wall segment should never have a unit strength greater than that of the entire wall; however, it may have a lower unit strength if  $h_w/\ell_w$  of the vertical segment is greater than that of the entire wall.

The nominal shear strength limit is equal to  $10A_{cw}\sqrt{f'_c}$  [In SI:  $0.83A_{cu}\sqrt{f'_c}$ ] for any one of the individual vertical wall segments in the group that resists the factored shear force where  $A_{cw}$  is the area of the concrete section of the individual vertical wall segment. This requirement is imposed to limit redistribution of the factored shear force to the vertical wall segments. Each vertical wall segment resists a percentage of the total factored shear force based on its flexural and shear rigidities, and that portion of the shear force must be less than or equal to the design shear strength of the vertical wall segment.

In the case of horizontal wall segments and coupling beams,  $V_n$  is limited to  $10A_{cw}\sqrt{f'_c}$  [In SI:  $0.83A_{cu}\sqrt{f'_c}$ ] where  $A_{cw}$  is the area of concrete section of the horizontal wall segment or coupling beam.

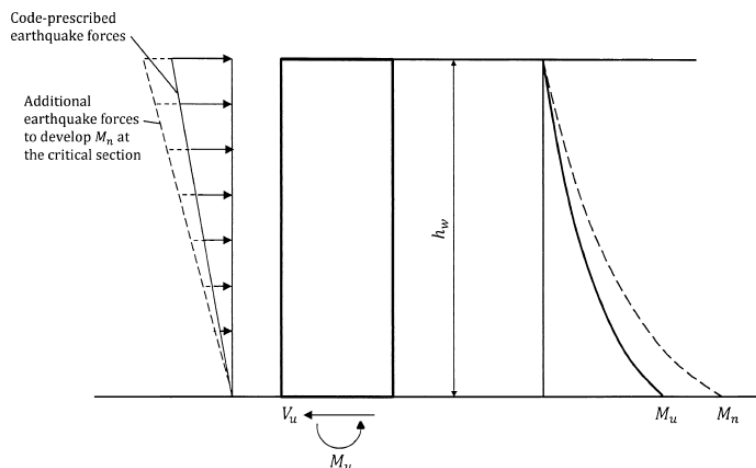
It is important at this time to discuss the appropriate strength reduction factor  $\phi$  to use when determining shear strength requirements. As noted previously,  $\phi$  for shear shall be 0.60 if the nominal shear strength of the member  $V_n$  is less than the shear corresponding to the development of the nominal flexural strength of the member  $M_n$  (ACI 21.2.4.1). Otherwise, it is equal to 0.75. For short, squat walls ( $h_w/\ell_w \leq 1.5$ ), designing for shear based on a strength reduction factor of 0.75 is usually not practical because the magnitude of the required shear force would be relatively large. Thus, for such walls, designing for shear using  $\phi = 0.60$  is appropriate.

In the case of more slender walls ( $h_w/\ell_w \geq 2.0$ ), it is recommended to design for shear using  $\phi = 0.75$ . Therefore, the walls are designed so that the design shear strength  $\phi V_n$  is at least equal to the shear force that corresponds to the development of  $M_n$

in the wall.

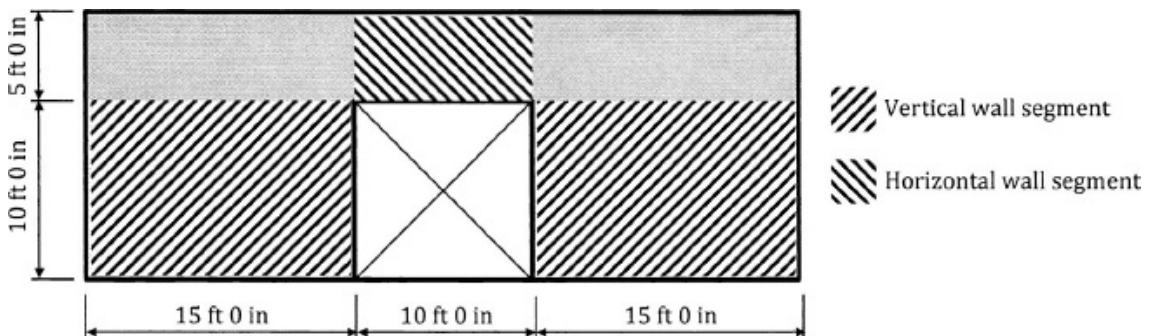
Reference 12 contains a method that can be used to design a structural wall for the effects of shear using a strength reduction factor equal to 0.75. Consider the structural wall depicted in Fig. 11.80. For purposes of discussion, assume that the wall is subjected to a triangular lateral load distribution, which approximates the distribution of seismic forces over the height of the wall. The factored moment diagram for the wall is shown in the figure and is obtained from the appropriate load combinations, which are based on the code-prescribed loads. Because multiple load combinations must be considered, the nominal flexural strength  $M_n$  will more than likely be greater than the required flexural strength  $M_u$  for one or both of the load combinations including seismic effects  $E$  over the height of the wall. Therefore, in order to yield the wall in flexure at the base of the wall which is the critical section, the lateral forces applied to the wall have to be larger than the code-prescribed values. This means the shear forces in the wall have to be larger than the design values as well. The ratio  $M_n/M_u$  is a measure of the flexural overstrength that is built into the wall. It can be assumed that the shear force that corresponds to the development of  $M_n$  at the critical section is equal to the design shear force at the critical section multiplied by the ratio  $M_n/M_u$  at the critical section. Because  $M_n$  depends on the magnitude of the axial force on the wall, which is different for different load combinations, the ratio that results in the most conservative shear force should be used. Walls designed for shear in this manner can use a strength reduction factor equal to 0.75.

Figure 11.80 Design and nominal shear forces and bending moments in a structural wall.



**Example 11.15** Check the shear strength requirements for the special structural wall depicted in Fig. 11.81. The thickness of the wall is 10 in and two curtains of No. 4 bars spaced at 12 in on center are provided for both the longitudinal and transverse reinforcement. From analysis,  $V_u = 500$  kips. Assume normal-weight concrete with  $f'_c = 4,000$  psi and Grade 60 reinforcement.

Figure 11.81 The special structural wall in Example 11.15.



**Solution** The design shear strength for the wall and the vertical segments of the wall are determined and compared to the applicable required shear strength.

For the entire wall,  $\phi V_n$  is determined by ACI Eq. (18.10.4.1):

$$\phi V_n = \phi A_{cv} (\alpha_c \lambda \sqrt{f'_c} + \rho_t f_y)$$

where  $A_{cv} = 2 \times [10 \times (15 \times 12)] = 3,600 \text{ in}^2$

$\alpha_c = 3.0$  for  $h_w/\ell_w = 15/40 = 0.4 < 1.5$

$\lambda = 1.0$  for normal-weight concrete

$\rho_t = (2 \times 0.2)/(10 \times 12) = 0.0033$

Because  $h_w/\ell_w < 1.5$ , use  $\phi = 0.60$ .

Therefore,

$$\begin{aligned} \phi V_n &= 0.6 \times 3,600 \times [(3.0 \times 1.0 \sqrt{4,000}) + (0.0033 \times 60,000)]/1,000 = 838 \text{ kips} \\ &> V_u = 500 \text{ kips} \end{aligned}$$

Thus, the shear strength requirements for the entire wall are satisfied. Note that because  $V_u = 500 \text{ kips}$  is greater than  $A_{cv} \lambda \sqrt{f'_c} = 228 \text{ kips}$  the longitudinal and transverse reinforcement ratios must be greater than or equal to 0.0025. In this example,  $\rho_t = 0.0033 > 0.0025$ . Also, two curtains of reinforcement are required because  $V_u = 500 \text{ kips} > 2A_{cv} \lambda \sqrt{f'_c} = 456 \text{ kips}$ .

For the two vertical wall segments on either side of the opening,

$\phi V_n = 0.6 \times 3,600 \times [(3.0 \times 1.0 \sqrt{4,000}) + (0.0033 \times 60,000)]/1,000 = 838 \text{ kips}$  where  $\alpha_c = 3.0$  for  $h_w/\ell_w = 10/15 = 0.7 < 1.5$  (the ratio  $h_w/\ell_w$  for the vertical wall segment was used because it is greater than that of the entire wall, which is equal to 0.4). It is evident that the design shear strength of the combined vertical wall segments is the same as that for the entire wall even though the height-to-length ratio of the vertical wall segment is greater than that of the entire wall.

The design shear strength  $\phi V_n$  for the combined vertical wall segments that share the 500-kip factored shear force must not be taken greater than  $\phi V_{n,max} = \phi 8A_{cv} \sqrt{f'_c} = 0.6 \times 8 \times (2 \times 10 \times 15 \times 12) \times \sqrt{4,000}/1,000 = 1,093 \text{ kips} > 838 \text{ kips}$ .

For each vertical wall segment, which is 15 ft wide, the design shear strength

$\phi V_n = 0.6 \times (10 \times 15 \times 12) \times [(3.0 \times 1.0 \sqrt{4,000}) + (0.0033 \times 60,000)]/1,000 = 419 \text{ kips}$  where  $\alpha_c = 3.0$  for  $h_w/\ell_w = 10/15 = 0.7 < 1.5$ .

The design shear strength  $\phi V_n$  for a vertical wall segment must not be taken greater than

$\phi V_{n,max} = \phi 10A_{cw} \sqrt{f'_c} = 0.6 \times 10 \times (10 \times 15 \times 12) \times \sqrt{4,000}/1,000 = 683 \text{ kips} > 419 \text{ kips}$ .

For the horizontal wall segment above the opening, the design shear strength

$\phi V_n = 0.6 \times (10 \times 10 \times 12) \times [(3.0 \times 1.0 \sqrt{4,000}) + (0.0033 \times 60,000)]/1,000 = 279 \text{ kips}$  where  $\alpha_c = 3.0$  for  $h_w/\ell_w = 5/10 = 0.5 < 1.5$ .

The design shear strength  $\phi V_n$  for the horizontal wall segment must not be taken greater than

$\phi 10A_{cw} \sqrt{f'_c} = 0.6 \times 10 \times (10 \times 10 \times 12) \times \sqrt{4,000}/1,000 = 455 \text{ kips} > 279 \text{ kips}$ .

If it is assumed that the vertical wall segments resist all of the factored shear force, all of the wall segments are adequate for shear. Note that the design shear capacity of the horizontal wall segment is almost adequate to carry the factored shear force alone. The exact distribution of shear force into each wall segment can be obtained from an analysis of the wall section considering both flexural and shear stiffness.

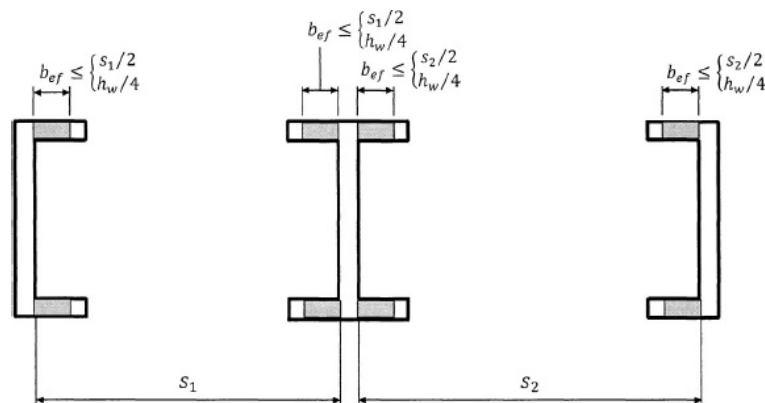
## 11.8.4. Design for Flexure and Axial Force

Structural walls are designed for combined flexure and axial forces in accordance with the provisions in ACI 22.4. An interaction diagram for a wall can be constructed using a strain compatibility analysis based on the reinforcement that is in the section. Just like in the design of reinforced concrete columns, all of the combined factored axial force and bending moment points that are obtained from the applicable load combinations in ACI Chap. 5 must fall within or on the design strength interaction diagram in order for strength requirements to be satisfied.

For walls with flanges like those depicted in Fig. 11.77, a portion of the flange is considered to be effective in resisting the effects from combined axial force and bending moment. In general, the effective width of a flange depends on the magnitude

of the axial force on the wall, the amount of lateral drift (effective flange width increases with increasing drift), and whether the flange is in tension or compression. In lieu of a rational analysis that takes these variables into account, ACI 18.10.5.2 contains a simple method to determine the effective flange width, which is illustrated in Fig. 11.82. Because the effective compression flange width has little effect on the strength and deformation capacity of a wall, the provisions in ACI 18.10.5.2 are based on the effective tension flange width only. It is assumed that the effective flange and the reinforcement in it fully contribute to the nominal strength of the wall section.

Figure 11.82 Effective flange width for special structural walls.



Openings in walls must be considered when determining the strength requirements of a special structural wall. Relatively large openings can dramatically change the overall behavior of a wall and it is important to understand how a wall will perform when one or more openings are introduced. Additional reinforcement around the openings is typically provided to counteract large tensile forces that may develop, especially at the corners.

In the case of slender walls, the goal is to achieve ductile flexural yielding at the base of the wall. For slender walls that are coupled, yielding of the coupling beams over the height of the wall should be achieved in addition to yielding at the base. To help ensure that a single critical section for flexure and axial force occurs at the base of the wall compared to any other section over the height of the wall (see Fig. 11.80). In structures with highly irregular walls and in very tall buildings, yielding may occur at sections other than the base, and special detailing is required at those locations.

One factor that helps improve ductility is to have the axial forces on a wall fall well below the balanced point on the interaction diagram. In the area below the balanced point, the flexural reinforcement yields prior to the compression zone reaching the compressive strain capacity. This factor should be considered when proportioning a special structural wall.

Short, squat walls are likely to have an inelastic response in shear rather than in flexure. As such, the shear strength requirements presented in Section 11.8.3 generally take precedence to those for strength under combined flexure and axial forces. It is common for the reinforcement in squat walls to be determined based on shear; the strength requirements for flexure and axial forces are subsequently checked using the reinforcement for shear and it is usually found that the requirements are satisfied.

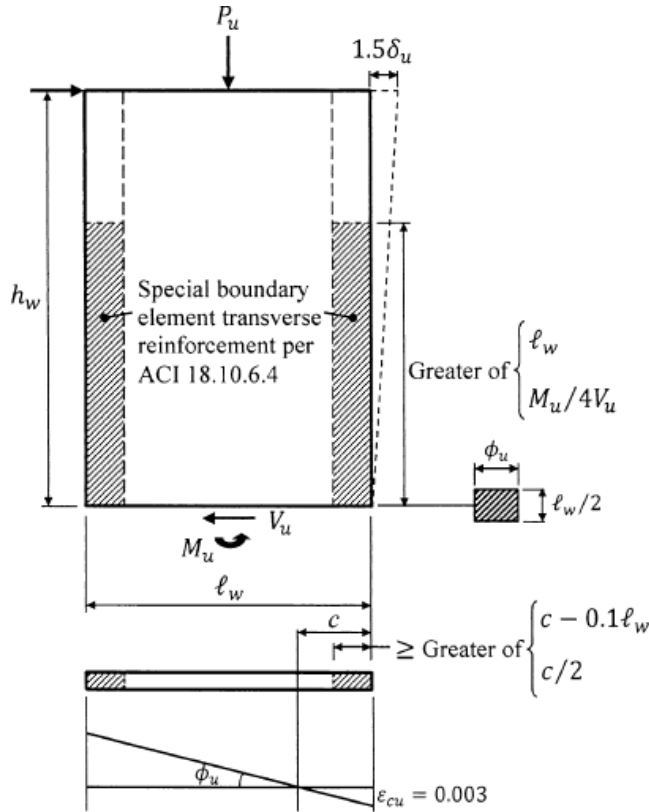
## 11.8.5. Boundary Elements

When a structural wall undergoes cyclic deformations caused by a seismic event, the ends of the walls and the edges adjacent to openings can be subjected to large compressive forces. Special transverse reinforcement may be required at these locations to confine the concrete and to restrain the longitudinal reinforcement in the wall so that buckling of the bars does not occur. Two design approaches for evaluating the need of special boundary elements are given in ACI 18.10.6. Detailing requirements for special boundary elements are also provided in that section as are detailing requirements for portions of the wall where special boundary elements are not required.

### 11.8.5.1. ACI 18.10.6.2—Displacement-based Approach

ACI 18.10.6.2 allows the use of a displacement-based approach, which is applicable to slender walls or wall piers ( $h_w/\ell_w \geq 2.0$ ) that are effectively continuous in cross-section over the entire height of the wall and are designed to have one critical section for flexure and axial loads, which typically occurs at the base of the wall. The provisions of this section are summarized in Fig. 11.83.

Figure 11.83 Special boundary element requirements in accordance with ACI 18.10.6.2.



$$\text{Special boundary elements required where } c \geq \frac{\ell_w}{600(1.5 \delta_u/h_w)}$$

Special boundary elements are required to confine the concrete where the strain at the extreme compression fiber of the wall exceeds a critical value when the wall is subjected to 1.5 times the design displacement  $\delta_u$ . Note that  $\delta_u$  is equal to the product of the displacement at the top of the wall determined from an analysis where the code-prescribed lateral earthquake forces are applied over the height of the structure and the deflection amplification factor  $C_d$ , which is given in ASCE/SEI Table 12.2-1 for the various SFRSs divided by the importance factor. The 1.5 multiplier on  $\delta_u$  results in a displacement that is consistent with the maximum considered earthquake, which is defined in ASCE/SEI 10 as the most severe earthquake effects considered by that standard. In order to obtain the design displacements, preliminary sizes are needed for the walls that are part of the SFRS.

Special boundary elements are required where Eq (18.10.6.2) is satisfied:

$$c \geq \frac{\ell_w}{600(1.5 \delta_u/h_w)}$$

(11.15)

where  $\delta_u/h_w \geq 0.005$ .



The neutral axis depth  $c$  in Eq. (11.15) corresponds to the largest neutral axis depth that is calculated for the factored axial force and nominal moment strength of the wall when it is displaced in the same direction as the design displacement  $\delta_u$ . The lower limit of 0.005 on the quantity  $\delta_u/h_w$  is specified to require moderate wall deformation capacity for stiff buildings.

Equation (11.15) is derived from the relationships shown in Fig. 11.83. It is assumed that the displacement  $1.5\delta_u$  at the top of the wall is due entirely to the curvature  $\phi_u$ , which is centered on the critical section of the wall at its base. It is also assumed that the length of the plastic hinge that forms at the base is equal to one-half of the length of the wall  $\ell_w$ . Assuming small curvatures,  $\phi_u = \varepsilon_{cu}/c = 0.003/c$ . Therefore,  $1.5\delta_u = 0.003h_w \ell_w/2c$ . Rearranging terms and rounding down the numerical constant in the denominator results in Eq. (11.15).

Reinforcement for slender structural walls is typically determined first for the combined effects of bending and axial forces in accordance with the provisions outlined above for all applicable load combinations. Based on the vertical reinforcement,  $c$  can be obtained from a strain compatibility analysis for each load combination that includes seismic effects, considering sidesway to the left and to the right. The largest  $c$  is used in Eq. (11.15) to determine if special boundary elements are required or not. The horizontal dimension of the boundary element extends over the length where the compression strain exceeds the critical value, while the height of the boundary element is based on upper bound estimates of plastic hinge length and extends beyond the zone over which spalling is likely to occur (see Fig. 11.83). Boundary elements do not necessarily require an increase in wall thickness.

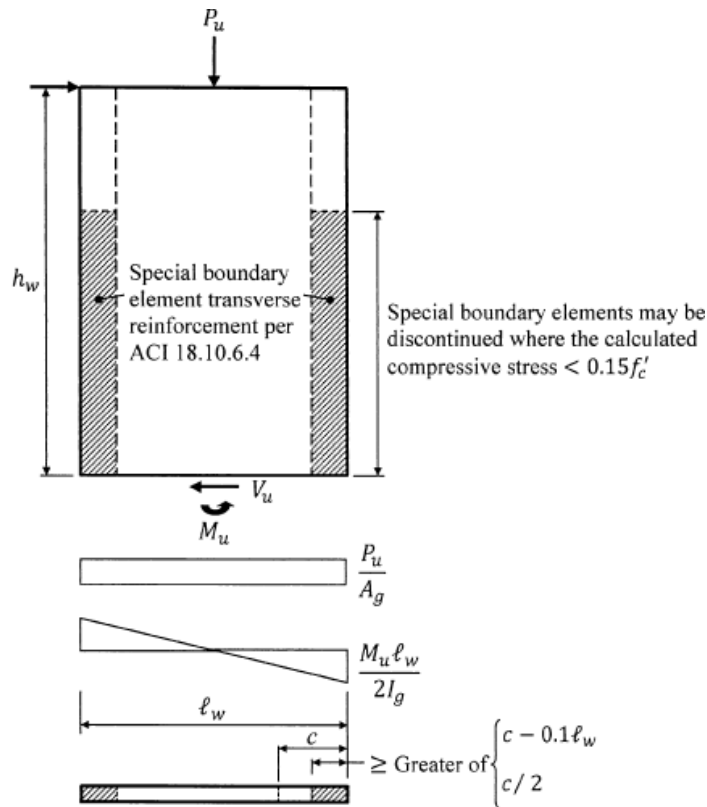
### 11.8.5.2. ACI 18.10.6.3—Compressive Stress Approach

The compressive stress approach in ACI 18.10.6.3 can be used to assess the need for special boundary elements for any structural wall. In this method, the wall is subjected to gravity loads and the maximum bending moments and shear forces due to earthquake effects in a given direction (see Fig. 11.84). The combined compressive stress due to gravity loads and bending moments is calculated assuming a linearly elastic model and gross section properties of the wall. Special boundary elements are required at the ends of the wall or at edges around openings where the maximum compressive stress  $f_{cu}$  exceeds  $0.2f'_c$ :

$$f_{cu} = \frac{P_u}{A_g} + \frac{M_u \ell_w}{2I_g} > 0.2f'_c$$

(11.16)

Figure 11.84 Special boundary element requirements in accordance with ACI 18.10.6.3.



Special boundary elements required where  $\frac{P_u}{A_g} + \frac{M_u \ell_w}{2I_g} > 0.2f'_c$

The special boundary elements may be discontinued where the combined stress is less than  $0.15f'_c$ .

### 11.8.5.3. Design and Detailing Requirements for Special Boundary Elements

The design and detailing requirements of ACI 18.10.6.4 must be satisfied where special boundary elements are required, regardless of the method that is used to determine whether or not such elements are required.

The horizontal length of a special boundary element must be greater than or equal to the greater of  $c - 0.1\ell_w$  and  $c/2$ . This length extends from the extreme compression fiber inward where  $c$  is the largest neutral axis depth determined in the same way as that in ACI 18.10.6.2.

The thickness of the wall within the required horizontal length of the special boundary element must be greater than or equal to  $h_u/16$  where  $h_u$  is the laterally unsupported height of a wall or pier at the extreme compression fiber. The intent of this thickness limit is to help prevent lateral instability failures of slender boundary elements.

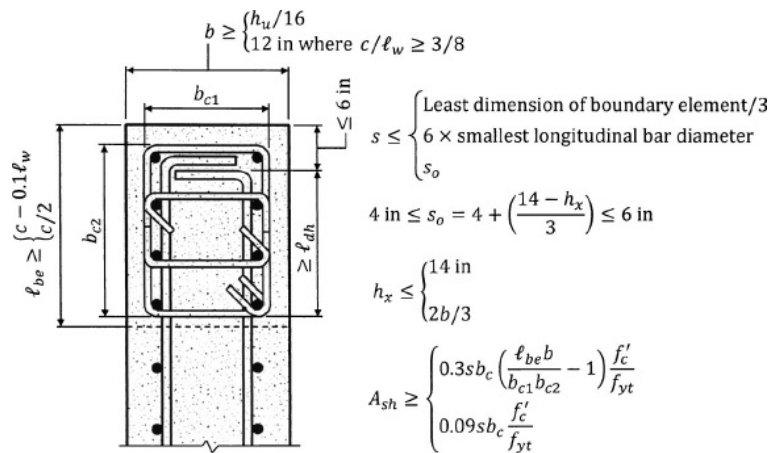
As was discussed in Chap. 5, a tension-controlled section is defined where the strain in the extreme tension reinforcement is greater than or equal to 0.005. It can be shown that at the strain limit of 0.005, the neutral axis is located a distance of  $3/8$  the distance from the extreme compression fiber to the centroid of the tension reinforcement. For slender walls ( $h_w/\ell_w \geq 2.0$ ) that are effectively continuous in cross-section over the entire height of the wall and are designed to have one critical section for flexure and axial loads, the thickness of the wall within the required horizontal length of the special boundary element must be greater than or equal to 12 in (300 mm) where  $c/\ell_w \geq 3/8$ . A value of  $c/\ell_w \geq 3/8$  is used to define a wall that is not tension-controlled. Such walls can be subjected to significant axial compression. The intent of this requirement is to reduce the likelihood of lateral instability of the compression zone after the concrete cover spalls.



For walls with flanges, the special boundary elements are to include the effective flange width in compression and are to extend at least 12 in (300 mm) into the web. The web-to-flange interface is subjected to large compression forces and may sustain local crushing failure unless the special boundary elements extend into the web of the wall.

Transverse reinforcement required in special boundary elements must conform to the requirements of ACI 18.7.5.2(a) through (e) and ACI 18.7.5.3 for columns in special moment frames. The following exceptions are applicable to walls and piers: (1) the value of  $h_x$  in ACI 18.7.5.2 must be less than or equal to the lesser of 14 in (350 mm) and two-thirds of the thickness of the special boundary element and (2) the transverse reinforcement spacing limit in ACI 18.7.5.3(a) shall be one-third of the least dimension of the special boundary element. The minimum amount of transverse reinforcement in ACI Table 18.10.6.4(f) is shown in Fig. 11.85 for a rectangular wall section with rectilinear hoops and cross ties. The main purpose of the transverse reinforcement is to confine the edges of walls that are subjected to large compressive forces.

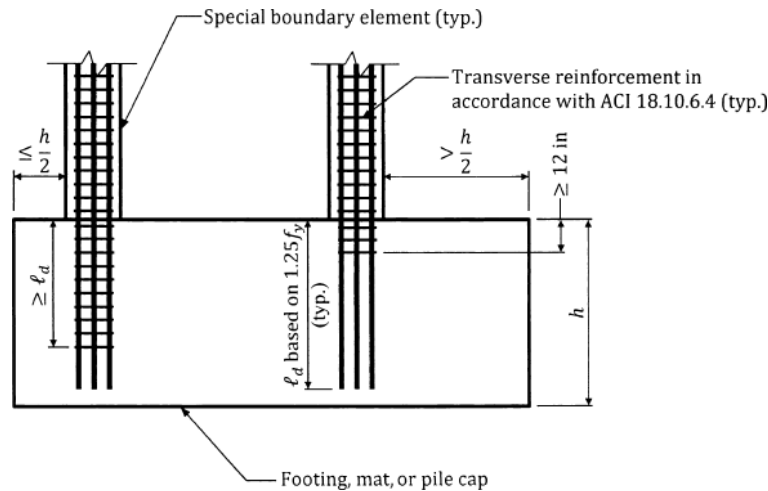
Figure 11.85 Design and detailing requirements for special boundary elements.



The longitudinal reinforcement of the special boundary elements must be fully developed in tension into the foundation in accordance with ACI 18.10.2.3 where the development length is calculated using  $1.25f_y$ . Where the foundation is not deep enough to accommodate the development of straight bars, standard hooks can be utilized.

In cases where a special boundary element is supported on a footing, mat, or pile cap, the transverse reinforcement in the special boundary element must extend into the supporting element a distance of at least 12 in (300 mm) provided the edge of the special boundary element is located greater than one-half the foundation depth from the edge of the foundation (see Fig. 11.86). For boundary elements that are located closer than that to the edge, which may occur, for example, near a property line, the transverse reinforcement must extend a length equal to the development length of the longitudinal reinforcement in the special boundary element (ACI 18.13.2.3). The intent of this detailing requirement is to help prevent an edge failure of the foundation.

Figure 11.86 Transverse reinforcement of special boundary elements at foundations.



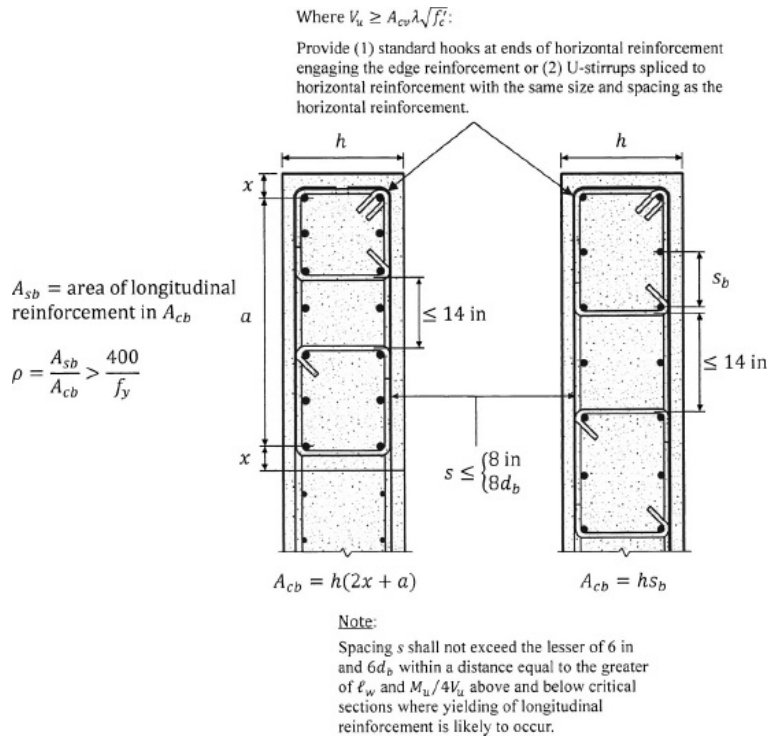
Horizontal reinforcement in the web of the special structural wall must extend to within 6 in (150 mm) of the end of the wall and must be anchored to develop  $f_y$  using standard hooks or headed bars (see Fig. 11.85). Anchoring and developing the horizontal bars in the web of a squat wall is especially important because these bars act like stirrups in a beam because of the truss action that resists shear in such walls. In cases where  $A_s f_{yt}/s$  of the horizontal web reinforcement is less than or equal to  $A_s f_{yt}/s$  of the special boundary element transverse reinforcement that is parallel to the horizontal web reinforcement, it is permitted to terminate the horizontal web reinforcement in the confined core of the special boundary element using straight bars provided the confined core has sufficient length to develop the horizontal web bars [see ACI Fig. R18.10.6.4.1(b)].

#### 11.8.5.4. Design and Detailing Requirements Where Special Boundary Elements Are Not Required

Even though special boundary elements may not be required in accordance with the provisions of ACI 18.10.6.2 or 18.10.6.3, cyclic load reversals may produce compressive forces large enough to buckle the longitudinal reinforcement at the ends of walls or adjacent to openings. According to ACI 18.10.6.5, transverse reinforcement in accordance with ACI 18.7.5.2(a) through (e) for columns in special moment frames is required where the longitudinal reinforcement ratio at the wall boundary is greater than  $400/f_y$  [In SI:  $2.8/f_y$ ]. The requirements of ACI 18.10.6.5 apply over the entire height of the wall and not just at the location of the critical section.

The longitudinal reinforcement ratio includes only the reinforcement at the wall boundary as indicated in Fig. 11.87 for the case where reinforcing bars are provided at the end of the wall that are larger than the uniformly distributed web longitudinal reinforcement (left portion of the figure) and where uniformly distributed longitudinal bars of the same size and spacing are provided throughout the length of the wall (right portion of the figure).

**Figure 11.87** Reinforcement details where special boundary elements are not required and  $\rho > 400/f_y$ .

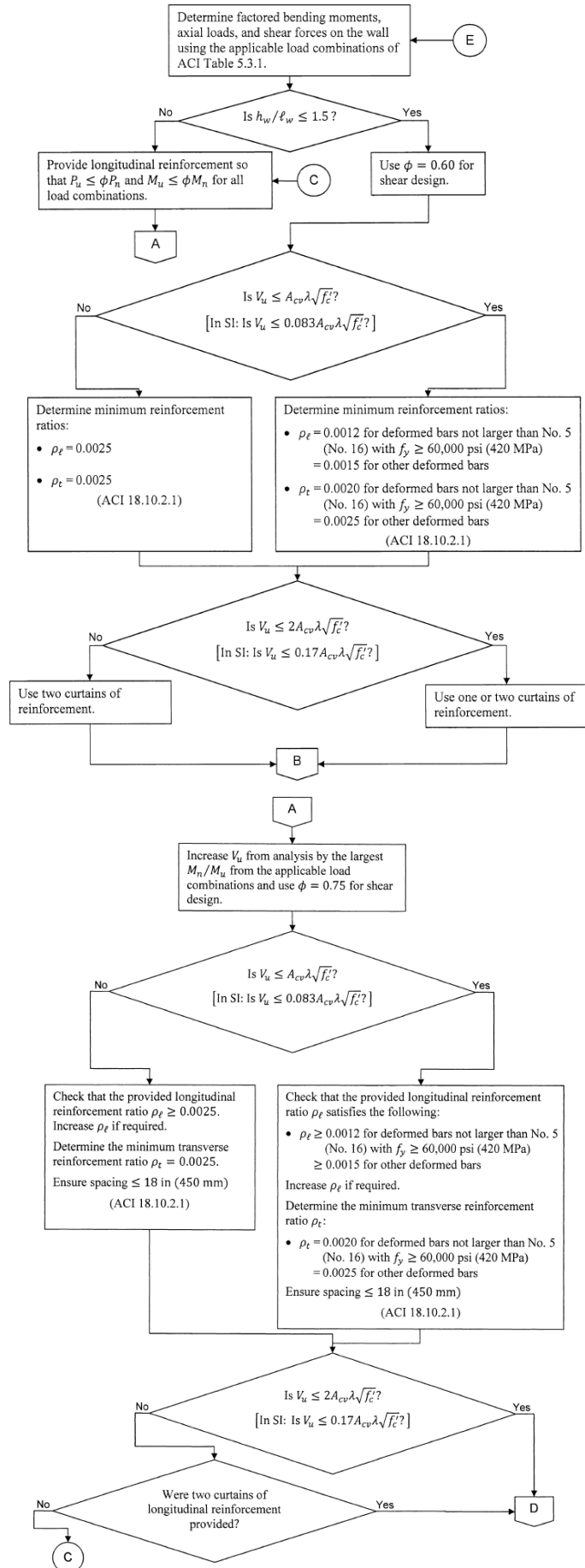


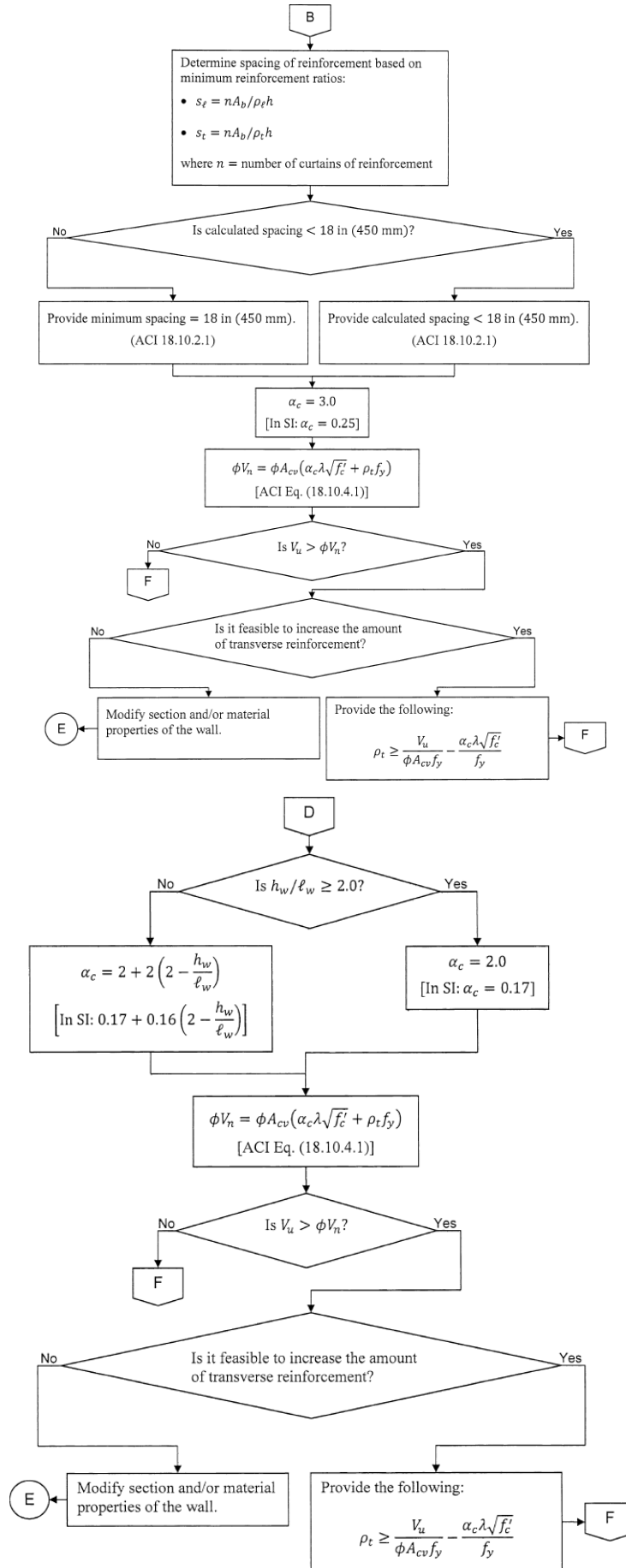
The required transverse reinforcement must extend over the distance determined in accordance with ACI 18.10.6.4(a), which is valid for special boundary elements. Spacing requirements for the transverse reinforcement are also shown in Fig. 11.87. It is evident that more stringent spacing requirements must be satisfied at sections within the plastic hinge region.

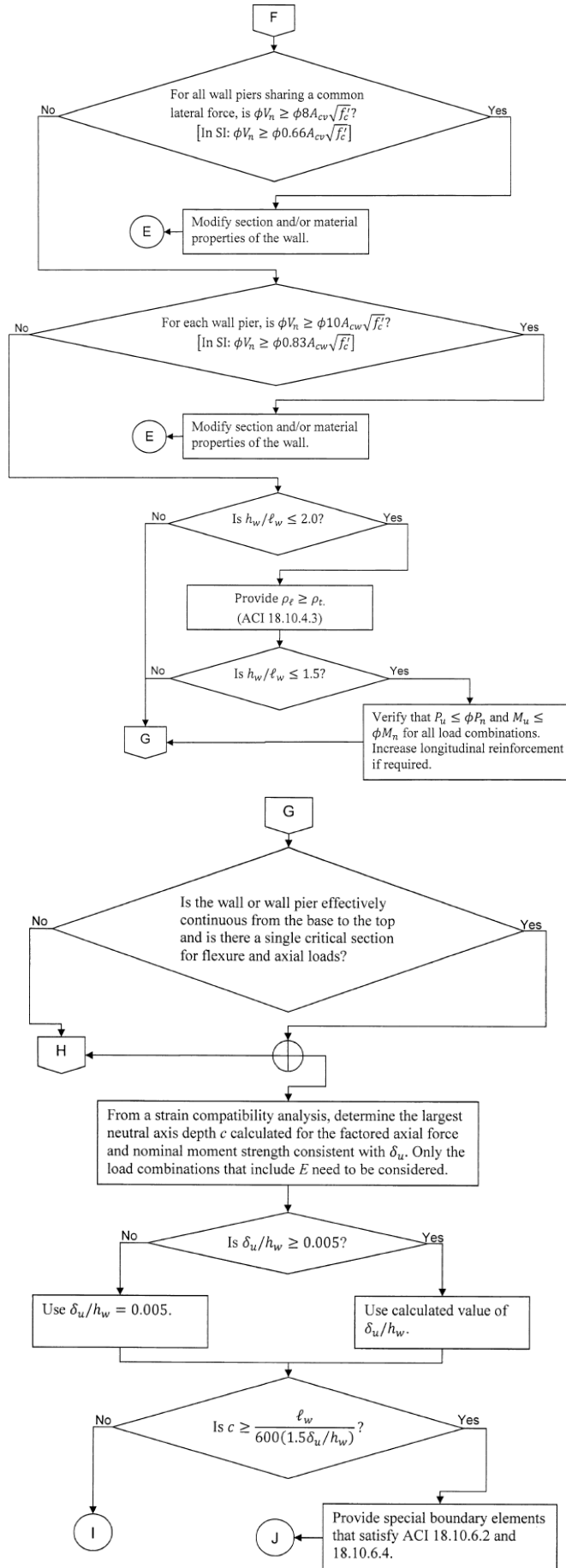
In walls with relatively large in-plane shear forces ( $V_u \geq A_{cv}\lambda\sqrt{f'_c}$ ), hooks or U-stirrups must be provided at the ends of the horizontal wall reinforcement to anchor these bars so that they can be effective in resisting the required shear force. The hooks or U-stirrups will also help prevent the longitudinal edge reinforcement in the wall from buckling.

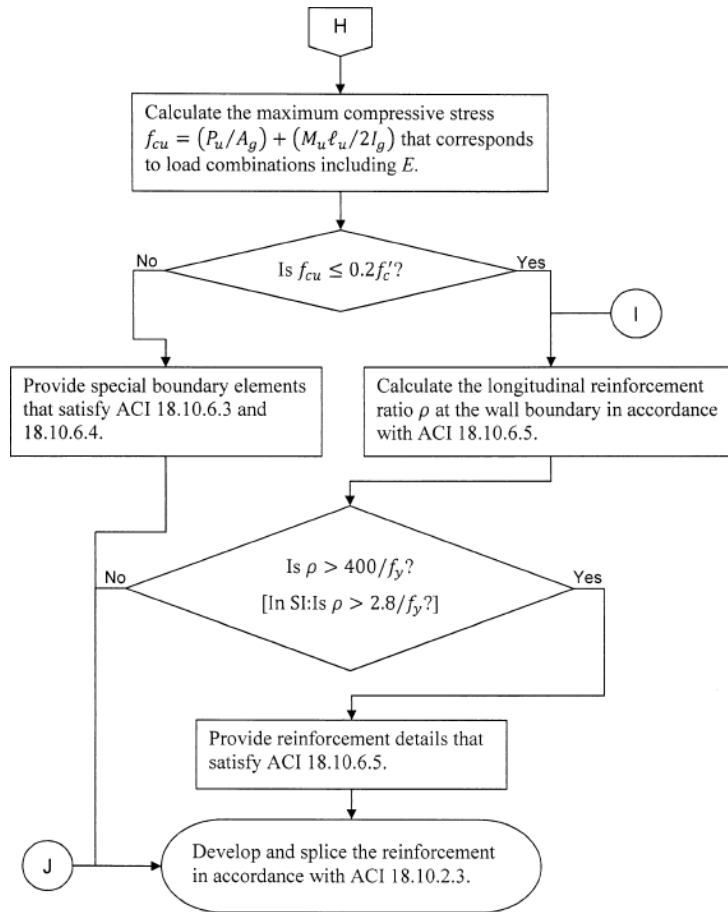
A summary of the overall design procedure for special structural walls is given in Fig. 11.88.

**Figure 11.88** Design procedure for special structural walls.



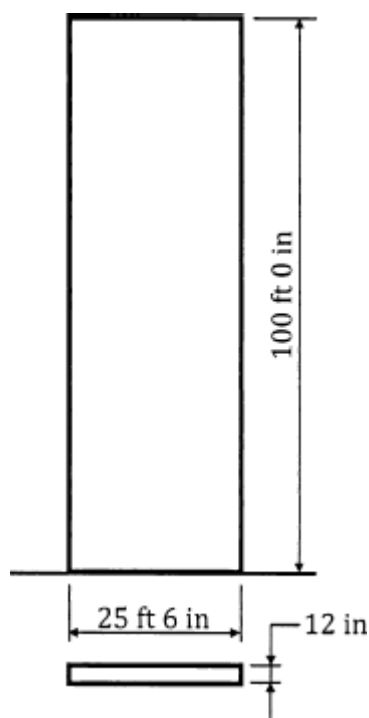






**Example 11.16** Depicted in Fig. 11.89 is a special structural wall that is part of the SFRS of a residential building located in SDC D. Design the wall given the factored load combinations in Table 11.21. From analysis of the building, it has been determined that  $\delta_u = 2.0$  in. Assume normal-weight concrete with  $f'_c = 4,000$  psi and Grade 60 reinforcement.

Figure 11.89 The special structural wall in Example 11.16.



**Table 11.21** Load Combinations for the Special Structural Wall in [Example 11.16](#)

Load Combination		Axial Force (kips)	Bending Moment (ft kips)	Shear Force (kips)
1.4D	ACI Eq. (5.3.1a)	1,290	0	0
1.2D + 1.6L	ACI Eq. (5.3.1b)	1,280	0	0
1.4D + 0.5L + Q <sub>E</sub>	ACI Eq. (5.3.1e)	1,345	20,000	440
0.7D + Q <sub>E</sub>	ACI Eq. (5.3.1g)	645	-20,000	-440

**Solution** The flowchart in [Fig. 11.88](#) is used to design the special structural wall.

**Step 1. Determine the factored bending moments, axial loads, and shear forces on the wall.** [Table 11.21](#) contains all of the applicable load combinations for this wall.

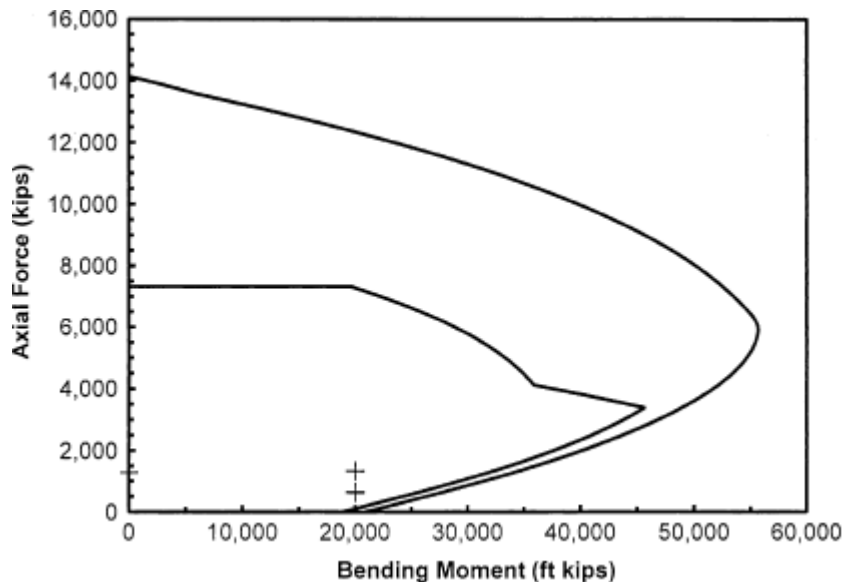
**Step 2. Determine  $h_w/\ell_w$ .**

$$h_w/\ell_w = 100/25.5 = 3.9 > 1.5$$

Because the height-to-length ratio is greater than 1.5, the wall will be designed for shear using a strength reduction factor of 0.75. Also, two curtains of reinforcement must be provided in the wall in accordance with ACI 18.10.2.2.

**Step 3. Determine required longitudinal reinforcement for flexure.** The design strength and nominal strength interaction diagrams for the 12-in-thick wall reinforced with eight No. 10 bars concentrated at each end of the wall and two curtains of No. 4 bars spaced at 12 in on center elsewhere are given in [Fig. 11.90](#). It is evident that the provided longitudinal reinforcement is adequate for all of the load combinations in [Table 11.21](#).

**Figure 11.90** Design and nominal strength interaction diagrams for the structural wall in [Example 11.16](#).



**Step 4. Determine  $V_u$  and check minimum longitudinal reinforcement ratio.** The maximum factored shear force from analysis will be multiplied by the largest  $M_n/M_u$  for the load combinations that include  $E$ . By doing so,  $\phi = 0.75$ .

The largest  $M_n/M_u$  occurs for the load combination from ACI Eq. (5.3.1e), which corresponds to an axial force  $P_n = P_u/\phi = 1,345/0.9 = 1,494$  kips. Using [Fig. 11.90](#),  $M_n/M_u = 35,940/20,000 = 1.8$ . Therefore,  $V_u = 1.8 \times 440 = 792$  kips.

$$V_u = 792 \text{ kips} > A_{cv}\lambda\sqrt{f'_c} = (12 \times 306) \times 1.0\sqrt{4,000}/1,000 = 232 \text{ kips}$$

Provided  $\rho_t = (2 \times 0.20)/(12 \times 12) = 0.0028 > 0.0025$  and the provided spacing of the No. 4 bars is less than 18 in.

**Step 5. Check shear strength requirements.** Assume two curtains of No. 4 bars spaced at 12 in for the transverse reinforcement ( $\rho_t = 0.0028$ ). Because  $h_w/\ell_w > 2.0$ ,  $\alpha_c = 2.0$ .



$$\begin{aligned}\phi V_n &= \phi A_{cv}(\alpha_c \lambda \sqrt{f'_c} + \rho_t f_y) \\ &= 0.75 \times (12 \times 306)[(2.0 \times 1.0 \sqrt{4,000}) + (0.0028 \times 60,000)]/1,000 \\ &= 811 \text{ kips} > 792 \text{ kips}\end{aligned}$$

Therefore, the two curtains of No. 4 bars spaced at 12 in are adequate for shear.

Check the upper limit on shear strength:

$$\phi V_n = 811 \text{ kips} < \phi 8 A_{cv} \lambda \sqrt{f'_c} = 1,393 \text{ kips}$$

**Step 6. Determine if special boundary elements are required.** The displacement-based approach of ACI 18.10.6.2 is used to determine the need for special boundary elements. Special boundary elements are required where Eq. (11.15) is satisfied:

$$c \geq \frac{\ell_w}{600(1.5\delta_u/h_w)}$$

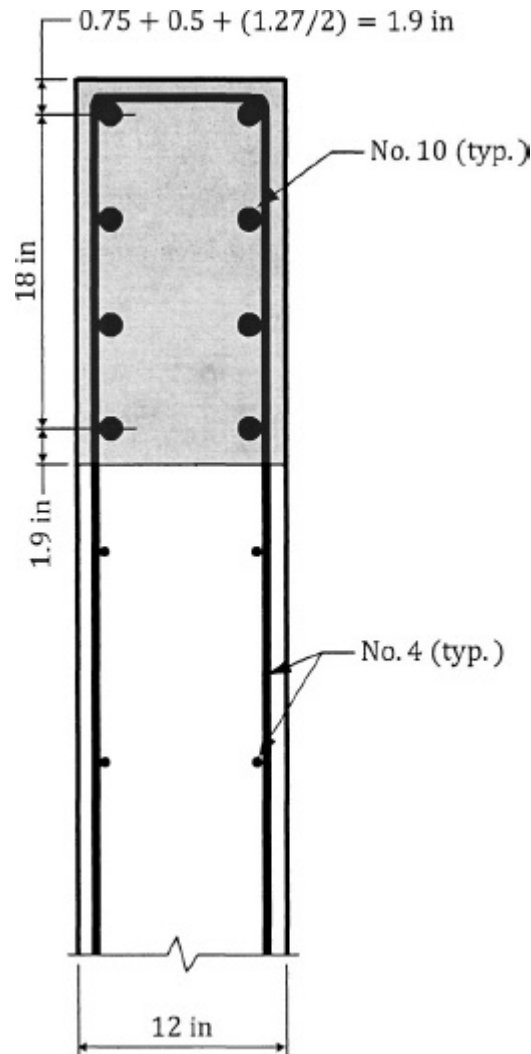
In this example,  $\ell_w = 306$  in and  $\delta_u/h_w = 2.0/(100 \times 12) = 0.002 < 0.005$ . Therefore, use  $\delta_u/h_w = 0.005$ . Special boundary elements are required if the neutral axis depth  $c$  is greater than or equal to  $306/(600 \times 1.5 \times 0.005) = 68.0$  in.

From strain compatibility analyses of the section, the largest neutral axis depth  $c$  is obtained from the load combination in ACI Eq. (5.3.1e), and is equal to 55.7 in, which is less than 68.0 in. Therefore, special boundary elements are not required.

**Step 7. Determine if transverse reinforcement required by ACI 18.10.6.5 is needed at the ends of the wall.** The reinforcement ratio at the wall boundary will be determined and compared to  $400/f_y = 0.0067$ . In this example, the reinforcement layout is similar to that shown on the left side of Fig. 11.87. Given a 6-in center-to-center spacing of the No. 10 bars and a 0.75-in clear cover to the No. 4 end reinforcement, the reinforcement ratio  $\rho$  is determined as follows (see Fig. 11.91):

$$\rho = \frac{A_{sb}}{A_{cb}} = \frac{8 \times 1.27}{12 \times [(3 \times 6) + (2 \times 1.9)]} = 0.0388 > 0.0067$$

Figure 11.91 Longitudinal reinforcement ratio at boundary of the structural wall in Example 11.16.



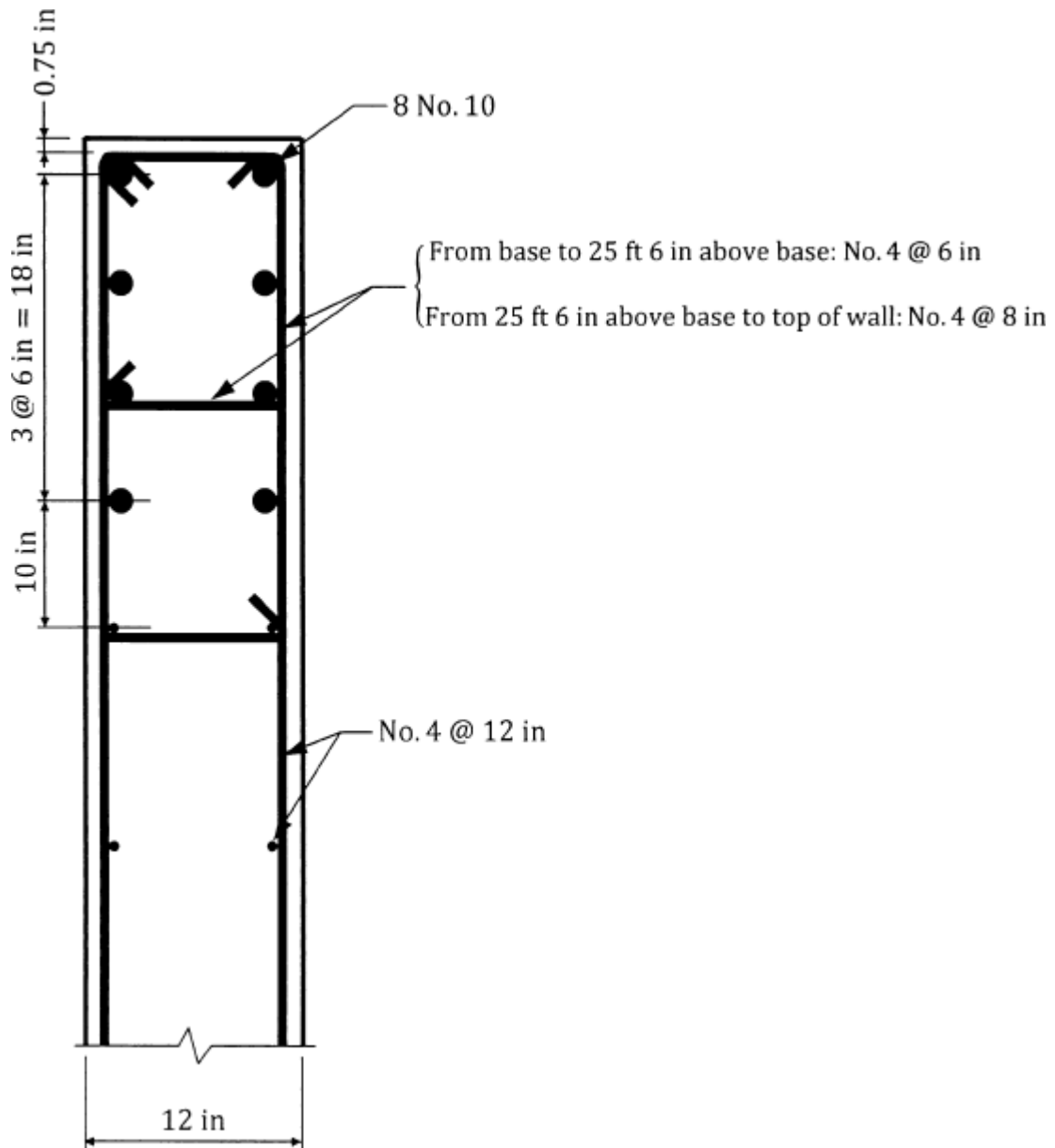
Therefore, provide transverse reinforcement in accordance with ACI 18.10.6.5. The maximum longitudinal spacing of the transverse reinforcement within a distance of the greater of  $\ell_w = 25.5$  ft (governs) or  $M_u/4V_u = 20,000/(4 \times 440) = 11.4$  ft from the base of the wall is the lesser of 6 in (governs) or  $6d_b = 7.6$  in. From 25.5 ft above the base of the wall to the top of the wall, the spacing is permitted to be the lesser of 8 in (governs) or  $8d_b = 10.2$  in.

The transverse reinforcement at the boundaries of the wall must extend a distance not less than the greater of  $c - 0.1\ell_w = 55.7 - (0.1 \times 306) = 25.1$  in or  $c/2 = 27.9$  in (governs). Based on the spacing of the No. 4 longitudinal bars adjacent to the No. 10 bars, provide the transverse reinforcement over a length of approximately 30 in at each end of the wall.

Because  $V_u > A_{cv}\lambda\sqrt{f'_c}$ , horizontal reinforcement for the web of the wall that is terminating at the edges of the wall must have a standard hook engaging the edge reinforcement or the edge reinforcement must be enclosed by U-stirrups that are spliced to the horizontal reinforcement. The U-stirrups must have the same size and spacing as the horizontal reinforcement.

Reinforcement details for the structural wall are given in Fig. 11.92.

Figure 11.92 Reinforcement details for the structural wall in Example 11.16.



**Comment** As an exercise, the compressive stress approach of ACI 18.10.6.3 will be utilized to determine if special boundary elements are required or not for the structural wall in this example. According to this method, special boundary elements are required where Eq. (11.16) is satisfied:

$$f_{cu} = \frac{P_u}{A_g} + \frac{M_u \ell_w}{2I_g} > 0.2f'_c$$

$$A_g = 12 \times 306 = 3,672 \text{ in}^2$$

$$I_g = \frac{1}{12} \times 12 \times 306^3 = 28,652,616 \text{ in}^4$$

Therefore,

$$f_{cu} = \frac{1,345,000}{3,672} + \frac{20,000 \times 12,000 \times 306}{2 \times 28,652,616} = 366 + 1,282 = 1,648 \text{ psi} > 800 \text{ psi}$$

According to this method, special boundary elements are required at the ends of the wall. It is common for this approach to yield conservative results.

**Example 11.17** Depicted in Fig. 11.40 is a typical plan of a reinforced concrete building, which utilizes special structural walls as part of the SFRS in the north-south direction (see Example 11.8). Design the wall on column line 4 given the factored load combinations in Table 11.22. The overall height of the wall is 28 ft. From analysis of the building, it has been determined that  $\delta_u = 0.8$  in. Assume normal-weight concrete with  $f'_c = 4,000$  psi and Grade 60 reinforcement.

Table 11.22 Load Combinations for Special Structural Wall in Example 11.17

Load Combination		Axial Force (kips)	Bending Moment (ft kips)	Shear Force (kips)
1.4D	ACI Eq. (5.3.1a)	476	0	0
1.2D + 1.6L + 0.5L <sub>r</sub>	ACI Eq. (5.3.1b)	485	0	0
1.4D + 0.5L + 1.3Q <sub>E</sub>	ACI Eq. (5.3.1e)	499	11,310	416
0.7D + 1.3Q <sub>E</sub>	ACI Eq. (5.3.1g)	238	-11,310	-416

Note: The 1.3 factor on Q<sub>E</sub> is the redundancy factor determined in accordance with ASCE/SEI 12.3.4.2.

**Solution** The flowchart in Fig. 11.88 is used to design the special structural wall.

**Step 1. Determine the factored bending moments, axial loads, and shear forces on the wall.** Table 11.22 contains all of the applicable load combinations for this wall.

**Step 2. Determine  $h_w/\ell_w$ .**

$$h_w/\ell_w = \frac{28}{19.7} = 1.4 < 1.5$$

Because the height-to-length ratio is less than 1.5, the wall will be designed for shear using a strength reduction factor of 0.60.

**Step 3. Determine the minimum reinforcement ratios for the web of the wall.**

$$V_u = 416 \text{ kips} > A_{cv}\lambda\sqrt{f'_c} = (10 \times 236) \times 1.0\sqrt{4,000}/1,000 = 149 \text{ kips}$$

Therefore,  $\rho_\ell = \rho_t = 0.0025$ .

**Step 4. Determine the number of curtains of reinforcement in the web of the wall.**

$$V_u = 416 \text{ kips} > 2A_{cv}\lambda\sqrt{f'_c} = 298 \text{ kips}$$

Therefore, provide two curtains of reinforcement.

**Step 5. Determine spacing of reinforcement based on minimum reinforcement ratios.** Assuming two curtains of No. 4 bars in the wall:

$$s_\ell = \frac{nA_b}{\rho_\ell h} = \frac{2 \times 0.20}{0.0025 \times 10} = 16 \text{ in} < 18 \text{ in}$$

The spacing for the transverse reinforcement is also 16 in.

**Step 6. Check shear strength requirements.** Assume two curtains of No. 4 bars spaced at 16 in for the transverse reinforcement. Because  $h_w/\ell_w < 1.5$ ,  $\alpha_c = 3.0$ .

$$\begin{aligned} \phi V_n &= \phi A_{cv}(\alpha_c \lambda \sqrt{f'_c} + \rho_t f_y) \\ &= 0.60 \times (10 \times 236)[(3.0 \times 1.0\sqrt{4,000}) + (0.0025 \times 60,000)]/1,000 \\ &= 481 \text{ kips} > 416 \text{ kips} \end{aligned}$$

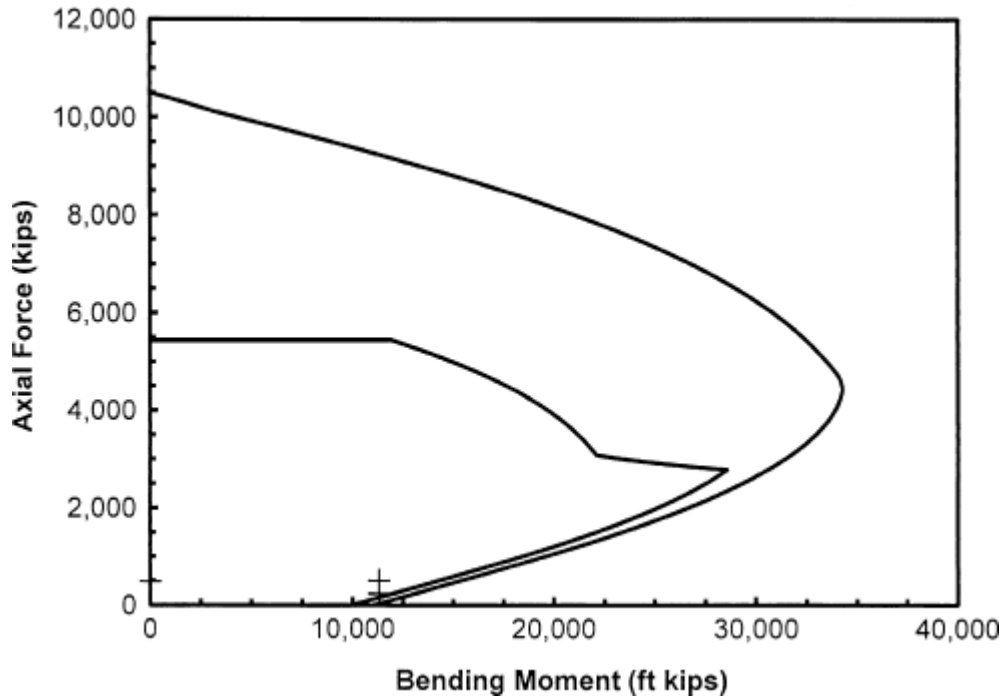
Therefore, the two curtains of No. 4 bars spaced at 16 in are adequate for shear.

Check the upper limit on shear strength:

$$\phi V_n = 481 \text{ kips} < \phi 8 A_{cv} \lambda \sqrt{f'_c} = 715 \text{ kips}$$

**Step 7. Check combined flexure and axial force strength.** The 20 × 20 in columns that attached to each end of the wall are utilized to resist combined bending and axial force. Assume the column is reinforced with 12 No. 7 bars (1.8% reinforcement ratio). The design strength and nominal strength interaction diagrams for the 10-in-thick wall reinforced with two curtains of No. 4 bars spaced at 16 in on center and the 20-in square columns reinforced with 12 No. 7 bars are given in Fig. 11.93. It is evident that the provided longitudinal reinforcement is adequate for all of the load combinations in Table 11.22.

Figure 11.93 Design and nominal strength interaction diagrams for the structural wall in Example 11.17.



**Step 8. Determine if special boundary elements are required.** Because  $h_w/\ell_w < 2.0$ , the displacement-based approach of ACI 18.10.6.2 cannot be used to determine the need for special boundary elements. Thus, the compressive stress approach is used. Special boundary elements are required at the ends of the wall or at edges around openings where the maximum compressive stress  $f_{cu}$  exceeds  $0.2f'_c$ :

$$f_{cu} = \frac{P_u}{A_g} + \frac{M_u \ell_w}{2I_g} > 0.2f'_c$$

$$A_g = (10 \times 196) + (2 \times 20^2) = 2,760 \text{ in}^2$$

$$I_g = \left( \frac{1}{12} \times 10 \times 196^3 \right) + \left( \frac{2}{12} \times 20^4 \right) + \left[ 2 \times (20 \times 20) \times \left( \frac{196}{2} + 10 \right)^2 \right] = 15,632,480 \text{ in}^4$$

Therefore,

$$f_{cu} = \frac{499,000}{2,760} + \frac{11,310 \times 12,000 \times 236}{2 \times 15,632,480} = 181 + 1,025 = 1,206 \text{ psi} > 800 \text{ psi}$$

According to this method, special boundary elements are required at the ends of the wall.

From strain compatibility analyses of the section, the largest neutral axis depth  $c$  is obtained from the load combination in ACI Eq. (5.3.1e), and is equal to 18.2 in.

**Step 9. Provide special boundary elements that satisfy ACI 18.10.6.3 and 18.10.6.4.** Design and detailing requirements of ACI 18.10.6.4:

- The boundary element shall extend horizontally from each end of the wall a distance not less than the greater of  $c - 0.1\ell_w = 18.2 - (0.1 \times 236) < 0$  or  $c/2 = 9.1$  in (governs). For simpler detailing, confine the entire column at both ends of the wall.
- Assuming a clear story height of  $14 - (8/12) = 13.3$  ft,  $h_w/16 = 13.3 \times 12/16 = 10$  in  $<$  provided flexural compression zone width = 20 in.
- Because  $h_w/\ell_w < 2.0$ , ACI 18.10.6.4(c) is not applicable in this example.

- d. ACI 18.10.6.4(d) is not applicable in this example.
- e. Spacing of the transverse reinforcement shall be less than or equal to the lesser of the following:
- Least dimension of special boundary element/3 = 20/3 = 6.7 in
  - $6d_b = 6 \times 0.875 = 5.3$  in (governs)
  - Assuming crossies engage all of the longitudinal bars in the column:

$$s_o = 4 + \left( \frac{14 - h_x}{3} \right) = 4 + \left( \frac{14 - 5.0}{3} \right) = 7.0 \text{ in} > 6 \text{ in, use 6 in}$$

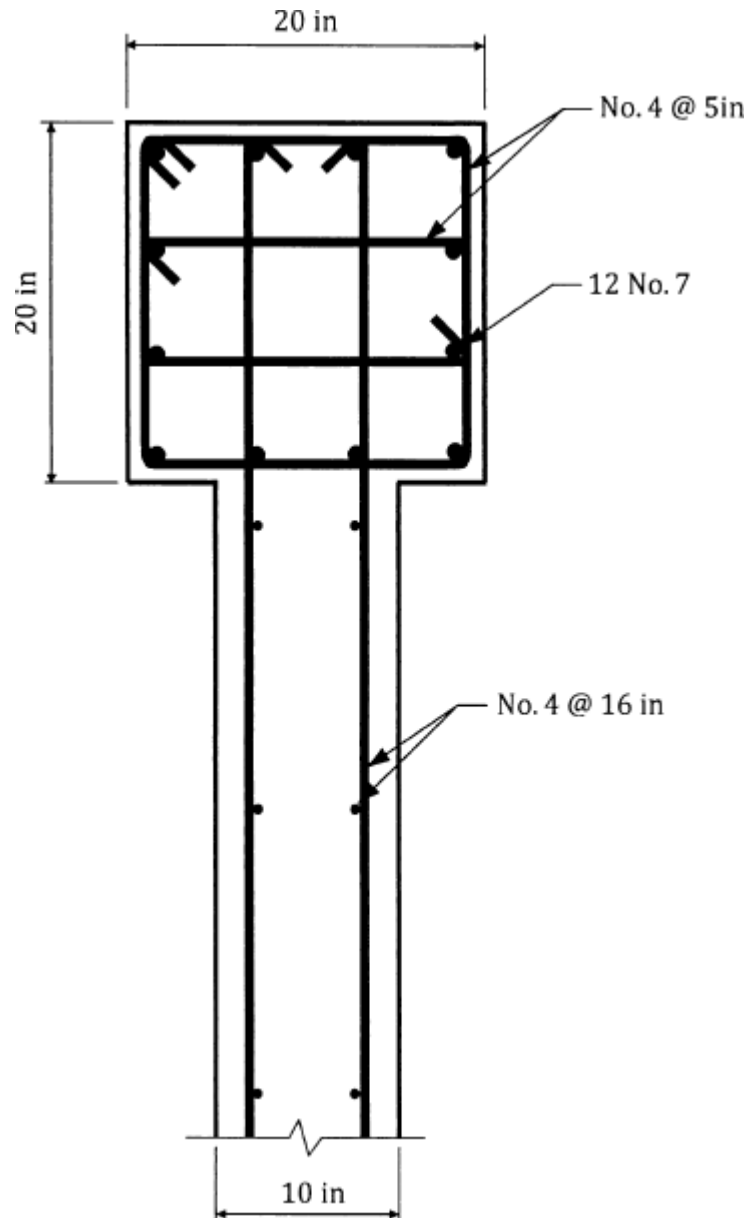
- f. Amount of transverse reinforcement is determined from ACI Table 18.10.6.4(f):

$$A_{sh} \geq \begin{cases} 0.3s_b c_c \left( \frac{A_g}{A_{ch}} - 1 \right) \frac{f'_c}{f_{yt}} = 0.3 \times 5 \times [20 - (2 \times 1.5)] \left( \frac{20^2}{17^2} - 1 \right) \times \frac{4}{60} = 0.65 \text{ in}^2 \text{ (governs)} \\ 0.09s_b c_c \frac{f'_c}{f_{yt}} = 0.09 \times 5 \times [20 - (2 \times 1.5)] \times \frac{4}{60} = 0.51 \text{ in}^2 \end{cases}$$

No. 4 hoops with crossies around every longitudinal bar provides  $A_{sh} = 4 \times 0.2 = 0.80 \text{ in}^2 > 0.65 \text{ in}^2$ . Therefore, use No. 4 hoops and crossies spaced at 5 in within the column. For simpler detailing, these hoops and crossies at the 5-in spacing is used over the entire height of the wall even though the special boundary elements can be discontinued where  $f_{cu} < 0.15f'_c$ .

Reinforcement details for the structural wall are given in [Fig. 11.94](#).

Figure 11.94 Reinforcement details for the structural wall in Example 11.17.



**Step 10. Determine the lap splice length of the longitudinal bars in special boundary elements in accordance with ACI 18.10.2.3.** From the base of the wall to where  $f_{cu} < 0.15f'_c$ , the longitudinal bars must be spliced to develop  $1.25 f_y$  in tension (ACI 18.10.2.3). For the No. 7 bars in the columns at the ends of the wall:

$$\ell_d = \left( \frac{3}{40} \frac{1.25 f_y}{\lambda \sqrt{f'_c}} \frac{\psi_t \psi_e \psi_s}{\left( \frac{c_b + K_{tr}}{d_b} \right)} \right) d_b$$

where  $\lambda = 1.0$  for normal-weight concrete

$\psi_t = 1.0$  for other than top bars

$\psi_e = 1.0$  for uncoated bars

$\psi_s = 1.0$  for No. 7 bars

$$c_b = \begin{cases} 1.5 + 0.5 + \frac{0.875}{2} = 2.4 \text{ in (governs)} \\ \frac{20 - 2(1.5 + 0.5) - 0.875}{2 \times 3} = 2.5 \text{ in} \end{cases}$$

$$K_{tr} = \frac{40A_{tr}}{sn} = \frac{40 \times (4 \times 0.2)}{5 \times 4} = 1.6$$

$$\frac{c_b + K_{tr}}{d_b} = \frac{2.4 + 1.6}{0.875} = 4.6 > 2.5, \text{ use } 2.5$$

Therefore,

$$\ell_d = \left( \frac{3}{40} \frac{1.25 \times 60,000}{1.0 \sqrt{4,000}} \frac{1.0 \times 1.0 \times 1.0}{2.5} \right) \times 0.875 = 31.1 \text{ in}$$

Class B splice length =  $1.3 \times 31.1 = 40.5$  in

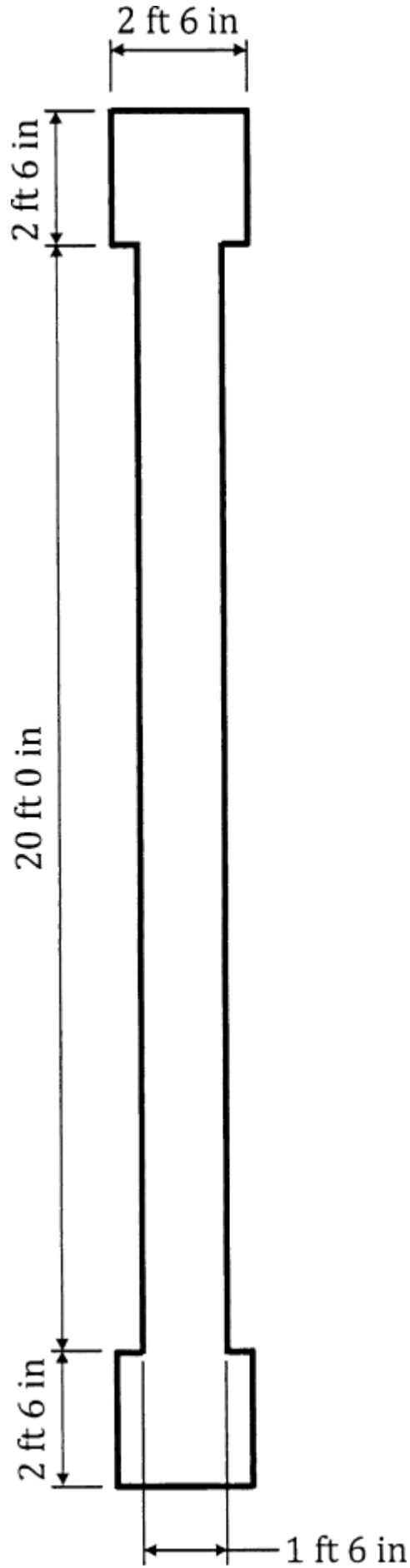
Use a 3 ft 6 in lap splice length in the plastic hinge zone for the No. 7 longitudinal bars in the columns. For simpler detailing, this splice length can be used over the entire height of the building.

Similar calculations can be performed for the splice length of the No. 4 bars in the web of the wall.

**Example 11.18** Depicted in Fig. 11.95 is a structural wall that is part of the SFRS in a multistory building located in SDC D. Determine if special boundary elements are required given the following information: normal-weight concrete with  $f'_c = 6,000$  psi and Grade 60 reinforcement;  $h_w = 150$  ft;  $\delta_u = 14.0$  in; governing load combination:  $P_u = 4,000$  kips,  $M_u = 50,000$  ft kips; and, 24 No. 10 bars in the column and No. 5 bars at 12-in spacing in the web (longitudinal and transverse).

Figure 11.95 Special structural wall of Example 11.18.





**Solution** The displacement-based approach of ACI 18.10.6.2 is used to determine the need for special boundary elements. Special boundary elements are required where Eq. (11.15) is satisfied:

$$c \geq \frac{\ell_w}{600(1.5\delta_u/h_w)}$$

In this example,  $\ell_w = 300$  in and  $\delta_u/h_w = 14/(150 \times 12) = 0.008 > 0.005$ . Special boundary elements are required if the neutral axis depth  $c$  is greater than or equal to  $300/(600 \times 1.5 \times 0.008) = 41.7$  in.

From strain compatibility analyses of the section, the neutral axis depth  $c$  corresponding to the governing load combination is equal to 47.6 in, which is greater than 41.7 in. Therefore, special boundary elements are required.

Design and detailing requirements of ACI 18.10.6.4:

- The transverse reinforcement at the boundaries of the wall must extend a distance not less than the greater of  $c - 0.1\ell_w = 47.6 - (0.1 \times 300) = 17.6$  in or  $c/2 = 23.8$  in (governs). For simpler detailing, confine the entire column at both ends of the wall.
- Assuming a clear story height of 12 ft,  $h_u/16 = 12 \times 12/16 = 9$  in  $<$  provided flexural compression zone width = 30 in.
- Because  $c/\ell_w = 0.16 < 0.375$ , ACI 18.10.6.4(c) is not applicable in this example.
- ACI 18.10.6.4(d) is not applicable in this example.
- Spacing of the transverse reinforcement shall be less than or equal to the lesser of the following:
  - Least dimension of special boundary element/3 =  $30/3 = 10$  in
  - $6d_b = 6 \times 1.27 = 7.6$  in
  - Assuming crossies engage all of the longitudinal bars in the column:

$$s_o = 4 + \left( \frac{14 - h_x}{3} \right) = 4 + \left( \frac{14 - 4.1}{3} \right) = 7.3 \text{ in} > 6 \text{ in, use 6 in (governs)}$$

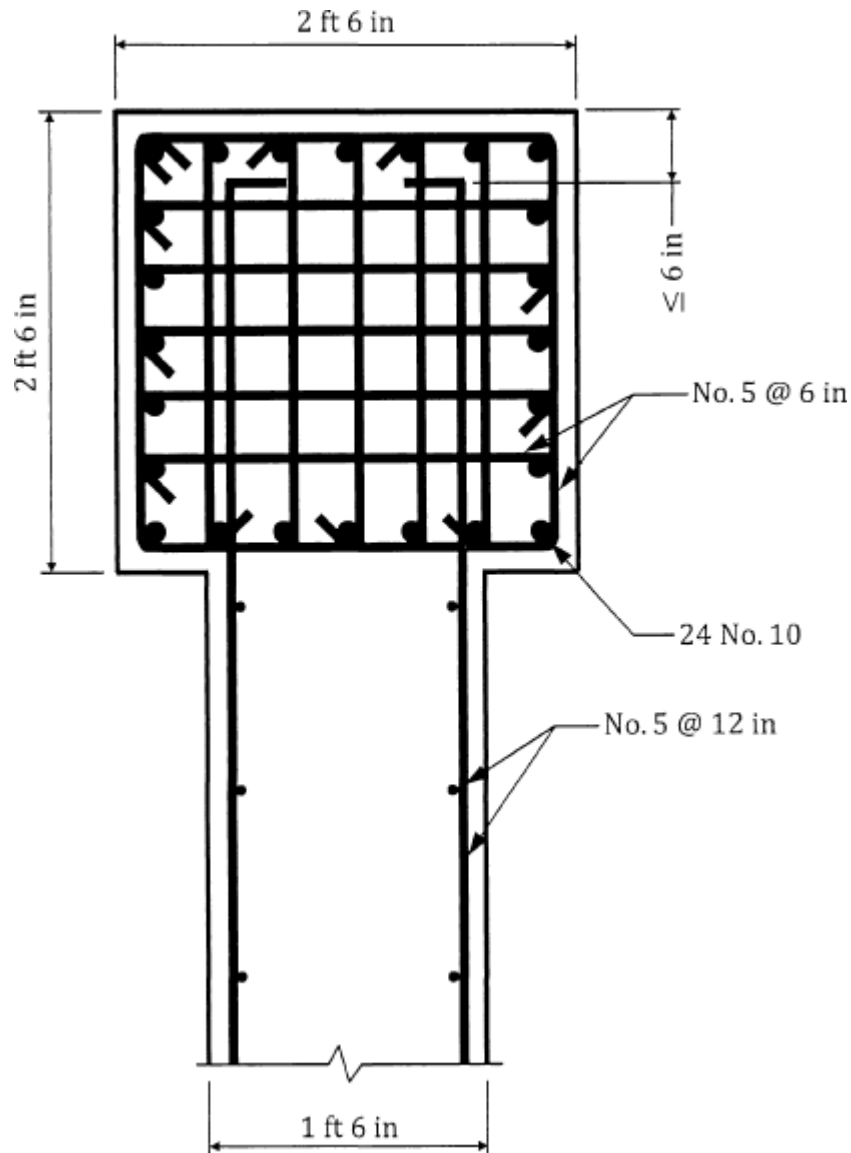
- Amount of transverse reinforcement is determined from ACI Table 18.10.6.4(f):

$$A_{sh} \geq \begin{cases} 0.3sb_c \left( \frac{A_g}{A_c h} - 1 \right) \frac{f'_c}{f_{yt}} = 0.3 \times 6 \times [30 - (2 \times 1.5)] \left( \frac{30^2}{27^2} - 1 \right) \times \frac{6}{60} = 1.1 \text{ in}^2 \\ 0.09sb_c \frac{f'_c}{f_{yt}} = 0.09 \times 6 \times [30 - (2 \times 1.5)] \times \frac{6}{60} = 1.5 \text{ in}^2 \text{ (governs)} \end{cases}$$

No. 5 hoops with crossies around every longitudinal bar provides  $A_{sh} = 7 \times 0.31 = 2.2 \text{ in}^2 > 1.5 \text{ in}^2$ . Therefore, use No. 5 hoops and crossies spaced at 6 in within the column.

Figure 11.96 shows the reinforcement details for this structural wall.

Figure 11.96 Reinforcement details for the structural wall in Example 11.18.

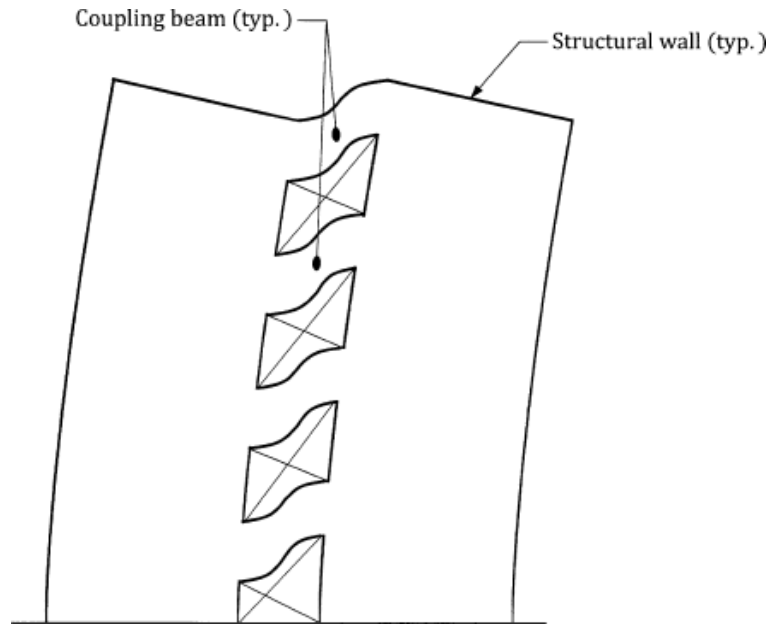


**Comments** Although it did not occur in this example, it is possible that the region where special boundary elements are required can extend into the web portion of a barbell structural wall. In such cases, the portion of the web within this region must be confined with the required transverse reinforcement.

## 11.8.6. Coupling Beams

Coupling beams are beams that connect two structural walls together and are typically aligned vertically above openings in the wall over its height (Fig. 11.97). When properly designed and detailed, such beams can provide an efficient means of energy dissipation when the structure is subjected to ground motion from a seismic event. When designing a coupled wall system, the objective is to develop plastic hinges (i.e., yield mechanisms) at the ends of the coupling beams over the height of the wall prior to flexural yielding of each wall at its base.

Figure 11.97 Behavior of coupled structural walls.



Because of geometric constraints, coupling beams usually have relatively large depth to span ratios. As such, the ends of the beams are typically subjected to large inelastic rotations and large shear forces. Sufficient shear reinforcement and proper detailing and confinement of all the reinforcement in the beam are needed to prevent shear failure and to ensure ductility and energy dissipation. Tests have shown that adequate resistance can be achieved by using confined diagonal reinforcement in deep coupling beams.

The design and detailing requirements depend on the clear span length to depth ratio  $\ell_n/h$  of the beam. A summary of the requirements follows.

- $\ell_n/h \geq 4$  (ACI 18.10.7.1). Coupling beams where  $\ell_n/h \geq 4$  are designed and detailed in accordance with the requirements in ACI 18.6 for beams in special moment frames. Note that wall boundaries take the place of columns in these provisions. The dimensional limits in ACI 18.6.2.1(b) and (c) need not be satisfied where analysis shows that the beam has adequate lateral stability. The use of diagonal reinforcement is not required in this case because the beams are considered to be too shallow for the diagonal bars to be efficient. Design of the coupling beams for flexure and shear is the same as for beams in special moment frames. The reinforcement details depicted in Fig 11.31 are applicable to coupling beams where  $\ell_n/h \geq 4$ . The longitudinal reinforcement must be embedded into the wall boundaries in accordance with the provisions in ACI 18.8.5 for development of straight bars.
- $\ell_n/h < 2$  and  $V_u > 4\lambda\sqrt{f'_c}A_{cw}$  [In SI:  $V_u > 0.33\lambda\sqrt{f'_c}A_{cw}$ ] (ACI 18.10.7.2). For relatively deep coupling beams subjected to relatively large factored shear forces, the reinforcement must consist of two intersecting groups of diagonally placed bars symmetrical about the midspan of the beam unless it can be demonstrated that safety and stability are not compromised. The diagonally placed bars are intended to provide both the shear and flexural strength of the beam. As noted above, experiments have shown that diagonal bars are only effective if they can be placed at a large inclination. The reinforcement details in ACI 18.10.7.4 for this case are covered below.
- **Coupling beams not governed by ACI 18.10.7.1 or 18.10.7.2 (ACI 1.10.7.3).** Coupling beams that are not governed by the aforementioned requirements may be reinforced with either two intersecting groups of diagonally placed bars symmetrical about the midspan of the beam or in accordance with the longitudinal and transverse reinforcement requirements of ACI 18.6.3 through 18.6.5. The choice between these reinforcement options leading to the most efficient design typically depends on geometry and the magnitude of the factored shear force.

The nominal shear strength of a coupling beam reinforced with two intersecting groups of diagonally placed bars symmetrical

about the midspan of the beam is determined by ACI Eq. (18.10.7.4):

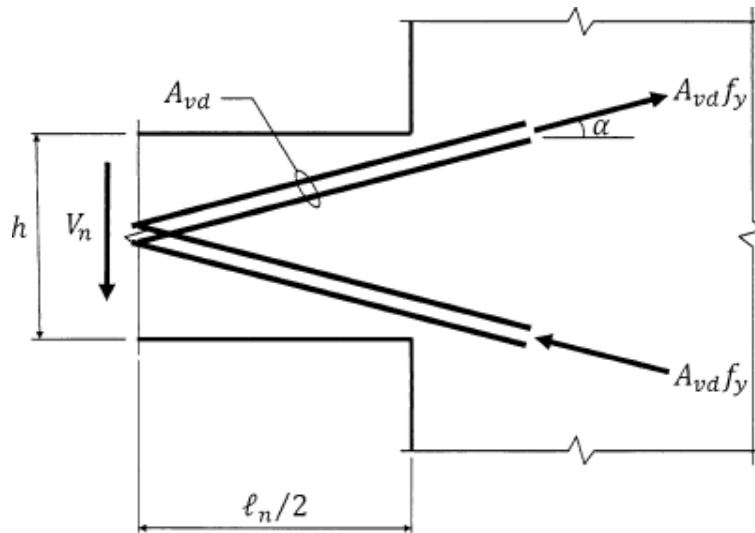
$$V_n = 2A_{vd} f_y \sin \alpha \leq 10\sqrt{f'_c} A_{cw}$$

$$[\text{In SI: } V_n = 2A_{vd} f_y \sin \alpha \leq 0.83\sqrt{f'_c} A_{cw}]$$

(11.17)

In this equation,  $A_{vd}$  is the total area of reinforcement in each group of diagonal bars,  $\alpha$  is the angle between the diagonal bars and the longitudinal axis of the coupling beam, and  $A_{cw}$  is the gross area of the coupling beam. The equality portion of this equation is based on vertical equilibrium of an idealized truss with tension and compression diagonals that act along the axes of the diagonal reinforcement (see Fig. 11.98). Note that each group of diagonal bars must have a minimum of four bars in two layers and all of the diagonal bars must be embedded into the wall boundaries at least 1.25 times the development length for  $f_y$  in tension. Headed reinforcement can be used to shorten development lengths and to help alleviate construction issues.

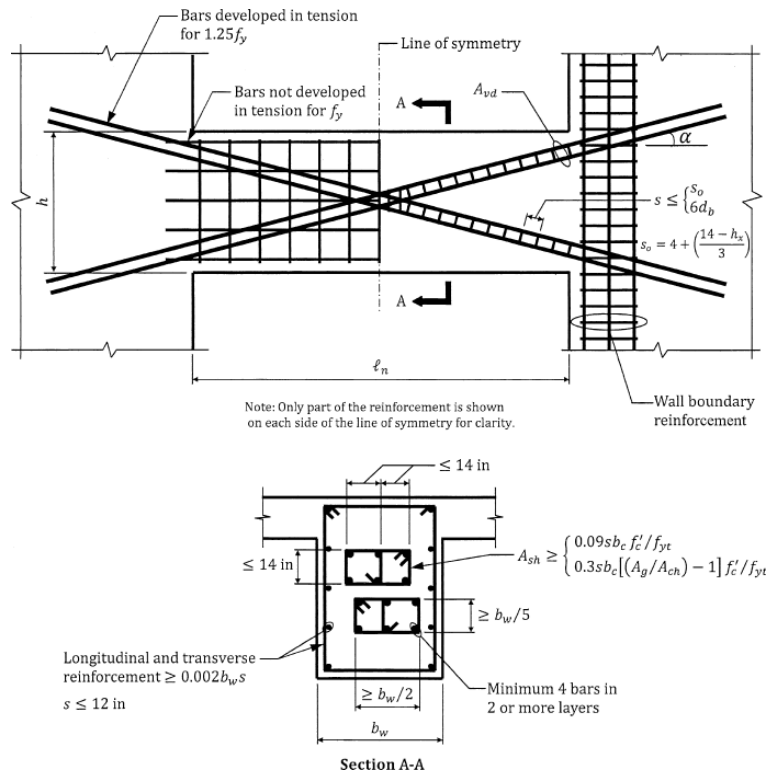
Figure 11.98 Idealized truss in a coupling beam.



The required shear strength  $V_u$  must be less than or equal to the design shear strength  $\phi V_n$  where the strength reduction factor  $\phi$  is equal to 0.85 for diagonally reinforced coupling beams (ACI 21.2.4.3). Moment resistance is automatically provided by the diagonal bars as well.

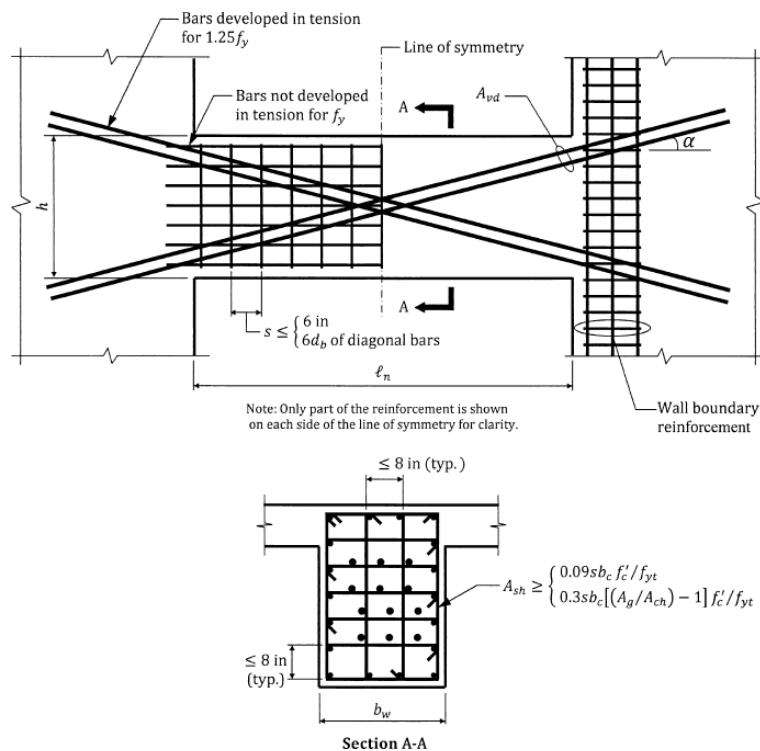
Two options are given regarding confinement of the diagonal bars. In the first option, each group of diagonal bars is confined by rectilinear transverse reinforcement that satisfies the requirements of ACI 18.10.7.4(c). These requirements are illustrated in Fig. 11.99. Note that the transverse reinforcement that is required along the entire length of the diagonal bars is the same as that at the ends of columns in special moment frames.

Figure 11.99 Detailing requirements for coupling beams—confinement of individual diagonals.



In the second option, transverse reinforcement in accordance with ACI 18.10.7.4(d) is provided for the entire cross-section of the coupling beam (see Fig. 11.100). This option helps facilitate placement of the reinforcing bars in the field, especially at the location where the diagonal bars intersect each other. Like in the first option, the required amount of transverse reinforcement is the same as that for columns in special moment frames.

Figure 11.100 Detailing requirements for coupling beams—confinement of the entire cross-section.



Transverse and longitudinal reinforcement must be provided around the beam section as shown in Figs. 11.99 and 11.100. The longitudinal reinforcement should not be fully developed into the wall so that it does not develop significant tensile stresses.

Regardless of the type of coupling beam that is utilized as part of the SFRS, it is important to keep in mind that a minimum coupling beam width, which is equal to the wall width, needs to be established so that all of the required reinforcement can be accommodated within the section. Scaled drawings of the cross-section should be produced to ensure that all of the bars adequately fit in the section considering the requirements for minimum cover and spacing between the bars that are prescribed in the Code. In the case of diagonally reinforced coupling beams, a minimum width of 14 in (350 mm) has been suggested.<sup>12</sup>

**Example 11.19** Determine the required reinforcement for a coupling beam that is 28 in deep and 16 in wide. The clear span  $\ell_n = 4$  ft. From analysis of the structure, the factored shear force is equal to 160 kips. Assume normal-weight concrete with  $f'_c = 5,000$  psi and Grade 60 reinforcement.

**Solution** Determine the clear span to height ratio:

$$\ell_n/h = (4 \times 12)/28 = 1.7 < 2$$

Check shear requirements to determine if diagonal bars are required:

$$V_u = 160 \text{ kips} > 4\lambda\sqrt{f'_c}A_{cw} = 4 \times 1.0\sqrt{5,000} \times 28 \times 16/1,000 = 127 \text{ kips}$$

Therefore, diagonal reinforcing bars must be provided.

In order to determine  $\alpha$ , the centroid of the diagonal bar bundles must be located. The minimum out-to-out dimensions of the transverse reinforcement are  $b_w/2 = 8.0$  in for the dimension parallel to  $b_w$  and  $b_w/5 = 3.2$  in along the other sides. Assuming  $8 \times 8$  in out-to-out dimensions of the transverse reinforcement around the diagonal bars and a 1.5-in cover to the transverse reinforcement near the ends of the beam, the distance from the top or bottom of the section to the centroid of the diagonal bar bundles is approximately  $1.5 + (8/2) = 5.5$  in. Thus, the distance from the centroid of the beam to the centroid of the diagonal bars is  $(28/2) - 5.5 = 8.5$  in. Therefore,

$$\alpha = \tan^{-1} \frac{8.5}{(48/2)} = 20 \text{ degrees}$$

Determine  $A_{vd}$  by ACI Eq. (18.10.7.2):

$$A_{vd} = \frac{V_u/\phi}{2f_y \sin \alpha} = \frac{160/0.85}{2 \times 60 \times \sin 20} = 4.6 \text{ in}^2$$

Provide six No. 8 bars in each group of diagonal bars (provided  $A_{vd} = 4.7 \text{ in}^2$ ).

Check upper limit on shear strength:

$$\phi V_n = 2\phi A_{vd} f_y \sin \alpha = 164 \text{ kips} < 10\phi\sqrt{f'_c}A_{cw} = 269 \text{ kips}$$

The required transverse reinforcement is determined using both options in ACI 18.10.7.4.

(1) Option 1—Confinement of Individual Diagonals

The required transverse reinforcement is determined in accordance with ACI 18.10.7.4(c). The area  $A_g$  is determined assuming a 0.75-in cover around the diagonal bar group:  $A_g = (1.5 + 8) \times (1.5 + 8) = 90.3 \text{ in}^2$ . Also,  $A_{ch} = 8 \times 8 = 64 \text{ in}^2$ . Assuming No. 4 bars for the transverse reinforcement around the diagonal bars, the spacing  $s$  is the smaller of the following:

$$s = \begin{cases} s_o = 4 + \left( \frac{14 - h_x}{3} \right) = 4 + \left[ \frac{14 - (8 - 1 - 1)}{3} \right] = 6.7 \text{ in} > 6.0 \text{ in, use } 6.0 \text{ in} \\ 6d_b = 6 \times 1.0 = 6.0 \text{ in} \end{cases}$$

In lieu of using  $s = 6$  in, specify a 4-in spacing to decrease the required area of the transverse reinforcement.

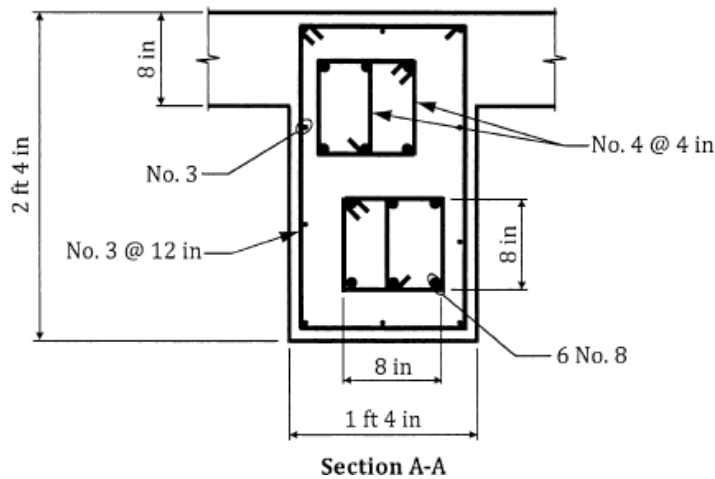
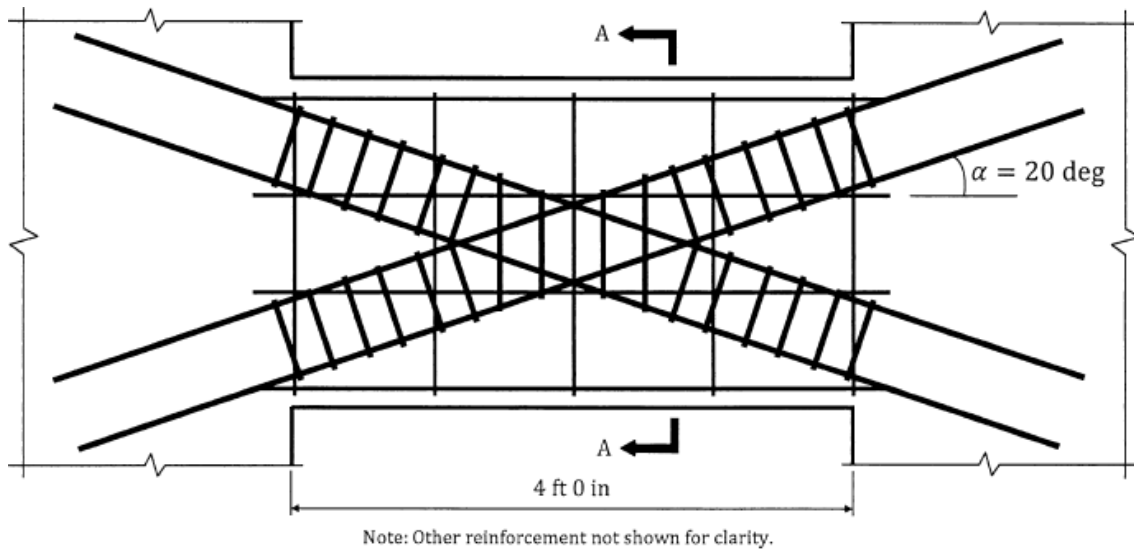
$$A_{sh} \geq \begin{cases} 0.09sb_c f'_c / f_{yt} = 0.09 \times 4 \times 8 \times 5 / 60 = 0.24 \text{ in}^2 \\ 0.3sb_c [(A_g / A_{ch}) - 1] f'_c / f_{yt} = 0.3 \times 4 \times 8 \times [(90.3 / 64) - 1] \times 5 / 60 = 0.33 \text{ in}^2. \end{cases}$$

Use No. 4 transverse reinforcement around the No. 8 diagonal bars ( $A_{sh,provided} = 0.4 \text{ in}^2$ ).

Additional longitudinal and transverse reinforcement is required around the beam perimeter with a spacing less than or equal to 12 in. Required area of steel =  $0.002b_w s = 0.002 \times 16 \times 12 = 0.38 \text{ in}^2$ . For the longitudinal reinforcement, provide No. 3 bars uniformly distributed around the perimeter of the beam. For the transverse reinforcement, use No. 3 bars spaced at 12 in on center.

Reinforcement details for this option are shown in Fig. 11.101.

Figure 11.101 Reinforcement details for the coupling beam in Example 11.19 (Option 1).

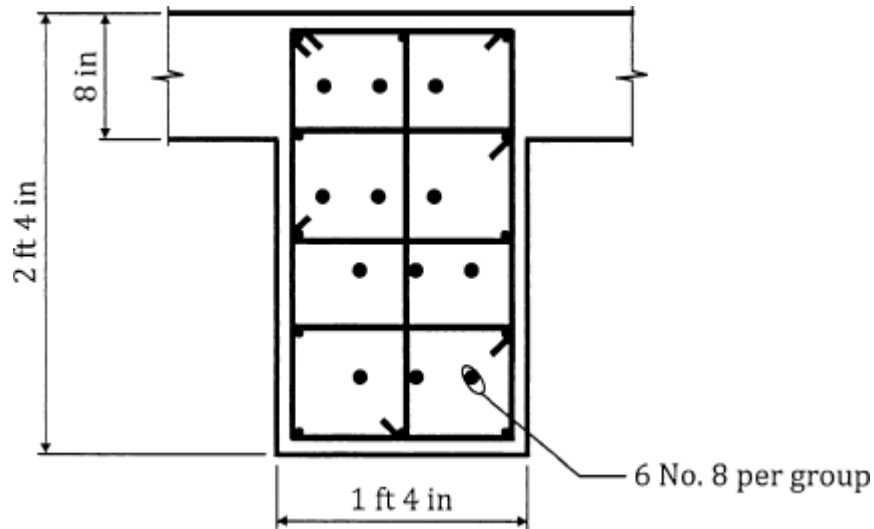


(2) Option 2—Confinement of the Entire Cross-section

The reinforcement layout shown in Fig. 11.102 satisfies the requirement in ACI 18.10.7.4(d) pertaining to the 8-in maximum spacing of crossties and legs of hoops.



Figure 11.102 Reinforcement layout for the coupling beam in Example 11.19 (Option 2).



Maximum spacing of the transverse reinforcement is 6 in or  $6d_b = 6 \times 1.0 = 6.0$  in. In lieu of using  $s = 6$  in, specify a 4-in spacing to decrease the required area of the transverse reinforcement. Assuming a 1.5-in cover to the transverse reinforcement, the required  $A_{sh}$  perpendicular to the long face of the beam is the following:

$$A_{sh} \geq \begin{cases} 0.09 \times 4 \times (28 - 3) \times 5/60 = 0.75 \text{ in}^2 \\ 0.3 \times 4 \times (28 - 3) \times [(448/325) - 1] \times 5/60 = 0.95 \text{ in}^2 \end{cases}$$

No. 4 hoops and crossies with a total of five legs provides  $1.0 \text{ in}^2 > 0.95 \text{ in}^2$ .

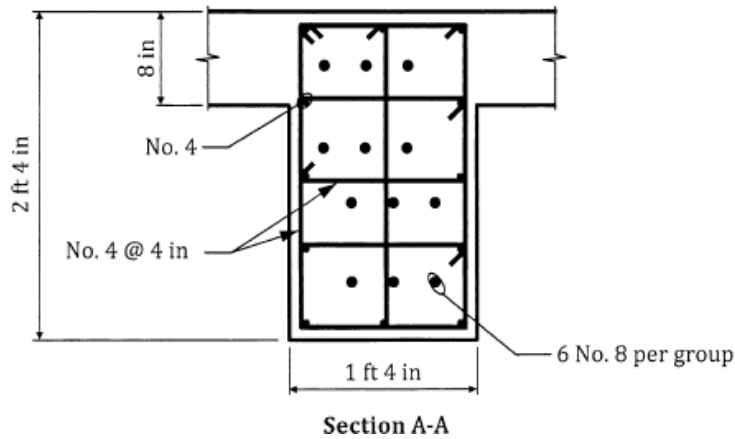
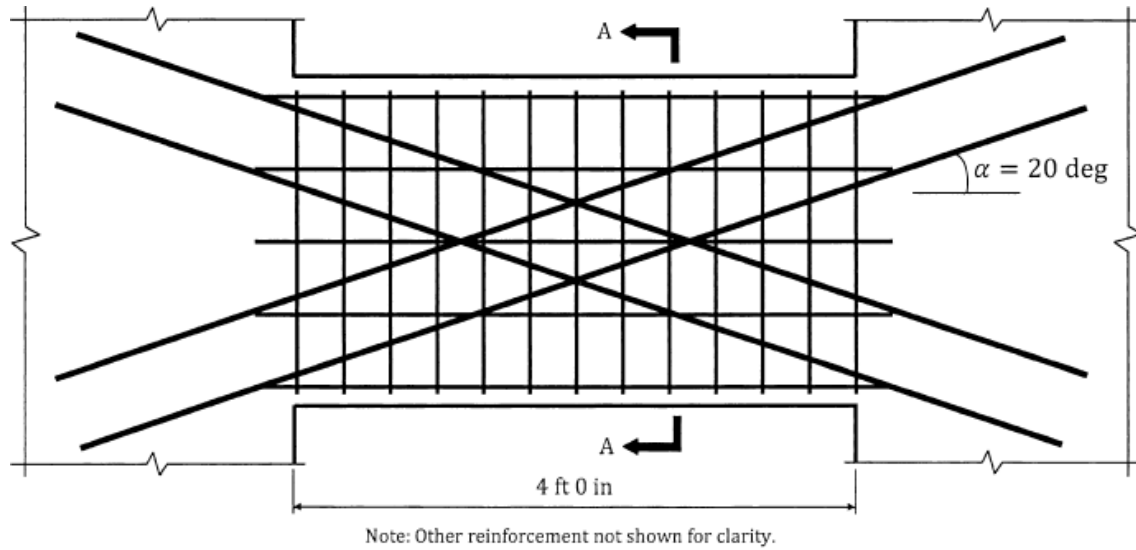
The required  $A_{sh}$  perpendicular to the short face of the beam is the following:

$$A_{sh} \geq \begin{cases} 0.09 \times 4 \times (16 - 3) \times 5/60 = 0.39 \text{ in}^2 \\ 0.3 \times 4 \times (16 - 3) \times [(448/325) - 1] \times 5/60 = 0.49 \text{ in}^2 \end{cases}$$

No. 4 hoops and a crossie with a total of three legs provides  $0.60 \text{ in}^2 > 0.49 \text{ in}^2$ .

Reinforcement details for this option are shown in Fig. 11.103.

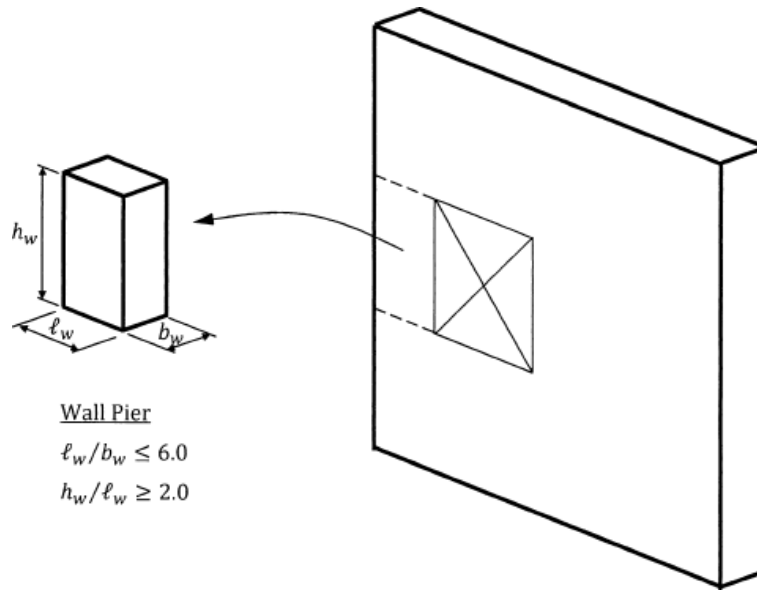
Figure 11.103 Reinforcement details for the coupling beam in Example 11.19 (Option 2).



## 11.8.7. Wall Piers

By definition, a wall pier is a vertical wall segment within a structural wall that is bounded horizontally by two openings or by an opening and an edge and that satisfies the following: (1) horizontal length to wall thickness  $l_w/b_w \leq 6.0$  and (2) clear height to horizontal length  $h_w/l_w \geq 2.0$  (see Fig. 11.104). These members are essentially columns, however the dimensions do not satisfy the requirements of columns in special moment frames.

Figure 11.104 Wall pier in accordance with ACI 18.10.8.



Wall piers are designed using the requirements for vertical wall segments as discussed in Section 11.8.3 and the following additional design and detailing requirements that are applicable for columns in special moment frames: (1) ACI 18.7.4 (longitudinal reinforcement), (2) ACI 18.7.5 (transverse reinforcement), and (3) ACI 18.7.6 (shear strength).

Alternatively, wall piers with  $l_w/b_w > 2.5$  are permitted to be designed and detailed according to the provisions of ACI 18.10.8.1(a) through (f). In regards to shear design, the factored shear force  $V_u$  is taken as the lesser of the following: (1) the shear force corresponding to the development of the probable flexural strength  $M_{pr}$  at both ends of the pier or (2) the shear force determined from analysis using code-prescribed earthquake to 366pt{forces multiplied by the overstrength factor  $\Omega_o$ . The design shear strength  $\phi V_n$  is determined in accordance with ACI 18.10.4 for structural walls and must be greater than or equal to  $V_u$ . Transverse reinforcement is required to be hoops. However, where one curtain of shear reinforcement is provided, which, according to ACI 18.10.2.2, is permitted only if  $V_u \leq 2A_{cv}\lambda\sqrt{f'_c}$  (In SI:  $0.17A_{cv}\lambda\sqrt{f'_c}$ ) single-leg shear reinforcement with 180-degree bends at each end that engages the wall pier boundary longitudinal reinforcement may be used. The vertical spacing of the transverse reinforcement is limited to 6 in (150 mm) and it must extend at least 12 in (300 mm) above and below the clear height of the wall pier. Special boundary elements at the ends of the pier must be provided if required by ACI 18.10.6.3.

For wall piers that are located at the edge of wall, as illustrated in ACI Fig. R18.10.8, horizontal reinforcement is required in the adjacent wall segments above and below the wall pier to prevent shear failure in these segments. The required length of the reinforcement into the adjacent wall is equal to the greater of the development length of the bars in tension and the shear strength of the wall segment. In the latter case, the total embedment length is equal to the factored shear force  $V_u$  in the wall pier divided by the design unit shear strength  $\phi v_n$  in the adjacent wall.

## 11.9. Diaphragms

### 11.9.1. Overview

According to IBC 202, a diaphragm is a horizontal or sloped system that transmits lateral forces to the vertical elements of the SFRS. They are usually treated as deep beams that span between these elements. The roof and floor systems act as the web of the beam, which resists the design shear forces that are uniform through the depth of the diaphragm. The boundaries of the diaphragm act as the flanges, which resist the flexural tension and compression design forces. The magnitude of these forces

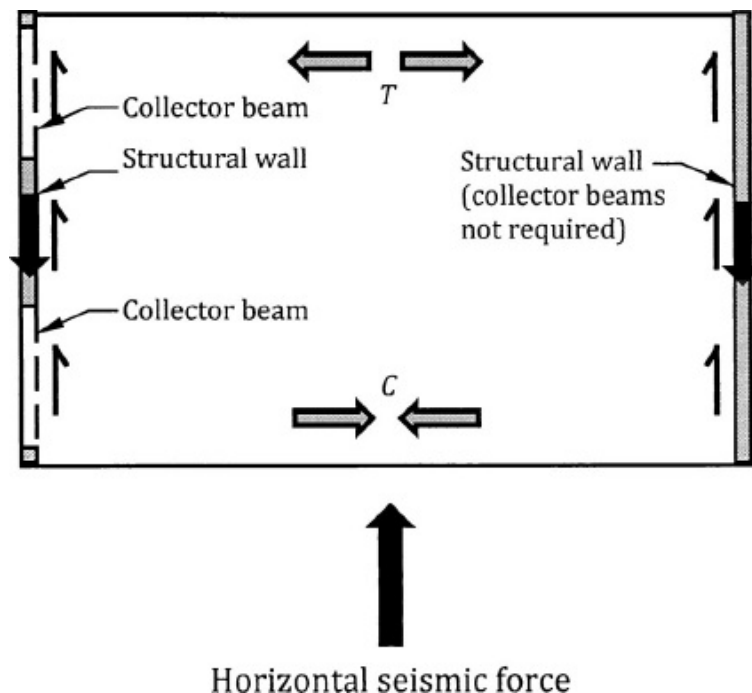
can be determined by dividing the bending moment in the diaphragm due to the lateral forces by the distance between the forces. This equivalent beam model is commonly used to design reinforced concrete diaphragms; ACI 12.5.1.3 provides other methods that can be used as well.

One of the main roles of a structural diaphragm is to transfer lateral inertial forces caused by seismic shaking to the vertical elements of the SFRS. It is common for reinforced concrete roof and floor slabs to be classified as rigid diaphragms, which means the lateral forces are transferred to the elements of the SFRS based on the stiffness of those elements (see ASCE/SEI 12.3.1 for a definition of a rigid diaphragm as well as ACI R12.4.2.3 and R12.4.2.4).

Diaphragms also transfer gravity loads that are on the diaphragm surface to floor or roof members (beams, joists, and columns), and provide lateral support to the vertical elements of the SFRS. These typical diaphragm actions and others are depicted in ACI Fig. R12.1.1.

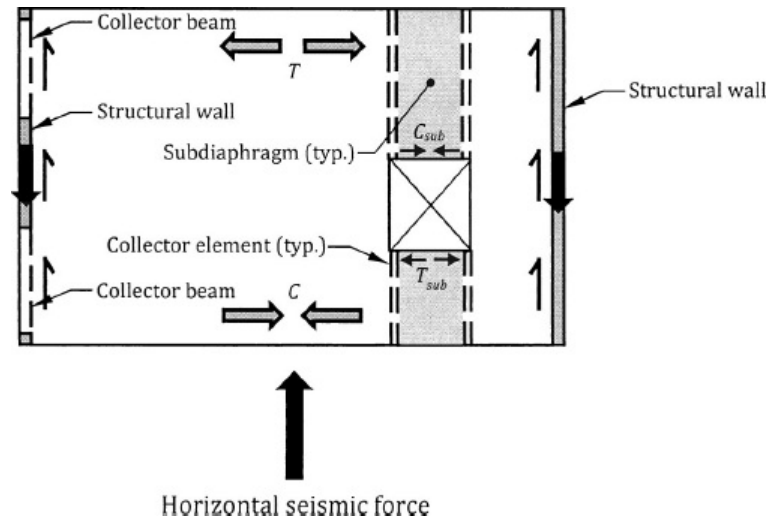
Illustrated in Fig. 11.105 is a diaphragm with a SFRS consisting of structural walls with collector beams. The horizontal seismic force is transferred through the web of the diaphragm to the walls, which act as supports for the diaphragm. Because the wall on the left does not extend the full depth of the diaphragm, collector beams are needed to collect the shear from the diaphragm and to transfer it to the wall. The diaphragm boundaries that are perpendicular to the seismic force (commonly referred to as chords) resist the tension and compression flexural forces that are induced in the diaphragm. Boundary reinforcement is concentrated along specifically defined widths along the edges of the diaphragm to resist the tensile forces (see Section 11.9.3 for more information).

Figure 11.105 Diaphragm force distribution.



Depicted in Fig. 11.106 is the same diaphragm system but now with an opening. In this case, forces develop in the subdiaphragms at the top and bottom of the opening. Collector elements on each side of the opening are required to transfer the diaphragm shear into the subdiaphragms.

Figure 11.106 Diaphragm force distribution with an opening.



ASCE/SEI 12.10.2.1 requires that collectors and their connections to the vertical elements of the SFRS in structures assigned to SDC C, D, E, and F be designed for the maximum of the three loads that are defined in that section. One of the loads that needs to be considered is obtained by multiplying the seismic load effects determined from analysis by the overstrength factor  $\Omega_o$ , which typically ranges between 2 and 3 for most concrete systems (see Table 11.6). The overstrength factor represents an upper bound lateral strength and is appropriate to use when estimating the maximum forces that can be developed in nonyielding elements of the SFRS during the design-basis earthquake. The intent of this requirement is to ensure that collectors and their connections have adequate strength and remain essentially elastic so that they can perform their vital role in the seismic load path, which is to transfer the seismic forces to the elements of the SFRS that have been properly detailed to yield during the anticipated ground motion.

ACI 18.12 contains the design and detailing requirements for diaphragms and trusses. These requirements are covered in the following sections.

## 11.9.2. Minimum Thickness

A 2-in (50 mm) minimum thickness is prescribed in ACI 18.12.6 for concrete slabs and composite topping slabs serving as diaphragms. This minimum thickness reflects the current practice in joist and waffle slab systems in cast-in-place construction. A thicker minimum slab is required for topping slabs placed over precast floor or roof systems.

Any element in a diaphragm that occurs around openings, edges, or other discontinuities must satisfy the requirements for collector elements in ACI 18.12.7.5 and 18.12.7.6, which are covered below. ACI Fig. R18.12.3.2 illustrates the case of an element adjacent to an opening in a diaphragm, which needs to conform to the requirements for collectors.

## 11.9.3. Reinforcement

The minimum reinforcement ratio for diaphragms corresponds to the minimum amount of temperature and shrinkage reinforcement prescribed in ACI Table 24.4.3.2. The maximum spacing of 18 in (450 mm) is intended to control the width of inclined cracks that may form.

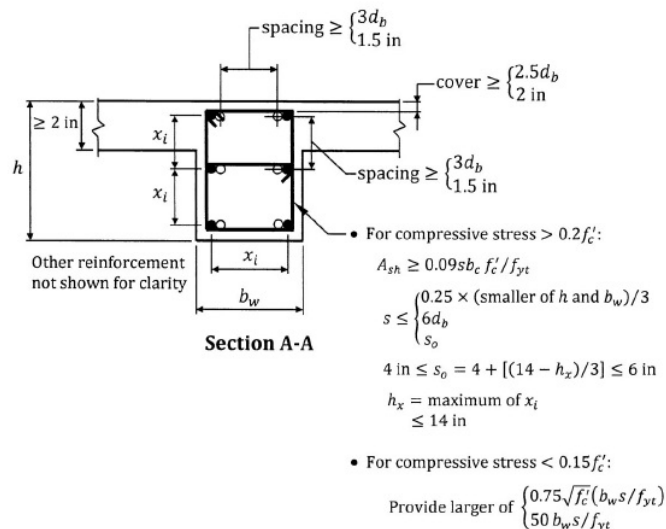
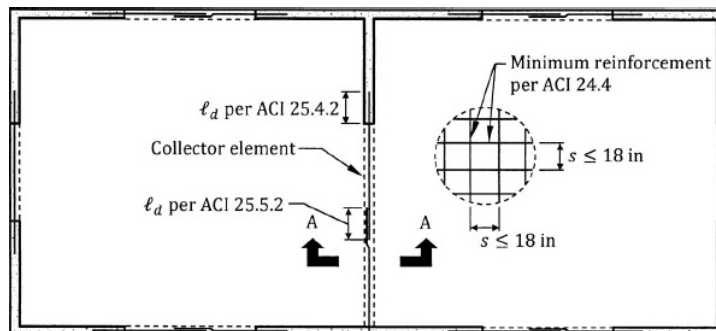
Bar development and lap splices in diaphragms and collectors are to be determined in accordance with ACI 25.4.2 and 25.5.2, respectively, for bars in tension. As indicated in ACI 25.4.10.2(e), reductions in development or splice lengths are not permitted in SFRSs in structures assigned to SDC D, E, or F.

In general, collectors must be designed for the effects due to flexure, shear, and axial compression or tension caused by the combinations of gravity and seismic load effects. At any section where the compressive stress is calculated to be greater than  $0.2f'_c$ , transverse reinforcement conforming to ACI 18.7.5.2(a) through (e) and ACI 18.7.5.3 for columns of special moment frames must be provided, except the spacing limit of ACI 18.7.5.3(a) shall be one-third of the least dimension of the collector (ACI 18.12.7.5). ACI Table 18.12.7.5 contains the required amounts of transverse reinforcement, which is intended to confine the concrete and reinforcement at those sections where substantial compressive forces under severe cyclic loading is anticipated. This confining reinforcement may be discontinued where the calculated compressive stress is less than  $0.15f'_c$ . Note that the compressive stresses in the collectors are calculated using the standard load combinations in ACI Chap. 5 and a linearly elastic model based on gross section properties. In cases where the forces have been amplified by  $\Omega_o$  (i.e., where the load combinations with overstrength factor of ASCE/SEI 12.4.3.2 have been used), the limits of  $0.2f'_c$  and  $0.15f'_c$  shall be increased to  $0.5f'_c$  and  $0.4f'_c$ , respectively.

Detailing requirements for the longitudinal reinforcement in collectors at splice and anchorage zone locations are given in ACI 18.12.7.6. The purpose of the spacing and cover requirements and the minimum transverse reinforcement requirements, the latter of which corresponds to those for beams, is to reduce the possibility of bar buckling and to provide adequate bar development in these regions. Note that either the spacing and cover requirements or the transverse reinforcement requirements need to be provided, not both.

Detailing requirements for diaphragms and collectors are given in Fig. 11.107.

Figure 11.107 Detailing requirements for diaphragms and collectors.

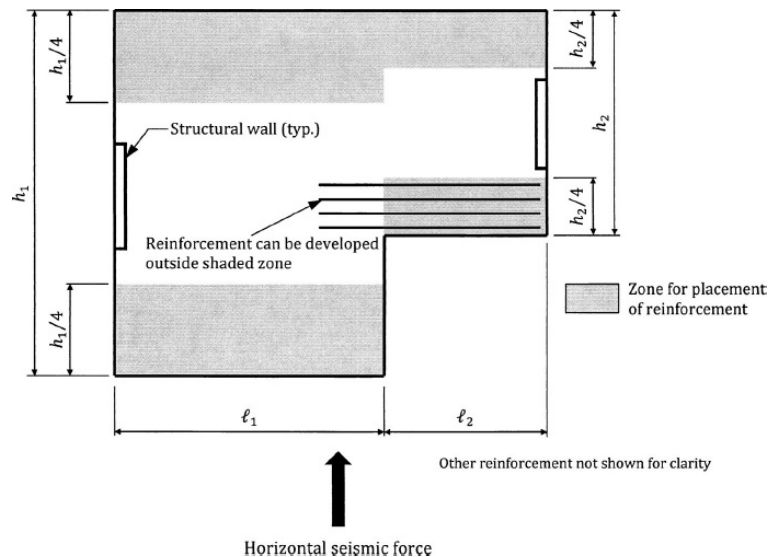


## 11.9.4. Flexural Strength

Flexural strength of diaphragms is calculated using the provisions in ACI Chap. 12. Design load combinations are determined in accordance with ACI Chap. 5, and the requirements for design strength for combined moment and axial force are given in ACI 12.5.2, which references the provisions in ACI 22.3 and 22.4 for flexural strength and combined axial force and flexural strength. As such, diaphragms are designed using the same assumptions that are used in the design of beams, columns, and walls, including the assumption that strains vary linearly through the depth of the diaphragm.

ACI 12.5.2.3 requires that nonprestressed reinforcement and mechanical connectors that resist tension due to moments caused by in-plane loading of the diaphragm be located within a distance of  $h/4$  from the edge of the diaphragm where  $h$  is the overall depth of the diaphragm in the direction of analysis. The main intent of this requirement is to control cracking at the boundaries of the diaphragm. Concentration of the flexural reinforcement at the edges also results in more uniform shear flow through the depth of the diaphragm, and it can be assumed that uniform shear stress occurs when the flexural reinforcement is concentrated within  $h/4$  from the edges. Where the depth of the diaphragm changes along the span perpendicular to the direction of analysis, it is permitted to develop the edge reinforcement into the adjacent diaphragm segment that is not within the  $h/4$  limit. Figure 11.108 illustrates the requirements of ACI 12.5.2.3.

**Figure 11.108** Location of reinforcement resisting tension due to moment and axial force according to ACI 12.5.2.3.



The Code does not require that the boundaries of a diaphragm that resist compression be detailed as columns. However, ACI R12.5.2.3 recommends using transverse reinforcement similar to column hoops where a boundary element resists a large compressive force compared to its axial strength.

ACI 18.12.3.2 requires that elements of a structural diaphragm that are subjected primarily to axial forces and that are used to transfer shear or flexural forces around openings or other discontinuities satisfy the requirements for collectors in ACI 18.12.7.5 and 18.12.7.6, which includes the transverse reinforcement requirements where the compressive stress exceeds  $0.2f'_c$ . An example of this so-called compressive strut where these provisions would be applicable is illustrated in ACI Fig. R18.12.3.2.

## 11.9.5. Shear Strength

The nominal shear strength  $V_n$  of diaphragms is given by ACI Eq. (18.12.9.1):



$$V_n = A_{cv}(2\lambda\sqrt{f'_c} + \rho_t f_y) \leq 8A_{cv}\sqrt{f'_c}$$

$$[\text{In SI: } V_n = A_{cv}(0.17\lambda\sqrt{f'_c} + \rho_t f_y) \leq 0.66A_{cv}\sqrt{f'_c}]$$

(11.18)

This equation is similar to that for slender structural walls. The term  $A_{cv}$  is the gross area of the diaphragm and is limited to the thickness times the width of the diaphragm in the direction of analysis; this corresponds to the gross area of the deep beam that forms the diaphragm.

The slab reinforcement  $\rho_t$  that contributes to the nominal shear strength is placed perpendicular to the flexural reinforcement in the diaphragm, or, equivalently, parallel to the shear force in the direction of analysis.

The strength reduction  $\phi$  that is to be used in calculating the design shear strength  $\phi V_n$  is 0.6 if the nominal shear strength is less than the shear corresponding to the flexural strength of the diaphragm (ACI 21.2.4.1). Otherwise, it is 0.75. However, according to ACI 21.2.4.2,  $\phi$  for shear in diaphragms shall not exceed the minimum  $\phi$  used for shear for the vertical components of the SFRS. Thus, where squat structural walls are used as the SFRS,  $\phi = 0.6$  for the shear design of the diaphragms in the structure.

## 11.9.6. Structural Trusses

Similar to structural diaphragms, structural truss elements must have transverse reinforcement in accordance with ACI 18.7.5.2, 18.7.5.3, 18.7.5.7, and ACI Table 18.12.11.1 at any section where the calculated compressive stress exceeds  $0.2f'_c$ . The expressions for the required transverse reinforcement helps ensure that the compression capacity of the equivalent column section is maintained after spalling of the concrete cover around the longitudinal reinforcement.

**Example 11.20** Design the roof diaphragm for the building depicted in Fig. 11.40 for seismic forces in the north-south direction. There is no opening in the roof of the building as shown in Fig. 11.40; the openings only occur on the typical floor levels. The seismic force at the roof level has been calculated to be 275 kips. Assume normal-weight concrete with  $f'_c = 4,000$  psi and Grade 60 reinforcement.

**Solution** The SFRS in the north-south direction is a building frame system where all of the seismic forces are assigned to the two structural walls (see Example 11.17 for the design of the structural walls). Collector beams are required along column lines 3 and 4 to transfer the horizontal seismic force from the diaphragm to the structural walls. The diaphragm and collectors are designed in this example.

**Design of Diaphragm** It is assumed that the roof diaphragm in this example is rigid (see ASCE/SEI 12.3.1.2); thus, the 275-kip seismic force in the diaphragm is distributed to the structural walls in proportion to their stiffness (Note: the seismic force in the diaphragm is determined in accordance with ASCE/SEI 12.10.1.1).

Because the walls are identical and the center of mass coincides with the center of rigidity, half of the diaphragm force is assigned to each wall. However, ASCE/SEI 12.8.4.2 requires that an accidental torsional moment be applied, which is caused by an assumed displacement of the center of mass each way from its actual location by a distance equal to 5% of the dimension of the structure perpendicular to the direction of the applied forces. For seismic forces in the north-south direction, the assumed displacement =  $0.05 \times 100 = 5$  ft. The center of mass is displaced 5 ft to the east, so that the wall on line 4 will have a larger force than the wall on line 3. Displacing the center of mass 5 ft to the west is also required, and the results for each wall are reversed because of symmetry.

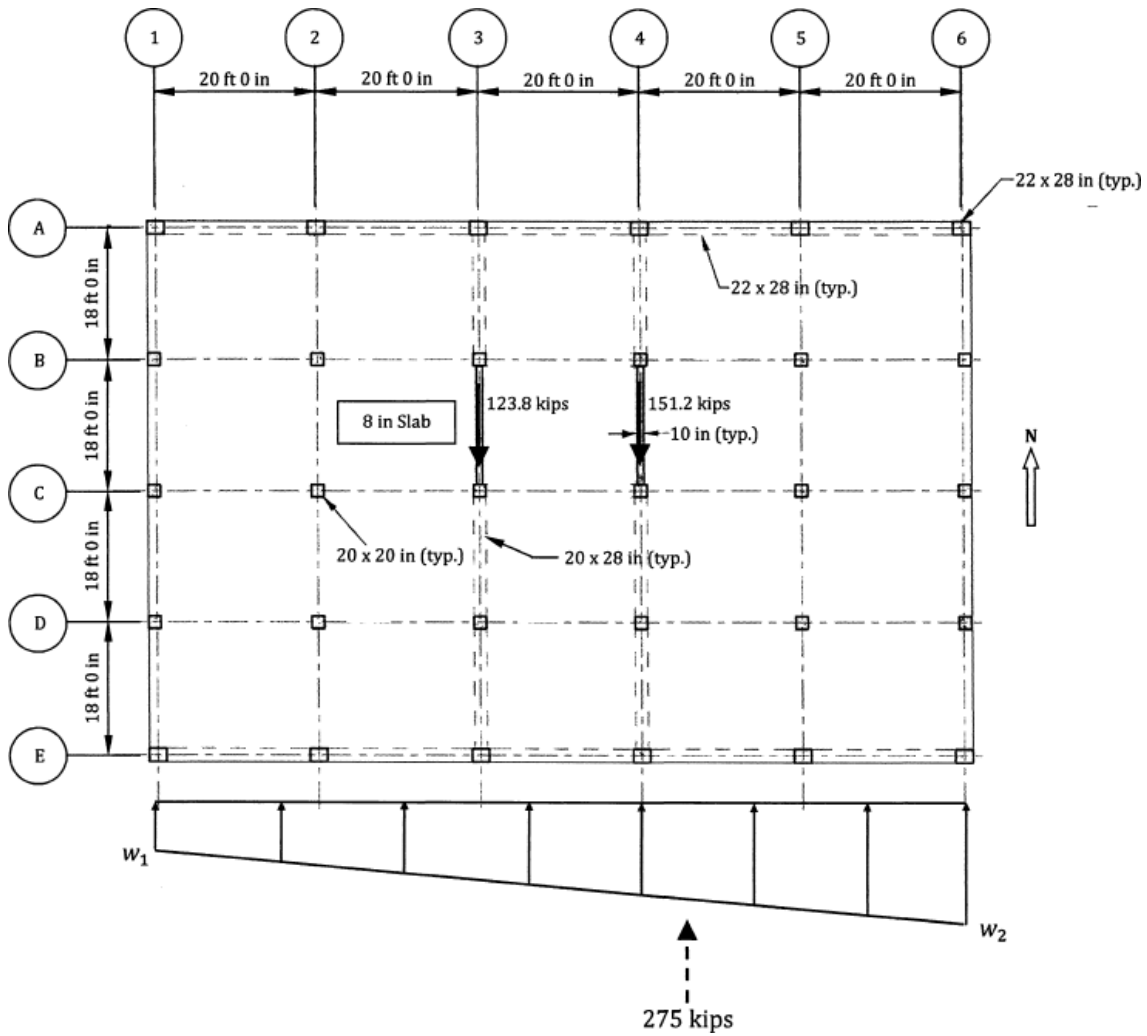
The 275-kip force is distributed to the walls considering the stiffness of the walls (shear component) and the combined stiffness of the walls and frames in the perpendicular direction (torsional component). The stiffness of the walls and frames can be obtained from a finite element analysis or from approximate methods that consider both the flexural and shear stiffness of the element. It can be determined that the force in the wall on line 4 is equal to 151.2 kips and the force in the wall along line 3 is equal to 123.8 kips (note that the sum of the forces in the walls is equal to the applied seismic force).

The concrete diaphragm is modeled as a beam with rigid supports along column lines 3 and 4. The reactions at these supports are equal to the forces in the walls.

It is assumed that the diaphragm force can be represented by a trapezoidal distributed load as illustrated in Fig. 11.109; this load distribution automatically takes into consideration the torsional component due to the accidental torsional moment. Because the reactions at the supports are known, the distributed loads  $w_1$  and  $w_2$  can be determined from statics. Once the distributed loads have been established, the maximum bending moment and shear force in the diaphragm can be determined.



Figure 11.109 Distributed load on roof diaphragm in Example 11.20.



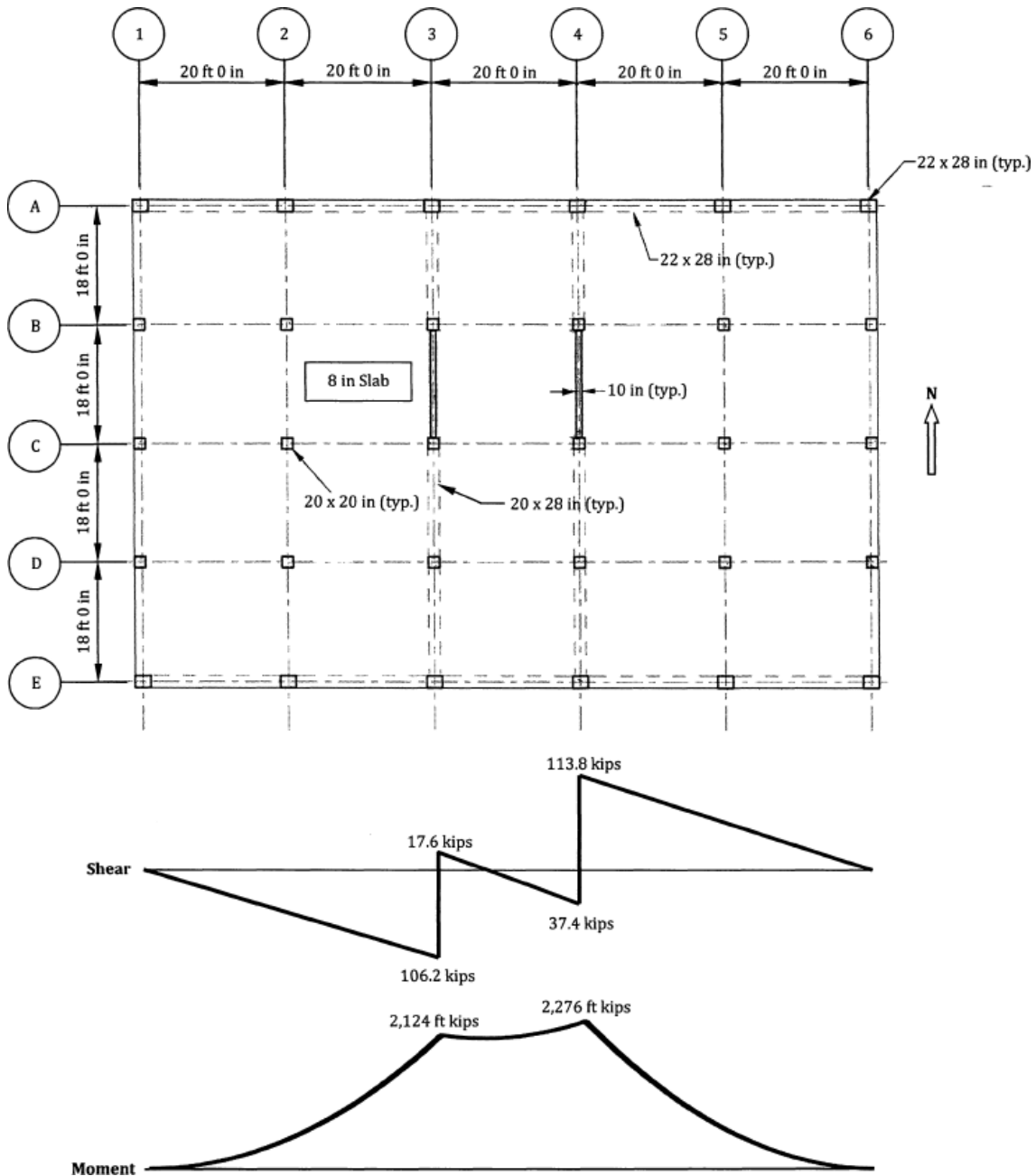
Summing forces in the north-south direction and summing moments about column line 3 results in the following two equations:

$$(w_1 \times 100) + \left[ \frac{1}{2}(w_2 - w_1) \times 100 \right] = 275$$

$$(w_1 \times 100 \times 10) + \left\{ \frac{1}{2}(w_2 - w_1) \times 100 \times \left[ \left( \frac{2}{3} \times 100 \right) - 40 \right] \right\} = 151.2 \times 20$$

Solving these equations simultaneously results in  $w_1 = 2.59$  kips/ft and  $w_2 = 2.91$  kips/ft. The shear and moment diagrams are shown in Fig. 11.110 where it can be seen that the maximum bending moment in the diaphragm is equal to 2,276 ft kips.

Figure 11.110 Shear and moment diagrams for the roof diaphragm in Example 11.20.



The maximum tension and compression forces within the  $h/4 = 72/4 = 18$  ft zones at the boundaries of the diaphragm that contain the reinforcement resisting tension due to the bending moment are equal to the following:

$$T = C = \frac{M_u}{h - \frac{h}{4}} = \frac{2,276}{72 - 18} = 42.2 \text{ kips}$$

where it is assumed that the distance between the resultant forces occurs at the midpoint of the zone at each end.

The required area of tension reinforcement is determined as follows:

$$A_s = \frac{T_u}{\phi f_y} = \frac{42.2}{0.9 \times 60} = 0.78 \text{ in}^2$$

Provide two No. 6 bars located near the edge of the diaphragm at each end. This reinforcement should be placed within the middle third of the slab and beam thickness in order to minimize interference with the reinforcement in those members.

The maximum shear force in the diaphragm occurs along column line 4 and is equal to 113.8 kips, which is uniformly distributed over the 72-ft

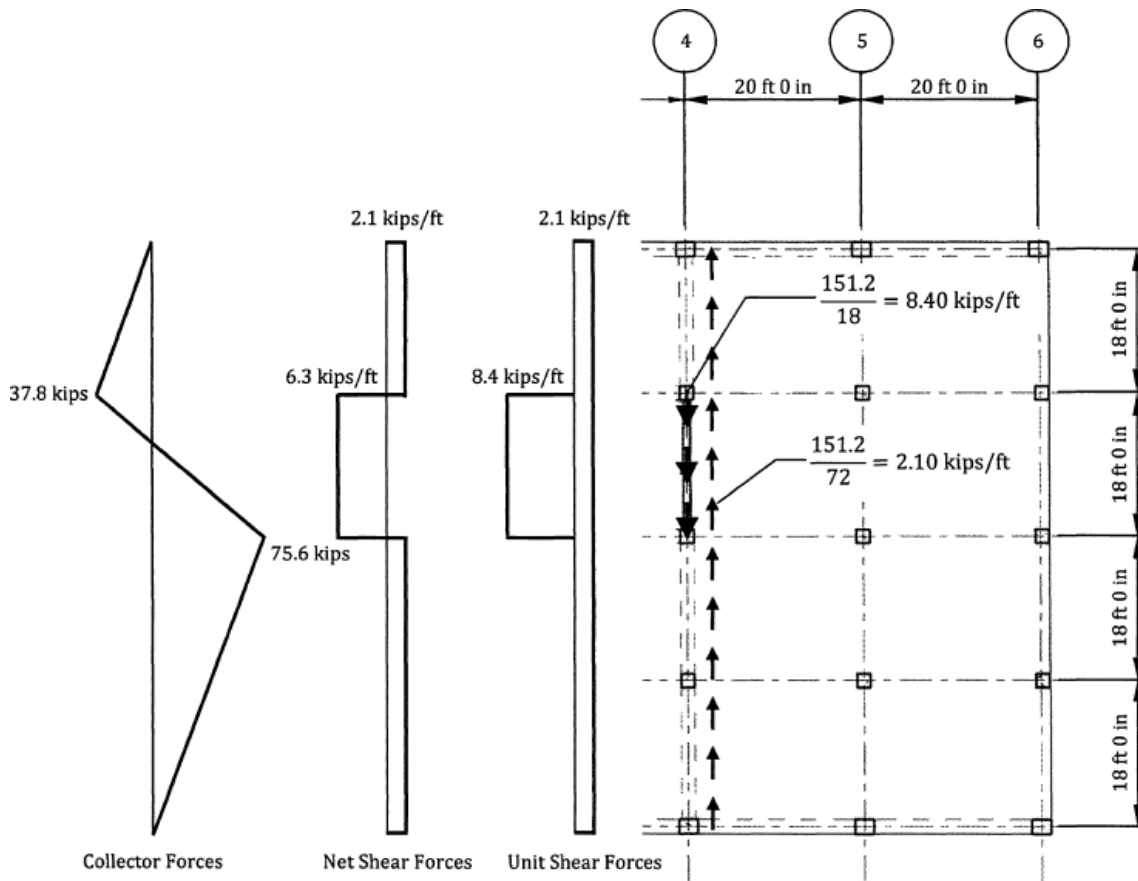
long depth of the diaphragm. The shear strength of the slab is determined by Eq. (11.18) conservatively assuming that the shear reinforcement in the slab  $\rho_t$  is equal to zero:

$$\begin{aligned}\phi V_n &= \phi A_{cv} 2\lambda \sqrt{f'_c} \\ &= 0.6 \times 8 \times (72 \times 12) \times 2 \times 1.0 \sqrt{4,000}/1,000 = 524.6 \text{ kips} > 113.8 \text{ kips} \\ &< \phi 8 A_{cv} \sqrt{f'_c} = 2,098.3 \text{ kips}\end{aligned}$$

The strength reduction factor was taken as 0.6 because this is the same value that was used in the design of the structural wall for shear (see ACI 21.2.4.2 and Example 11.17).

**Design of Collectors** The beams along lines 3 and 4 are utilized as collectors that transfer the shear forces in the diaphragm into the walls along those lines. It was determined above that the total diaphragm force along line 4 is equal to 151.2 kips. The total unit shear force in the diaphragm is equal to  $151.2/72 = 2.10$  kips/ft. Similarly, the unit shear force along the length of the wall is equal to  $151.2/18 = 8.40$  kips/ft. The unit shear forces and net shear forces are depicted in Fig. 11.111. The collector forces, which are also shown in this figure, are determined by summing the areas in the net shear force diagram. For example, at the north end of the collector between lines A and B, the force is equal to zero. At a distance of 18 ft from the end, the force is equal to  $2.1 \times 18 = 37.8$  kips. The maximum axial force in the collector is equal to  $(6.3 \times 18) - 37.8 = 75.6$  kips or, equivalently,  $2.1 \times 36 = 75.6$  kips.

Figure 11.111 Unit shear forces, net shear forces, and collector force diagram for the roof diaphragm in Example 11.20.



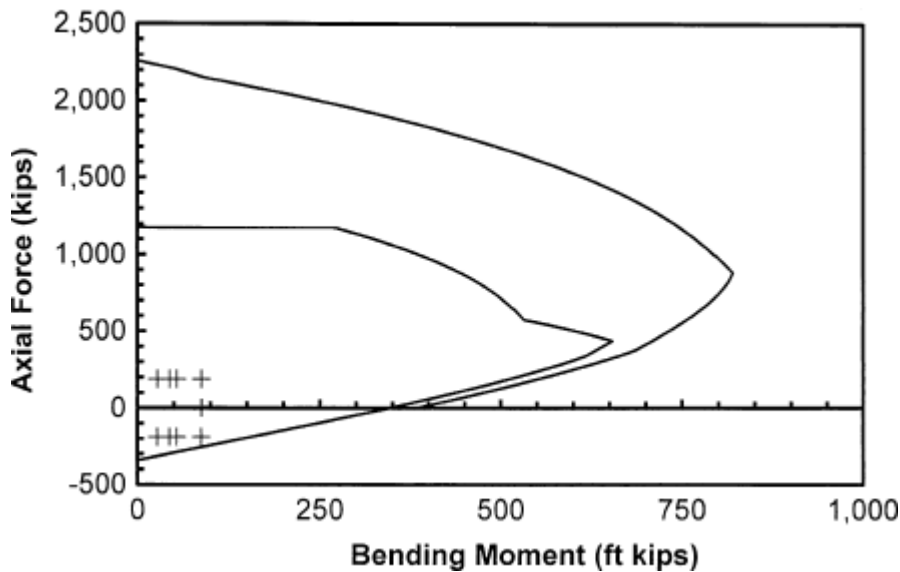
The collectors must be designed to resist the combined effects from gravity forces (bending moments and shear forces) and earthquake forces (axial compression and tension). Table 11.23 contains a summary of the axial forces, bending moments, and shear forces for the collector along line 4. The 75.6-kip axial force in the collector can be tension or compression. Using ASCE/SEI 12.10.2.1, it has been determined that the maximum axial force in the collector is obtained by multiplying the axial force determined from analysis ( $Q_E = 75.6$  kips) by the overstrength factor  $\Omega_o$ , which is equal to 2.5 for a building frame system with special reinforced concrete structural walls (see ASCE/SEI Table 12.2-1). Therefore, according to ASCE/SEI 12.4.3,  $E$  in ACI Eq. (5.3.1e) shall be taken equal to  $E_m = \Omega_o Q_E + 0.2S_{DS} D$ , and  $E$  in ACI Eq. (5.3.1g) shall be taken equal to  $E_m = \Omega_o Q_E - 0.2S_{DS} D$ . From Example 11.8,  $S_{DS}$  was given as 1.0. A summary of the applicable load combinations is also given in Table 11.23.

**Table 11.23** Summary of Design Axial Forces, Bending Moments, and Shear Forces for the Collectors in [Example 11.20](#)

Load Case		Axial Force (kips)	Bending Moment (ft kips)		Shear Force (kips)
			Negative	Positive	
Dead ( $D$ )		0	62.5	38.6	17.5
Roof Live ( $L_r$ )		0	8.0	5.1	2.2
Seismic ( $2.5Q_E$ )		$\pm 189.0$	0	0	0
<b>Load Combination</b>					
1.4D	ACI Eq. (5.3.1a)	0	87.5	54.0	24.5
1.2D + 1.6L <sub>r</sub>	ACI Eq. (5.3.1c)	0	87.8	54.5	24.5
1.4D + 2.5Q <sub>E</sub>	ACI Eq. (5.3.1e)	$\pm 189.0$	87.5	54.0	24.5
0.7D + 2.5Q <sub>E</sub>	ACI Eq. (5.3.1g)	$\pm 189.0$	43.8	27.0	12.3

Based on the load combinations in [Table 11.23](#), a 20 × 28 in collector reinforced with three No. 8 top bars, three No. 8 bottom bars, and two No. 8 side bars is adequate ( $A_{st} = 0.011A_g$ ). The interaction diagram for this collector is given in [Fig. 11.112](#).

**Figure 11.112** Design and nominal strength interaction diagrams for the collectors in [Example 11.20](#).



Check if transverse reinforcement satisfying ACI 18.7.5.2(a) through (e) and ACI 18.7.5.3 must be provided in the collector (ACI 18.12.7.5):

$$\text{Compressive stress} = \frac{2.5Q_E}{A_g} = \frac{189}{20 \times 28} = 0.34 \text{ ksi} < 0.5f'_c = 2.0 \text{ ksi}$$

Therefore, transverse reinforcement satisfying the aforementioned sections need not be provided. The limit of  $0.5f'_c$  was used because the axial force in the collector was amplified by  $\Omega_o$ .

The maximum shear force  $V_u$  in the collector is 24.5 kips and because it is subjected to significant axial tension, it has been decided to set the design shear strength of the concrete  $\phi V_c$  equal to zero. Thus, assuming No. 3 ties, the required spacing is the following:

$$s = \frac{A_v f_{yt} d}{\frac{V_u}{\phi} - V_c} = \frac{(2 \times 0.11) \times 60 \times 25.5}{\frac{24.5}{0.6} - 0} = 8.2 \text{ in} < \frac{d}{2} = 12.8 \text{ in}$$

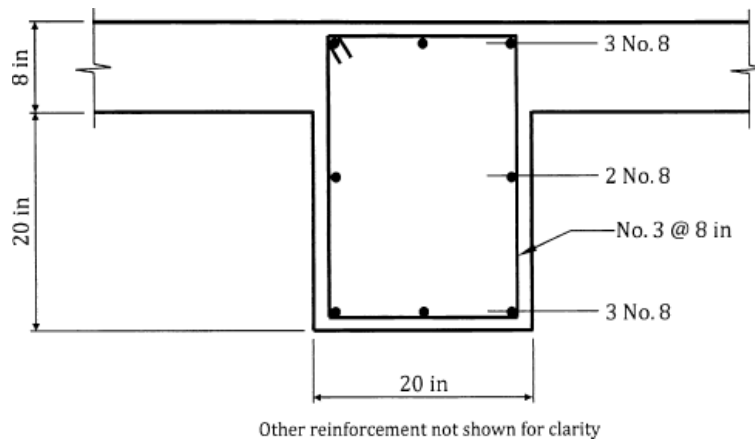
$$\leq \frac{A_v f_{yt}}{0.75 \sqrt{f'_c} b_w} = \frac{(2 \times 0.11) \times 60,000}{0.75 \sqrt{4,000} \times 20} = 13.9 \text{ in}$$

$$\leq \frac{A_v f_{yt}}{50 b_w} = \frac{(2 \times 0.11) \times 60,000}{50 \times 20} = 13.2 \text{ in}$$

Provide No. 3 ties spaced at 8.0 in on center over the entire length of the collectors.

Reinforcement details for the collectors are shown in Fig. 11.113. Top, bottom, and side longitudinal bars are continuous over the entire length and are spliced and anchored in accordance with ACI 18.12.7.6.

Figure 11.113 Reinforcement details for the collectors in Example 11.20.



## 11.10. Foundations

### 11.10.1. Overview

Requirements for foundations supporting buildings assigned to SDC D, E, or F are contained in ACI 18.13. Foundations must also comply with all of the other applicable provisions of the Code. It is desirable that foundations do not undergo any significant inelastic response when subjected to strong ground motion because repair of foundations is difficult and expensive.

Chapter 18 of the IBC also contains provisions for the design of foundations. It is important to check with the local jurisdiction to make sure the correct requirements are being implemented in the design and detailing of these elements.

### 11.10.2. Footings, Foundation Mats, and Pile Caps

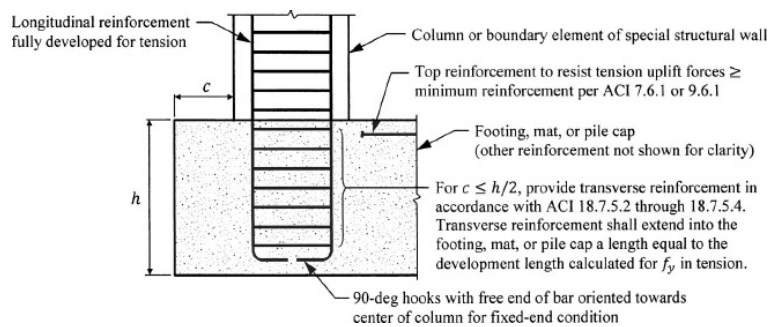
Longitudinal reinforcement of columns and structural walls that are part of the SFRS must be fully developed for tension into the footings, mats, and pile caps that support them. Where columns are assumed to be fixed at the top of the foundation, tests have demonstrated that longitudinal reinforcement that is developed into the foundation from a supported flexural member (column or wall) should have 90-degree hooks turned inwards toward the axis of the member in order for the joint to be able to resist flexure at this location.

ACI 18.13.2.3 contains requirements for columns or boundary elements of special structural walls that are located near the edge of the foundation. These requirements have been discussed in Section 11.8.5 and are illustrated in Fig. 11.86.

Flexural reinforcement must be provided in the top of the foundation where uplift forces are generated in columns and boundary elements of special structural walls. The reinforcement must be designed for the applicable load combinations in ACI Chap. 5 and must be greater than or equal to the applicable minimum reinforcement for one-way slabs in ACI 7.6.1 or for beams in ACI 9.6.1.

A summary of these provisions is illustrated in Fig. 11.114.

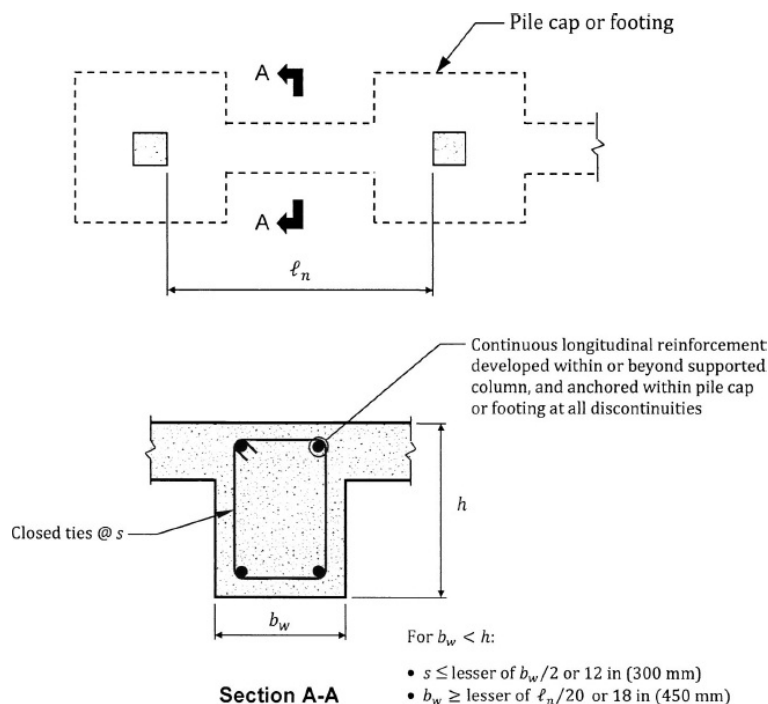
Figure 11.114 Requirements for footings, foundation mats, and pile caps.



### 11.10.3. Grade Beams and Slabs-on-ground

The cross-section limitations and the minimum closed tie requirements of ACI 18.13.3 for grade beams between pile caps or footings are intended to provide reasonable beam proportions. Grade beams that are part of a mat foundation that resists flexural stresses from columns that are part of the SFRS must conform to the provisions of ACI 18.6 for beams of special moment frames. Requirements for grade beams are shown in Fig. 11.115.

Figure 11.115 Requirements for grade beams.



A slab-on-ground that is not subjected to earthquake effects is generally considered nonstructural, and the Code does not govern its design and construction (ACI 1.4.7). However, for structures assigned to SDC D, E, or F, a slab-on-ground is often part of the SFRS, acting as a diaphragm that holds the structure together at the ground level and minimizes the effects of out-of-phase ground motion that may occur over the footprint of the structure. In such cases, it must be designed and detailed in accordance with the provisions of ACI 18.12 for diaphragms.

## 11.10.4. Piles, Piers, and Caissons

The provisions of ACI 18.13.4 apply to concrete piles, piers, and caissons that support structures subjected to earthquake effects. Adequate performance of these supporting members for seismic loads requires that these provisions be met, as well as other applicable standards and guidelines, which are contained in ACI R1.4.6.

For piles, piers, or caissons resisting tension loads, a load path is necessary to transfer the tension forces from the longitudinal reinforcing bars in a column or boundary member of a special structural wall through the pile cap to the reinforcement of the pile, pier, or caisson.

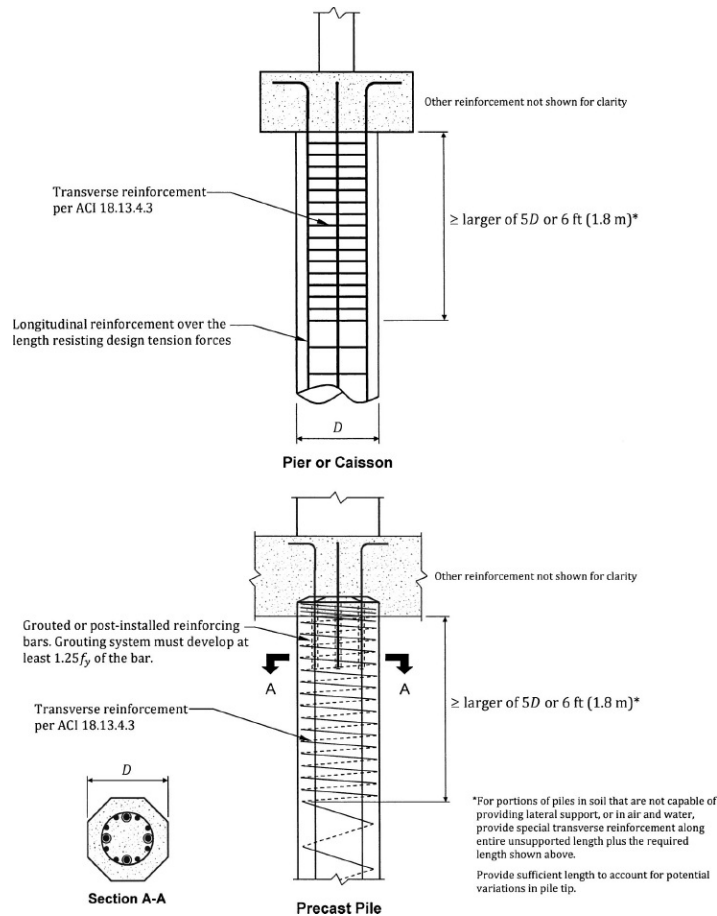
Piles can be subjected to extremely high flexural stresses just below the pile cap and near the base of soft or loose soil deposits during an earthquake. The requirement of ACI 18.13.4.3 for transverse reinforcement at the specified locations is based on numerous failures that were observed after recent earthquakes.

It is important to consider that the tip of a precast pile may be driven to an elevation different than that specified in the design drawings. For example, a pile may reach refusal at a shallower depth than anticipated. When this occurs, the excess pile length needs to be cut off, which results in a portion of transverse reinforcement required by ACI 18.13.4.3 being cut off as well. If this possible situation is not anticipated, transverse reinforcement may not be provided over the length that is required.

Extensive structural damage has been observed at the junction of batter piles and buildings after some recent earthquakes. Pile caps and surrounding structure must be designed for the potentially large forces that can be developed in batter piles.

Requirements for piles, piers, and caissons are given in [Fig. 11.116](#).

Figure 11.116 Requirements for piles, piers, and caissons.



## 11.11. Members Not Designated as Part of the SFRS

### 11.11.1. Overview

ACI 18.14 is applicable to the design and detailing of those members in a structure that have not been assigned to the SFRS. For example, these provisions would be applicable to the beams and columns in a building frame system where it is assumed that the structural walls provide the total resistance to the earthquake effects. Similarly, in a moment frame system, the beams and columns in the frames that have not been assigned to the SFRS would be designed and detailed in accordance with these requirements.

Regardless of the type of SFRS that is utilized and the members that have been designated not to be a part of it, all of the members in a structure are subjected to the effects from an earthquake because they are all tied together by the diaphragms at each level. The entire structure undergoes displacements and the members in the structure that are not part of the SFRS are subjected to these displacements. In short, the purpose of these requirements is to ensure that the members that are not part of the SFRS can support their respective gravity loads when subjected to the design displacement  $\delta_u$ , which is the total calculated lateral displacement expected for the design-basis earthquake. In ASCE/SEI 12.8.6,  $\delta_u$  is determined by multiplying the displacement due to the code-prescribed forces by the deflection amplification factor  $C_d$ , which is given in ASCE/SEI Table 12.2-1 for the various systems, and then dividing by the seismic importance factor  $I_e$  given in ASCE/SEI 11.5.1.

The provisions of ACI 18.14 are applicable to columns, beams, slabs, and wall piers. The purpose of these requirements is to enable ductile flexural yielding of these members by providing sufficient confinement and shear strength. Columns and



beams are assumed to yield if the combined effects from factored gravity loads and design displacement exceed the strengths specified, or if the effects from the design displacements are not calculated.

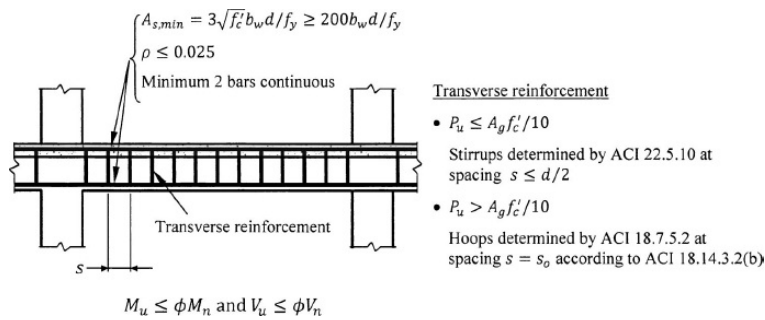
Members that are not part of the SFRS are to be evaluated for the gravity load combinations of  $(1.2D + 1.0L + 0.2S)$  or  $0.9D$ , whichever is critical, acting simultaneously with the design displacement  $\delta_u$ . The load factor on  $L$  can be reduced to 0.5 except for garages, areas occupied as places of public assembly, and all areas where  $L$  exceeds 100 psf ( $4.8 \text{ kN/m}^2$ ).

The detailing requirements for beams and columns depend on the magnitude of the bending moments and shear forces that are induced in those members when they are subjected to  $\delta_u$ . Specific detailing requirements are given where the induced moments and shears do not exceed the corresponding design moment and shear strength, and other requirements are applicable where they do. If it has been decided that the induced moments and shears caused by the design displacement  $\delta_u$  are not going to be calculated, then the set of design requirements where the induced moments and shears exceed the corresponding design moment and shear strength must be satisfied.

## 11.11.2. Beams

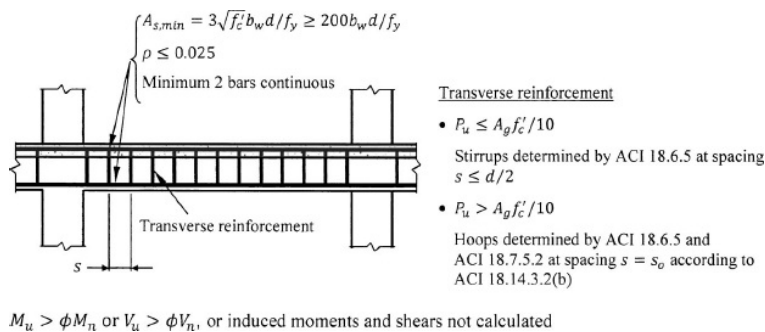
Where the induced moments and shears do not exceed the design moment and shear strength of the beam, the detailing requirements of ACI 18.14.3.2(a) must be satisfied. These requirements are illustrated in Fig. 11.117.

Figure 11.117 Requirements of ACI 18.14.3.2(a) for beams.



Where the induced moments and shears exceed the design moment and shear strength of the beam, or where the induced moments or shears are not calculated, the detailing requirements of ACI 18.14.3.3 must be satisfied, which are given in Fig. 11.118. Note that the materials, mechanical splices, and welded splices must comply with the requirements in ACI 18.2.5 through 18.2.8 for special moment frames in such cases.

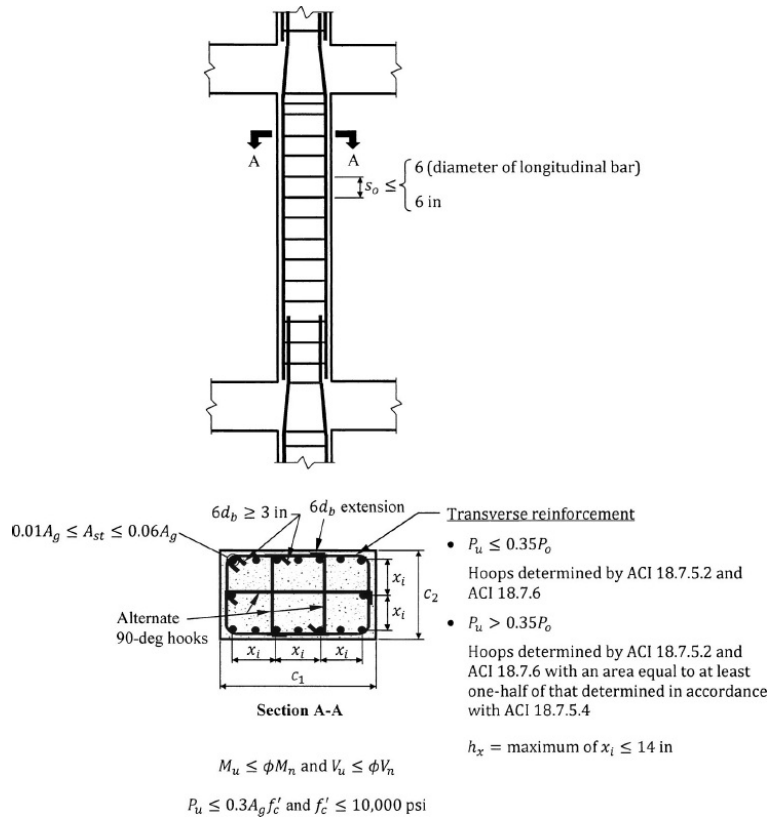
Figure 11.118 Requirements of ACI 18.14.3.3(b) for beams.



## 11.11.3. Columns

Requirements for columns where the induced moments and shears do not exceed the design moment and shear strength of the column are given in Fig. 11.119 where  $P_u \leq 0.3A_g f'_c$  and  $f'_c \leq 10,000$  psi (70 MPa). In cases where induced moments and shears exceed the design moment and shear strength of the column or where induced moments and shears are not calculated, the detailing requirements of ACI 18.14.3.3(c) are essentially the same as those for columns in special moment frames (see Figs. 11.51 through 11.53).

Figure 11.119 Requirements of ACI 18.14.3.2(b) and (c) for columns.



## 11.11.4. Joints

Requirements for joints where the induced moments and shears exceed the respective design strengths or where the induced moments and shears due to  $\delta_u$  are not calculated are given in ACI 18.14.3.3(d). This section refers to the provisions of ACI 18.8.3.1, which are applicable to joints in special moment frames.

## 11.11.5. Slab–Column Connections

Provisions for slab–column connections in two-way slabs without beams are given in ACI 18.14.5. These requirements are intended to reduce the likelihood of punching shear failure in cases where the design story drift exceeds the specified value.

Slab shear reinforcement conforming to ACI 8.7.6 (stirrups) or 8.7.7 (headed studs) and providing a nominal shear strength  $v_s$  greater than or equal to  $3.5\sqrt{f'_c}$  must be provided at all slab–column connections of two-way slabs without beams where  $\Delta_x/h_{sx} \geq 0.035 - (1/20)(v_{ug}/\phi v_c)$ . In these equations,  $\Delta_x$  is the relative lateral deflection between the top and bottom of a story (i.e., the story drift),  $h_{sx}$  is the story height under consideration,  $v_{ug}$  is the factored shear stress at the slab critical section due to the gravity loads without moment transfer, and  $\phi v_c$  is the design two-way shear strength provided by the concrete calculated in accordance with ACI 22.6.5. The shear reinforcement must extend a minimum of four times the slab thickness

from the face of the support adjacent to the critical section of the slab. Note that the shear reinforcement provisions of this section are not applicable where  $\Delta_x/h_{sx} \leq 0.005$ .

The value of  $\Delta_x/h_{sx}$  to use in the aforementioned inequalities is the greater of the values of the adjacent stories above and below the slab–column connection under consideration.

ACI Fig. R18.14.5.1 illustrates the criterion of ACI 18.14.5.1. If adding shear reinforcement as noted above is not practical or desirable, certain parameters can be modified so that such reinforcement is not required.

Note that the above requirements must be evaluated at any critical section adjacent to the slab–column connection where there are changes in slab thickness, such as around drop panels or shear caps, in accordance with ACI 22.6.5.1.

## 11.11.6. Wall Piers

The requirements of ACI 18.10.8 must be satisfied for any wall pier that is not part of the SFRS (ACI 18.14.6.1). Because ACI 18.10.8 requires that the design shear force be determined in accordance with ACI 18.7.6.1 for columns in special moment frames, which in some cases may result in large shear forces that are unrealistic, the design shear force may be obtained by multiplying the shear force induced in the pier when it is subjected to the design displacement  $\delta_u$  by the overstrength factor  $\Omega_o$ .

## 11.12. References

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